Third ODE Integrator

Consider ODE

$$\frac{d}{dt}\vec{z} = \vec{f}(\vec{z}, t).$$

We are interested in how function $g(\vec{z}, t)$ changes along solution curve.

$$\frac{d}{dt}g(\vec{z}(t), t) = \vec{f} \cdot \vec{\nabla} g + \frac{\partial}{\partial t} g = L_{\vec{f}} g$$

$$\frac{d^n}{dt^n} g = L^n_{\vec{f}} g.$$

One can get an nth order time step algorithm by forming

$$g_{new} = g_{old} + \sum_{i=1}^{n} \frac{L^i_{\vec{f}}}{i!} g \cdot \Delta t^i.$$

$\vec{f} \cdot \vec{\nabla}$ is computed OK in DA if $\vec{f}(0) = \vec{0}$. Successively choosing $[g] = [z_i]$, one can propagate $[z_i]$ by Δt. For autonomous systems, the integrator works to any order, and even allows dynamic adjustment of order.