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Design of a fifth-order achromat

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A repetitive system free of all aberrations up to the fifth order was designed based on a recently developed analytical theory that, in principle, allows the design of such achromats to an arbitrary order ([1]). It serves as an example to show that complete correction of aberrations is possible beyound order three, which is the highest order achieved before ([2] [3]). Instead of repetition of identical cells, which is widely used in achromat design based on normal form theory, we utilize cells which are obtained from the the original ones through mirror imaging about the x-y plane, which corresponds to a reversion. In our design, the second half of the ring is the reversion of the first one, and two turns make a fifth-order achromat. A possible application of repetitive high-order achromats being time-of-flight spectroscopy, the resulting ring was analyzed with respect to dynamic aperture and energy resolution using maps of orders nine and higher.

1 Introduction

In the past few years, various third-order achromatic systems containing as few as seven identical cells have been designed using normal form theory ([2] [3] [4]). The number of bending magnets needed ranges from 7 to 300. Each solution requires a specific number of cells depending on the choice of the tunes of a cell.

By introducing mirror symmetry into the consideration, we developed a new theory which requires only four cells and as few as one bend per cell to obtain in principle achromats of arbitrary order ([1] [8]). The use of mirror symmetry enables us to choose from four kinds of cells, namely the forward cell (F), the cell in which the order of elements is reversed (R), the cell in which the direction of bend is switched (S), and the cell where reversion and switching is combined (C). According to the theory, the minimum number of conditions required for a four-cell fifth-order achromat with an arbitrary forward cell are five for the first order, four for the second order, fifteen for the third and the fourth order and thirty nine for the fifth order. The optimal four-cell systems which require only the minimum number of conditions are listed in Table 1, together with the first-order requirements.

Systems	Linear Conditions
F R S C	$(a\|\delta)=0$, $(x\|a)=(a\|x)=0$
F R F R	$(a\|\delta)=0$, $(x\|x)=(a\|a)=0$
F C S R	$(x\|\delta)=0$, $(x\|a)=(a\|x)=0$
F C F C	$(x\|\delta)=0$, $(x\|x)=(a\|a)=0$

Table 1: The optimal four-cell systems. Additional linear conditions are $(y|y)=(b|b)=0$ or $(y|b)=(b|y)=0$ for each case.

As it turns out and is shown in table 1, achromaticity always implies that the tune for each cell can only be either an integer, a half integer or a quarter integer. This fact certainly limits the possible applications of these kind of systems, yet there are several applications where the concept of high-order achromats is useful. On the one hand, there are single-pass beamlines and achromatic bending arcs of storage rings. On the other hand, there are the time-of-flight mass spectrometers, including both single-pass and multi-pass systems. In fact, these are the areas where second-order achromats have been widely used in the past ([5] [6] [7]).

For single-pass systems, the resonances do not affect the beam at all. For storage rings with achromatic bending arcs, the resonances are avoided by adjusting the tunes of the straight sections to suitable values. Finally, for multi-pass mass spectrometers, the particles are usually stored for less than about one hundred turns, and therefore resonances

tend to have only limited influence.

To verify the analytical theory and to begin exploring the possibility of using a high-order achromat as a multi-pass time-of-flight energy spectrometer, we present a conceptual design of a ring which is achromatic to fifth order after two turns. Since the main emphasis of this paper is of theoretical nature, we restrict ourselves to the actual correction of aberrations and forego a detailed error analysis of the influence of present-day instrumental constraints.

The details of the design is presented in Section 2 which is followed by the analysis of the system (Section 3). In Section 3, the repetitive stability and the time-of-flight energy resolution as a function of the number of turns are studied through tracking. Conclusions are given in Section 4.

2 Design of the Achromat

2.1 First- and Second-Order Design

In order to design an achromatic system for a circular layout, no switched (S) or switched-reverse (C) sections can be used, the only choice is FRFR. The first-order layout should avoid large changes in the beta function in order to minimize nonlinear aberrations; furthermore, there should be room for the insertion of correction multipoles and a reasonable average distance between multipoles of the same order. Another consideration is that, if possible, the number of first-order conditions should be further reduced through symmetry arrangements inside a cell.

The result of these thoughts is the ring shown in Figure 1. It consists of sixteen FODO cells plus two dispersion correction sections each including two quadrupoles. The left half is the forward cell (F) and the right half is the reversed cell (R). To reduce the size and the cost, the ring is designed so that achromaticity is obtained after two turns, which corresponds to the choice of a tune of 1/4 per cell. The forward cell itself also consists of two parts, where one is the reversion of the other. This guarantees that, at the end of the system, $(x|x) = (a|a)$ and $(y|y) = (b|b)$. Here $a = p_x/p_0$ and $b = p_y/p_0$ are normalized canonical momenta, the use of which is necessary within the symplectic theory of arbitrary-order achromats ([1]). All four FODO cells within one part of a cell are identical except that the last one has an extra quadrupole for dispersion correction. So there are three knobs for the first-order design which can zero $(x|x)$, $(a|a)$, $(y|y)$, $(b|b)$, $(x|\delta)$ and $(a|\delta)$ at the same time. Figure 1 shows beta functions and dispersion of the beam around the ring; the apparent uniformity of these functions suggests that the layout is benign, and also facilitates the subsequent higher order correction.

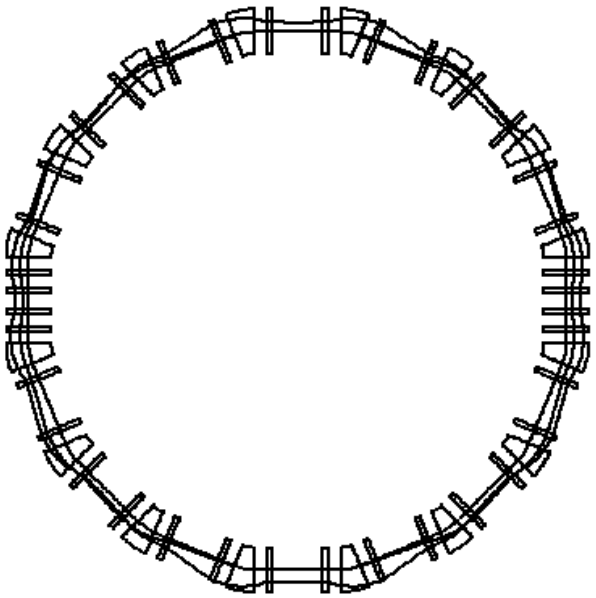

Figure 1: Layout, beam envelope and dispersive ray of the ring. Circumference: 266.64 m; Acceptance: $30\pi\ mm \cdot mr$; Energy acceptance: 0.3%.

According to the analytical theory ([1]), four independent sextupoles are required to obtain a second-order achromat. However, because the cell R is identical to the cell F to the first order, a simplification is possible based on Brown's theory of second-order achromat design ([9] [10]). In this theory it is shown that a second-order achromat can be achieved by placing two pairs of sextupoles in dispersive regions and separating the two in each pair by a negative identity in both transverse planes. In our case, the first-order one-turn map is a negative identity, and thus the same sextupole can be used in two consecutive turns to satisfy the requirements above.

So in principle a second-order achromat can be achieved on the ring using two sextupoles per cell (half ring). In our case it turned out to be necessary to split the sextupoles into symmetrically excited

pairs to ensure that up to the second order the second half still is the reversion of the first. The strengths of the quadrupoles and sextupoles are shown in Table 2.

Strengths of the Multipoles (Aperture 10cm)	
Quadrupoles (Length 75cm)	
Gradient (kG/cm)	Field (kG)
-0.162869	-0.814344
0.134119	0.670597
-0.131803	-0.659013
Sextupoles (Length 24cm)	
Gradient (kG/cm^2)	Field (kG)
-0.718659E-03	-0.179665E-01
0.364420E-03	0.911050E-02

Table 2: The field strengths of the quadrupoles and sextupoles.

2.2 Higher Order Design

After the investment in a careful first-order layout, the third-, fourth- and fifth-order corrections actually turn out to be rather straightforward conceptually and a direct application of the methodology outlined in ([1]), although the computational demands are of course substantially higher than for the low order design. For the whole process of nonlinear optimization, two aspects seem worth to note. First, experience showed that the required multipole strengths strongly depend on the average distance between multipoles of the same order. So in order to keep their strengths limited, it is important to dimension the total size of the ring and the dispersive region sufficiently large, which was done in the first-order design stage, and distribute multipoles of the same order roughly uniformly.

Secondly, all the decapoles have to be placed in regions with sufficient dispersion because all the fourth-order aberrations remaining after third-order achromaticity is achieved are of chromatic type. Thus it is advantageous to use a substantial dispersive region.

The combination of these considerations results in rather limited multipole strengths for third-, fourth- and fifth-order corrections. Assuming that the aperture and length are 10cm and 24cm, respectively, the pole-tip field strengths of the octupoles, decapoles and duodecapoles are in the range of up to ten Gauss. The details of the layout and setting of the multipoles can be obtained from reference [12].

3 Analysis of Stability and Resolution

To study the repetitive stability of the ring, which is of importance for the use as a time-of-flight spectrograph, high order one-turn maps were generated using COSY INFINITY ([11]). By comparing tracking results of various orders, it was concluded that the 11th-order map is sufficient, and the resulting map was then used for symplectic tracking. Because the tune shifts of the horizontal and vertical motions vanish due to achromaticity, the system has unavoidable low-order resonances. However, because achromaticity entails that all low-order resonance driving terms vanish, still substantial regions of phase space are stable for several hundred turns.

To be specific, we calculated the short-term dynamic aperture for 12000 turns for various momenta with a total spread of $\pm 0.5\%$. As the measure of dynamic aperture of a given momentum, we consider the smallest of the maximum distances from the origin that a particle survives along 16 different directions in the x-y plane.

Figure 2 shows a comparison of the dynamic aperture for various energies of the third order achromat and the fifth order achromat. The fifth order achromat has a stability region that peaks near momentum spread zero, while that of the third order achromat is substantially smaller for on-momentum particles and rather skewed to the left. This effect is apparently due to remaining fourth and fifth order chromatic nonlinearities in the third order design. In the fifth order design, these nonlinearities are compensated, resulting in a more linear behavior with increased acceptance around the origin.

It is rather illuminating to study a tracking picture of the fifth order achromat. Figure 3 shows the horizontal motion of on-energy particles up to 1000 turns. The absence of linear effects as well as any nonlinearities up to order five leads to very unusual behavior of particles staying nearly in place, and only very gradually drifting away due to nonlinearities of orders six and higher.

The time-of-flight energy resolution of the fifth order achromat is determined in a statistical manner. It turns out that while detailed tracking requires the use of 11th-order maps, for the purpose of resolution computation, no significant changes were observed for orders beyond nine, and thus the 9th-order one-turn map is used for this computationally intensive task.

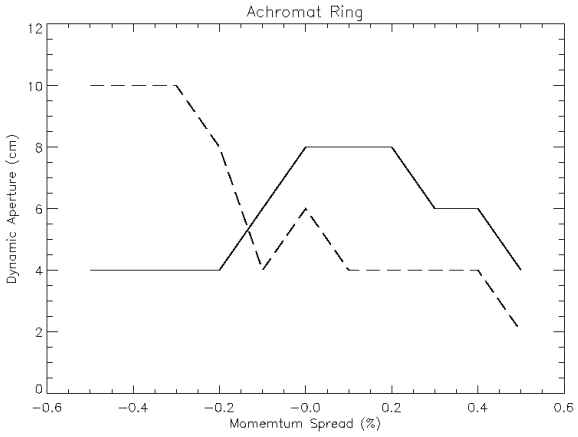

Figure 2: The dynamic aperture footprint of the third- and fifth order achromats.

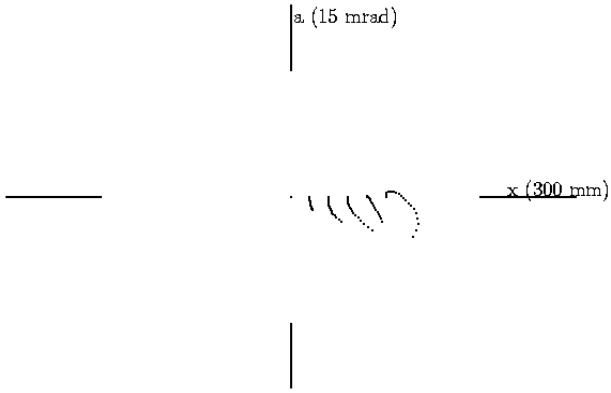

Figure 3: 1000-turn tracking of the $x - a$ motion of on-energy particles in the fifth order achromat.

For the actual resolution calculation, 10,000 particles were randomly placed inside the phase space area of interest. Then these particles were sent through the one turn map n times, and thus the n-turn time-of-flight of each particle was computed. To this value, the random error of the detector, which was assumed to be 100 ps, was added, resulting in a simulation for the measured time-of-flight. This value was used to estimate the energy of the particle by inverting the time-energy dependence, and the result was compared with the known actual value. The resolution of the ring was determined by calculating the inverse of the average differences. The dependence of the resolution on the number of turns is presented in Fig. 4.

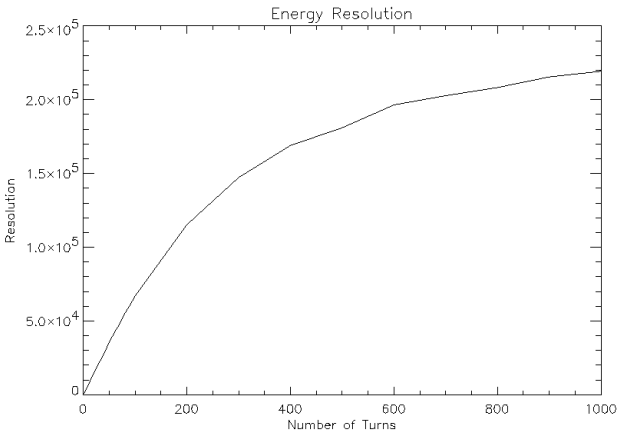

Figure 4: Resolution versus the number of turns. Gaussian distribution is assumed for the initial conditions and cutoff is set at 2.5 σ. $\sigma_{x,y} = 10mm$, $\sigma_{a,b} = 1mrad$ and $\sigma_\delta = 0.1\%$.

4 Conclusion

It has been shown that, using the method outlined in [1], from a computational point of view it is possible to correct all aberrations up to order five. Careful first-order considerations allow the use of relatively weak correction elements, and thus also small nonlinearities due to uncorrected higher-order terms. The remaining nonlinearities were estimated through repetitive tracking and found to be small enough for particles to survive several hundred turns.

As a proof of principle, the fifth-order achromat was applied to serve as a time-of-flight spectrograph, yielding resolutions of about 200,000 over significant phase space volumes.

Acknowledgment

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