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ADVANCES IN NONLINEAR NON-SCALING FFAGs

C. JOHNSTONE
Fermilab, Batavia, IL 60510, USA
cjj@fnal.gov

M. BERZ and K. MAKINO
Michigan State University, East Lansing, MI 48824, USA

S. KOSCIELNIAK
TRIUMF, Vancouver, BC 60439, Canada

P. SNOPOK
Illinois Institute of Technology, Chicago, IL 60616, USA

Accelerators are playing increasingly important roles in basic science, technology, and medicine. Ultra high-intensity and high-energy (GeV) proton drivers are a critical technology for accelerator-driven sub-critical reactors (ADS) and many HEP programs (Muon Collider) but remain particularly challenging, encountering duty cycle and space-charge limits in the synchrotron and machine size concerns in the weaker-focusing cyclotrons; a 10–20 MW proton driver is not presently considered technically achievable with conventional re-circulating accelerators. One, as-yet, unexplored re-circulating accelerator, the Fixed-field Alternating Gradient or FFAG, is an attractive alternative to the other approaches to a high-power beam source. Its strong focusing optics can mitigate space charge effects and achieve higher bunch charges than are possible in a cyclotron, and a recent innovation in design has coupled stable tunes with isochronous orbits, making the FFAG capable of fixed-frequency, CW acceleration, as in the classical cyclotron but beyond their energy reach, well into the relativistic regime. This new concept has been advanced in non-scaling nonlinear FFAGs using powerful new methodologies developed for FFAG accelerator design and simulation. The machine described here has the high average current advantage and duty cycle of the cyclotron (without using broadband RF frequencies) in combination with the strong focusing, smaller losses, and energy variability that are more typical of the synchrotron. The current industrial and medical standard is a cyclotron, but a competing CW FFAG could promote a shift in this baseline. This paper reports on these new advances in FFAG accelerator technology and presents advanced modeling tools for fixed-field accelerators unique to the code COSY INFINITY.[1]

1. Introduction

Accelerators are playing increasingly important roles in basic science, technology, and medicine including accelerator-driven subcritical reactors, industrial irradiation, material science, neutrino production, and provide one of the most effective treatments for many types of cancer. Multi-MW proton driver capability remains a challenging, critical technology for many core HEP programs, particularly the neutrino ones such as the Muon Collider and Neutrino Factory, and for high-profile energy applications such as Accelerator Driven Subcritical Reactors (ADS) and Accelerator Transmutation of Waste (ATW) for nuclear power and waste management.[2–4] Aggressive, coordinated and funded programs are underway in Europe[5] (Belgium,[6] the U.K.[7]), Asia (India, Japan,[8] South Korea,[9] recently China[10]), also Russia, the Ukraine, Belarus, and Brazil on proton driver technologies.

Such high-intensity, multi-GeV accelerators however encounter duty cycle and space-charge limits in synchrotrons and machine size concerns in the weaker-focusing cyclotrons. A 10–20 MW proton driver is challenging at best, if even technically feasible, with conventional accelerators. Only an SRF linac, which has the highest associated cost and footprint, is presently considered realizable. Work is focused almost exclusively on an SRF linac, as, to date, no re-circulating accelerator can attain the high intensities necessary for the nuclear applications. However, the ultra-high reliability issues required in the nuclear applications complicate the accelerator and dramatically impact the cost and drive the technology to the limits in the linac solution. The only unexplored, potential candidate for ultra-high intensity, high energy applications is the Fixed-Field Alternating Gradient accelerator (FFAG).[2]

This increasing demand for higher and higher beam power, duty cycle, and reliability at reasonable cost has thus focused world attention on fixed-field accelerators, notably a broad class of accelerators termed Fixed-field Alternating Gradient (FFAGs). (Cyclotrons can be considered a specific expression or sub-class of FFAGs which employ a predominately constant rather than gradient magnetic field.) The fixed magnetic fields, strong focusing (coupled to recent advances in tune stability), a large dynamic aperture, compact footprint, and, importantly, the capacity for isochronous or CW operation have attracted international attention.[11] These new breeds of FFAGs have been tagged by energy collaborations for serious study.[2,12]

Advanced FFAG designs, however, are not mature and their ultimate limits in performance are just beginning to be explored. Recently, the concept of isochronous orbits has been proposed and developed for the most general type of FFAG (termed non-scaling) using powerful new methodologies in fixed-field accelerator design], with the concept recently achieved in non-scaling nonlinear FFAG designs.[13,14] The property of isochronous orbits enables the simplicity of fixed RF and by inference, CW operation. By tailoring a nonlinear radial field profile, the FFAG can remain isochronous, well into the relativistic regime. Therefore one application is high-intensity, and, in particular, high-energy (GeV) proton drivers. (Continuous beam delivery and ultra-high reliability are required to avoid thermal shock to the reactor in the nuclear application, mandating CW operation capability.[2]) With isochronous orbits, the FFAG has the high average current and duty cycle advantages of a cyclotron in combination with the strong focusing, smaller losses, and energy variability that are more typical of a synchrotron.

FFAG accelerator technology has been recently transferred to industry, with electron scaling FFAGs rapidly becoming commercially available. NHV and Mitsuibishi Corporations in Japan are building compact high current electron FFAGs which have a proven size advantage over other industrial electron accelerators.[15] Although the cyclotron is the current industrial and medical standard, a competing CW FFAG would have a broad impact on medical accelerators, proton drivers for neutron production, accelerator-driven nuclear reactors, waste transmutation, the production of radiopharmaceuticals, and open up a range of as-yet unexplored industrial applications. A high-energy, high-intensity CW re-circulating proton accelerator would have specific impact on HEP facilities and nuclear power. This paper reports on new advances in FFAG accelerator technology, design, and simulation, and also presents advanced tools developed for all fixed-field accelerators recently added the high-order code COSY INFINITY.[1]

2. Background

The FFAG concept in acceleration was invented in the 1950s independently in Japan,[16] Russia[17] and the U.S[18] (T. Ohkawa[16] in Japan, H.S. Snyder[18] at Brookhaven, and A.A. Kolomenskij[17] in the Soviet Union). The field is weak at the inner radius and strong at the outer radius, thus accommodating all orbits from injection to final energy. Focusing is provided by an alternating gradient. An extensive discussion of the various FFAG configurations, including derivations of the formulas relating the various accelerator and orbit parameters can be found in the references.[19] The configuration initially proposed was called a radial sector FFAG accelerator.[20] A spiral sector configuration[21] was also invented consisting of magnets twisted in a spiral as the radius increases, such that a beam crossing the magnet edges experiences alternating gradients. With no reverse-bending magnets, the orbit circumference of the spiral-sector scaling FFAG is about twice that for a circular orbit in a uniform field. These machines are the so-called scaling FFAGs (either spiral or radial-sector FFAGs) and are characterized by geometrically similar orbits of increasing radius. Direct application of high-order magnetic fields and edge focusing maintains a constant tune and optical functions during the acceleration cycle and avoids low-order resonances. The magnetic field follows the law B $\propto r^k$, with r as the radius and k as the constant field index.

The non-scaling FFAG was invented in 1997 (C. Johnstone and F. Mills) and a working lattice published in 1999[22] as a solution for the rapid acceleration of muon beams. The non-scaling FFAG proposed for muon acceleration utilizes simple, combined function magnets like a synchrotron. However, it does not maintain a constant tune and is not suitable for an accelerator with a modest RF system and requiring many turns for acceleration.

Recently, innovative solutions were discovered (C. Johnstone, Particle Accelerator Corp.) for non-scaling FFAGs which approximated the constant tune feature of the scaling FFAG without applying the scaling principle. This new non-scaling

FFAG accelerator applied weak and alternating gradient focusing principles (both edge and field-gradient focusing) in a specific configuration to a fixed-field combined-function magnet to stabilize tunes.[23–25] Note that, stable tunes, however, do not imply isochronous orbits or CW operation.

Isochronous orbits are achievable only at relativistic energies in a synchrotron and predominately non-relativistic energies in a cyclotron. In a synchrotron, the magnetic field increases proportional to energy and therefore particles are confined about a laboratory-based reference trajectory independent of energy. Since the path-length is fixed independent of energy, a frequency change is required except at highly-relativistic energies, so swept-frequency RF is unavoidable even at GeV energies. In a fixed-field machine, such as a FFAG or cyclotron, the reference orbit moves outward transversely with energy. The orbital path length and the orbital frequency change with energy (at energies approximately $\geq \frac{1}{4}$ GeV in a cyclotron). The challenges faced in a fixed-field accelerator, a FFAG or a high-energy cyclotron, are therefore twofold: the RF frequency must change or "sweep" to match the beam revolution time and the large orbit swing implies a cavity design with a large transverse aperture.

Recently, the concept of isochronous orbits coupled with constant machine tunes has been researched and successfully developed for the non-scaling FFAG using powerful new methodologies in fixed-field accelerator design.[26] By tailoring a nonlinear radial field profile, the FFAG can remain isochronous with stable tune, well into the relativistic, multi-GeV regime for protons. Specifically, the problem of isochronous, or CW operation, has been solved for a non-scaling FFAG design in an energy regime of a GeV and higher. This property of isochronous orbits enables the simplicity of fixed-frequency RF and by inference, CW operation, as in the cyclotron, but with strong focusing. (More quasi-isochronous orbits permit more rapid, > 100 Hz, swept-frequency RF, operation, a rate not achievable in synchrotrons and synchrocyclotrons.) Designing and demonstrating performance for the FFAGs with their complex field profiles and edge contours required new advances in accelerator modeling.[13,14] The strong focusing attribute, particularly in the vertical plane of the FFAG as compared to cyclotrons, further implies stronger vertical envelope control and some degree of mitigation of space-charge effects when compared to the cyclotron. These new innovations will be discussed in detail in later sections after broader concepts describing properties of fixed-field accelerators are presented.

3. Dynamics of Fixed-Field Accelerators

Tune is perhaps the most important optical indicator of stable particle motion, since it determines when particles in the beam, executing periodic motion around the accelerator, return to the same transverse position relative to a central, or reference orbit in the machine. In a fixed-field machine such as an FFAG or cyclotron, this reference orbit moves with energy so the tune is controlled through radial and azimuthal variations in the magnetic field as described below.

Three conventional techniques exist for controlling the beam envelope and corresponding tune, or phase advance, in a magnetic field. The first confinement technique is the weak focusing principle used in classical cyclotrons in which changes in path-length through the magnetic field as a function of transverse position focus the beam, but only in the bend plane (which is typically horizontal). Weak focusing by the dipole component of the field in the body of the magnet does not affect the vertical plane.

The second arises from the field falloff at the physical edge of a magnet. A vertically-oriented (horizontally-bending) dipole field presents either a horizontally focusing or defocusing effect or no effect depending on the on the angle through which the beam traverses the fringe field. This edge effect is essentially equivalent to a quadrupole-like element located at each magnet edge: it can be either focusing horizontally and defocusing vertically, or the reverse for a non-normal crossing angle. (A normal entrance angle has no focusing effect.) In a cyclotron, vertical control is established via edge focusing through deliberate radial shaping of the pole-tip combined with a non-normal edge-crossing angle. The use of an edge-crossing angle in a cyclotron for vertical envelope control is normally weaker than focusing from path-length differences in the horizontal plane.

The remaining technique used in synchrotrons involves application of strong-focusing, alternating gradients in consecutive ring magnets. Strong-focusing techniques are capable of focusing equally in both planes with much stronger strengths resulting in larger phase advances, shorter focal lengths, and corresponding higher machine tunes than achievable in weak-focusing machines, i.e. stronger and more versatile envelope control. Contrary to cyclotrons, edge focusing effects are kept deliberately small in large multi-cell synchrotron rings. This term becomes increasingly important for and often causes difficulties in the dynamics of small synchrotron rings.

All three principles are applied in FFAGs—scaling machines specifically require edges plus gradient fields in relatively constant strengths to achieve similar orbits and corresponding constant tunes. In the non-scaling FFAG, the different focusing principles are combined in different and generally varying composition through the acceleration cycle—the varying composition can be exploited to control the machine tune without applying the field scaling law.

In particular, and unlike a cyclotron, the strength of the edge focusing and centripetal terms can be enhanced in the presence of a gradient - importantly their strength can increase with radius and therefore with energy. Understanding the powerful interplay between gradient and the centripetal and edge focusing is critical to understanding the dynamics and potential of the FFAG accelerator.

3.1. *Thin lens formalism*

The application of the transverse focusing terms and their inter-dependence can be understood conceptually using the thin-lens approximation. This approximation provides direct insight into the transverse dynamics of both FFAGs and traditional accelerators

The dynamics of most accelerators can be expressed and understood almost completely in terms of the three "conventional" transverse focusing principles outlined above. To understand the interplay between strong, weak and edge focusing, a simple linear, thin-lens matrix model serves as a guiding example. The approach is most easily rendered using a simple sector magnet matrix, adding a gradient term to the focusing, and then applying an edge angle to the entrance and exit. The following is the first order matrix for a horizontally-focusing sector magnet with a gradient and an edge angle, η.

$$M = \begin{bmatrix} 1 & 0 \\ -\frac{\tan\eta}{\rho_0} & 1 \end{bmatrix} \begin{bmatrix} \cos\Theta & \frac{1}{\sqrt{K}}\sin\Theta \\ -\sqrt{K}\sin\Theta & \cos\Theta \end{bmatrix}, \tag{1}$$

where $\Theta = \sqrt{K}l$ and $K = k_0 + \frac{1}{\rho_0^2}$ for a combined function sector magnet with a linear gradient. For the edge angle we adopt the sign convention to be: $\eta > 0$ is outward, or away from the body of the magnet and thus it increases the net horizontal focusing. Reducing to thin lens, the matrices from the center of the gradient magnet through the edge are:

$$\begin{aligned} &\begin{bmatrix} 1 & 0 \\ -\frac{\tan\eta}{\rho_0} & 1 \end{bmatrix} \begin{bmatrix} 1 & l \\ -Kl & 1 \end{bmatrix} \approx \begin{bmatrix} 1 & 0 \\ -\frac{\eta}{\rho_0} & 1 \end{bmatrix} \begin{bmatrix} 1 & l \\ -Kl & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & l \\ -\left(k_F l + \frac{l}{\rho_F^2} + \frac{\eta}{\rho_F}\right) & -\frac{\eta l}{\rho_F} + 1 \end{bmatrix} \\ &\cong \begin{bmatrix} 1 & l \\ -\left(k_F l + \frac{l}{\rho_F^2} + \frac{\eta}{\rho_F}\right) & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & l \\ -\left(k_F l + \frac{(\vartheta+\eta)}{\rho_F}\right) & 1 \end{bmatrix} = \begin{bmatrix} 1 & l \\ -\frac{1}{f_F} & 1 \end{bmatrix}, \end{aligned} \tag{2}$$

where $k_0 \to K_F$ and $\rho_0 \to \rho_F$ for a horizontally focusing gradient and $\frac{l}{\rho_F^2} \cong \frac{\vartheta}{\rho_F}$, where ϑ is the sector bend angle and the length l is the half-magnet length. The edge angle here has been assumed small to allow the tangent function to be approximated. Note that the gradient does not necessarily have to be linear; this thin lens derivation applies "locally" even in the presence of a nonlinear gradient. For the case of a nonlinear gradient, the local focusing strength (B') is simply evaluated at each orbital location.

From Eq. (3) for the thin-lens focal length, one can immediately see that the sector angle and edge angle term increase the focusing in the horizontal plane for a positive bend angle or dipole component. The choice of dipole component—which, in the presence of a gradient, changes at each reference orbit as a function of energy—has very important consequences. If the dipole component increases with radius, then focusing increases with energy relative to injection. Both the centripetal and edge-angle term add constructively with the strong-focusing. The integrated strength of the strong-focusing term can also increase if

a) the edge angle increases the path length through the magnetic field, and/or
b) if the gradient itself increases with radius (for a non-constant gradient; i.e. higher or quadrupole).

When the integrated strong focusing strength increases as a function of energy, it serves to stabilize the tune. Both planes are not identical, however, for in the vertical only the strong focusing and edge-angle terms contribute to a change in focusing strength.

$$1/f_F = k_F l + \frac{\vartheta}{\rho_F} + \frac{\eta}{\rho_F}. \tag{3}$$

Therefore, in the vertical version of Eq. (3), only the gradient, $k_D l$, and the edge term η/ρ_D apply so two terms contribute to the vertical machine tune. The following summarizes tune and envelope control in conventional accelerators.

- Centripetal (Cyclotrons + FFAGs):
  - bend plane only, horizontally defocusing or focusing;
  - strength $\propto \theta/\rho$ (bend angle/bend radius of dipole field component on reference orbit);
- Edge focusing (Cyclotrons + FFAGs):
  - horizontally focusing / vertically defocusing, vice versa, or no focusing;
  - strength $\propto \tan\eta/\rho$, or $\sim \eta/\rho$ for a small edge-crossing angle (edge crossing angle/bend radius of dipole field component at entrance to magnet;
- Gradient focusing (Synchrotrons + FFAGs):
  - body field components > dipole: $B = a + bx + cx^2 + dx^3 + \ldots$; $B' = b + 2cx + 3dx^2 + \ldots$;
  - constant gradient: synchrotrons, linear-field non-scaling FFAGs (muon FFAGs);
  - scaled nonlinear field, gradient increases with r or energy: scaling FFAGs;
  - arbitrary nonlinear field, gradient increases with r or energy: nonlinear, non-scaling FFAGs.

4. FFAG Design Principles

In a scaling FFAG, the field-scaling law predetermines that the reference beam trajectories remain parallel implying that much of the optics remain constant with energy—in particular the tunes remain fixed. The non-scaling FFAG relaxes this condition and aims only for stable beam during acceleration. If the acceleration is quick, then tune variations can be tolerated. If the acceleration is slow the tune must be more controlled (although some tune variation can be accommodated or compensated for if the acceleration cycle is slow enough).

Non-scaling in its simplest terms implies nonparallel reference orbits in a FFAG. Although parallelism automatically implies constant tune (through fixed number of betatron oscillations), it is not a necessary condition. In the non-scaling FFAG, the different focusing terms can be varied independently to control tune and further optimize machine parameters. This last point is very important for FFAGs because it allows the field, orbit location, and important machine parameters such as tune, footprint, and aperture to be more independent and strongly controlled than in cyclotron.

The constant + linear-gradient field case serves as an instructive example. Interestingly, this case remains a valid "local" interpretation of FFAG dynamics even in the presence of a strongly nonlinear global field. The local "quadrupole" strength parameter, k, is simply the "local" derivative of the field profile evaluated at the reference orbit for a specific energy. Even in the case of only linear gradient field profiles, a sextupole component[27] arises when the quadrupole body field is combined with an edge angle. The presence of higher-order field components contributes still higher nonlinear terms in combination with an edge angle. Therefore, even in the linear case, the dynamics do not obey linear optics. However, a local interpretation in terms of linear optics and dynamics remains valid and is critical to designing and understanding compact FFAG accelerators.

4.1. *FFAG lattice design*

FFAG lattices are completely periodic, like a cyclotron. Periodicity permits closed geometry and repetitive, adiabatic optical solutions over a tremendous range in momentum. However, the strong-focusing does allow stable, "long" straights to be integrated into the base unit cell. (Specialized utility insertions are under development but are nontrivial to properly match over the large dynamical momentum range of the FFAG).

All lattices are simple, single lens structures based on the FODO cell. That is, the maximum and minimum beam envelopes alternate between opposing planes—even in the so-called doublet and triplet FFAGs. Single lens structures are optically stable over a large range in momentum; there are no telescope-based FFAGs with any significant dynamical range.

FFAGs utilize short cells to achieve short focal lengths. The stronger the focusing and the shorter the focal length; the more adiabatic the optical functions, and the larger the stable momentum range. FFAG designs exploit combined function magnets to minimize unit cell length and optimize dynamic range. Long straights are inserted at points of reflection symmetry in the lattice (at points where the derivatives of optics functions are zero) thereby causing little disruption to the periodic optics.

4.2. *Progression of non-scaling FFAG design*

Initial non-scaling FFAG lattices (EMMA project)[11] utilized a linear fields/constant gradient and rectangular magnets. However, it does not maintain a constant tune and is not suitable for an accelerator with a modest RF system and therefore a slower acceleration cycle.

With tune the strongest indicator of stable particle motion, simply constraining the machine tune can be sufficient to design a stable machine. In all fixed-field accelerators, the FFAG or the cyclotron, the reference orbit moves with energy. Using this property, tune can be controlled in a linear or nonlinear gradient FFAG by shaping the edges of the magnets.

All three focusing terms are impacted by the edge contour and their interaction can be used to manipulate the machine tune in the horizontal. Two terms, gradient and edge focusing, are available for tune control in the vertical. For example, use of a gradient plus an edge angle on a linear-gradient magnet enhances not only the integrated strong-focusing strength, but also weak (centripetal), and edge focusing as a function of radius (and therefore energy). Further, in a non-scaling FFAG, contributions from the different strength terms can vary with radial position and can also be independent in the F and D magnets. In a non-scaling FFAG the edge crossing angle often changes with energy resulting in non-similar orbits. This increase in strength of all the terms tracks the increase in momentum and stabilizes the tune. The result is a dramatic increase in the momentum reach of the machine, from 2–3 to a factor of 6 utilizing a simple edge contour on a constant-gradient magnet. Figures 1 and 2 indicate the improvement in tune control in a constant-gradient non-scaling FFAG through application of a simple linear edge contour.

Completely stable tunes, and compact machines in footprint and aperture, however, required higher-order, field profiles tailored to reach the advanced specifications. An arbitrary field expansion has been exceptionally successful in controlling both tunes and physical attributes of a machine. An order of magnitude increase

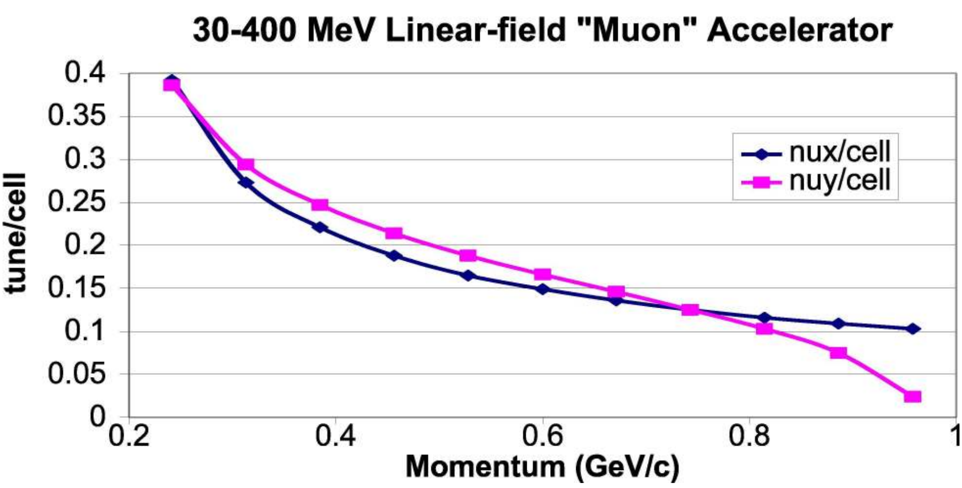

Fig. 1. Variation of tune in a linear gradient, large acceptance non-scaling FFAG for rapid acceleration.

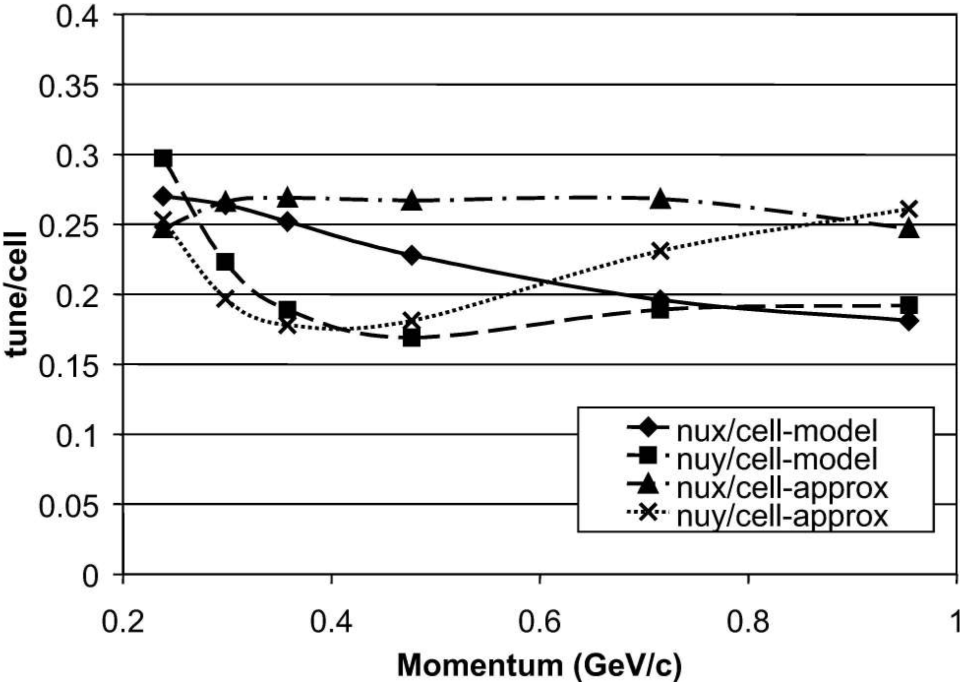

Fig. 2. A constant gradient non-scaling FFAG with an edge contour to stabilize tune. Approx curves indicate the results of a thin lens solution of linear dynamical equations and model curves give the results of a calculation in the accelerator code, MAD.[30]

has been achieved in momentum range relative to the initial non-scaling concept (an acceleration range of a factor of 44 has actually been achieved in one ultra-compact nonlinear design). Even in predominately nonlinear fields, the strong focusing permits adjustment of cell tunes to produce a large dynamic acceptance and surprisingly linear performance (elliptical phase space portraits).

Further, isochronous orbits have been achieved in a non-scaling FFAG by applying both a nonlinear gradient and edge contour. Isochronous implies CW operation and simple RF systems.

Isochronous orbit path lengths are proportional to velocity. However, the orbital path length of a particular momentum follows the B field and thus is not necessarily proportional to velocity. At relativistic energies, the momentum which defines the trajectory and ultimately the path length is an increasingly nonlinear function of velocity. Therefore, the integrated B field must be a nonlinear function of radius to keep it proportional to the relativistic velocity. A nonlinear field expansion combined with an appropriate edge angle can constrain the orbit at each momentum to be proportional to velocity and simultaneously control the tune. Unlike the cyclotron which relies on a dipole field and is therefore limited in adapting path length to match relativistic velocities, the non-scaling FFAG can maintain isochronous orbits well into strongly relativistic energy regimes as shown in Fig. 3.

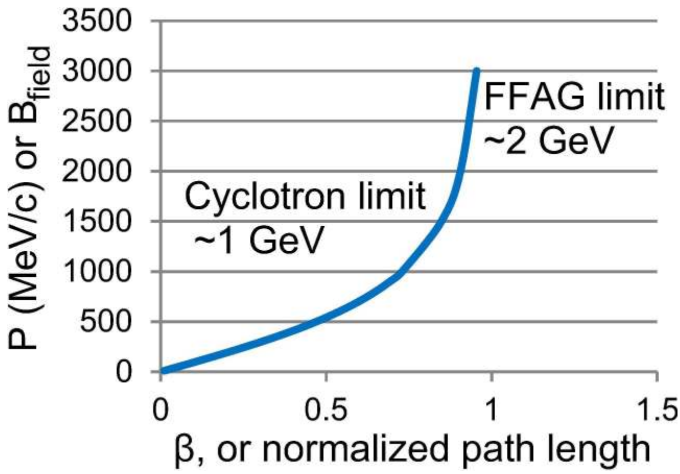

Fig. 3. Momentum dependence ($\propto \langle B \rangle$ field) on velocity (or path length) to maintain isochronous condition.

Further, the nonlinear gradient required to achieve this decreasing change in path length with increasing momentum at relativistic energies has the advantage of providing increasing focusing in both transverse planes as a function of energy.

5. Isochronous FFAG Design

In general, conventional accelerator codes provide too little flexibility in field description and are limited to low order in the dynamics; as such they cannot adequately formulate and predict FFAG accelerators, especially in the presence of the strong nonlinearities from edge contours and fields along with other high-order effects.

Powerful new methodologies in accelerator design and simulation have been pioneered using control theory and optimizers in advanced design scripts with final simulation in COSY INFINITY.[1] COSY INFINITY now has a full complement of sophisticated simulation tools to fully and accurately describe both conventional accelerators and the FFAG's complex electromagnetic fields. Specifically, new tools were developed for the study and analysis of synchrotron, cyclotron, and FFAG dynamics based on transfer map techniques unique to the code COSY INFINITY. With these new tools, closed orbits, transverse amplitude dependencies, and dynamic aperture are determined inclusive of full nonlinear fields and kinematics to arbitrary order. Various methods of describing complex fields and components are now supported including representation in radius-dependent Fourier modes, complex magnet edge contours, as well as the capability to interject calculated or measured 3D field data from a magnet design code or actual components, respectively. These new advanced tools fulfill a critical need in advanced accelerator design.

6. Design and Simulation Tools

A major prerequisite for advanced accelerator design is the existence of reliable, easy to use optimization and simulation tools. Such tools are different in nature for FFAGs than those used in other kinds of accelerators; the rapidly azimuthally varying fields entail significant fringe field effects and out-of-plane nonlinearities. Tracking of orbits for assessment of dynamic aperture needs to be carried out with careful consideration of the nonlinearities, with modern methods of symplectification to insure phase space volume conservation. Further, space charge effects inherent in the high-power operation of the devices produce very novel challenges due to the necessity to treat crosstalk with neighboring orbits. Optimization challenges are difficult since they always affect many orbits at the same time and hence need to be of a global nature.

The ability to model FFAGs—both scaling and non-scaling—with conventional codes is limited. Often new prototypes of accelerators including FFAGs are simulated with codes like MAD[29] and Optim[30] as the standard codes for modeling, but these codes do not provide much flexibility in the description of the available fields and are limited to low order. This limitation can be inadequate to fully demonstrate performance including dynamic aperture, where strong nonlinearities due to edge fields and other high-order effects appear. The significant size of the beam emittance nominally invalidates the paraxial representation (kinematical, or angle effects in the Hamiltonian are significant), which implies that codes that fully represent the kinematics are necessary.

The cyclotron code CYCLOPS[31] has been used to describe the FFAG, but has limited accuracy in this application primarily due to lack of out-of-plane expansion order, which specifically impacts the ability to describe dynamic aperture especially in the case of edge effects with rapid field fall-off—a condition that appears in the FFAG but is not normally present in cyclotrons. Field expansion codes such as ZGOUBI[32] can accurately track the kinematics of such machines, but they have limitations when field profiles become very complex and include significant nonlinear effects. Further, ZGOUBI requires dedicated effort and expertise in order to implement a FFAG design accurately, cannot easily deal with the large transverse emittances required, and lacks some modern analysis tools for symplectic tracking, global optimization, tune shifts and chromaticities, and resonance analysis. In particular, field map codes are difficult to use when one wants to study parameter dependencies, perform detailed study of dynamic aperture, extract advanced optical functions such as high-order resonances, or use optimization routines to study the most advantageous combination of multipole correction schemes, for example.

Modern extensions of the transfer map-based philosophy,[33] as implemented in the arbitrary order code COSY INFINITY,[1] can remedy the limitation in order and in the accuracy of the dynamics. It is particularly suitable for accurate, high-order descriptions of accelerators. Yet in their standard configuration based on pre-selected field elements like combined function magnets with edge angles, they are still not sufficient to describe in full detail the richness of the nonlinearities that can arise in the fields.

Significant enhancements of the code COSY INFINITY for the particularly challenging case of FFAG accelerators have been implemented. Based on the Differential Algebra (DA) approach,[33] unconventional arbitrary-field elements comprising the machine can now be described in a conventional matrix formulation to any order, without any approximations in the dynamics. The following is a list of the powerful features developed for accelerators and native to COSY INFINITY.

Arbitrary Order Maps COSY allows the computation of all dynamics of the system to arbitrary order, including out-of-plane expansions of fields and any nonlinear terms in the Hamiltonian.

Arbitrary Fields There is no limitation in principle of the fields COSY can treat, as long as they can be modeled in a reasonable way. For efficient initial simulation and optimization, it is particularly useful to utilize very high order out-of-plane expansions of suitable mid-plane models.

Symplectic Tracking There are various methods to perform tracking in COSY that preserve the symplectic symmetry inherent in Hamiltonian systems, including methods that do so with minimal modifications based on the EXPO approach described in Ref. 34. These allow a very faithful estimation of dynamic aperture.

Nonlinear Analysis Tools In addition to the mere empirical study, there are various tools for analysis of nonlinear effects, including the normal form-based computation of high-order amplitude-dependent tune shifts and resonances.

Sophisticated Global Optimization COSY allows the automatic adjustment and optimization of arbitrary system parameters; and different from other tools, the search uses methods of global optimization with constraints over a prespecified search region, and not merely local optimization from a starting parameter setting.

In practice, different accelerators are described accurately by different orders in the matrix, or map. For design studies, often orders around 5 or 7 are sufficient; however, once a specific or optimal configuration is chosen, final tracking studies are usually pursued at the 11^{th}–15^{th} order for required accuracy in predicting performance.

6.1. *Examples: 6-fold symmetric FFAG*

To provide an illustrative example, a sample FFAG having six-fold symmetry was studied, with focusing supplied by an azimuthal field variation expressed as a single Fourier mode, as well as edge focusing. The system is studied to various orders of out-of-plane expansion, so that conclusions about dynamic aperture can be drawn. The results for orders three and five, which are typical for the situation of conventional out-of-plane expansion in codes like CYCLOPs, are shown. Since the method used in COSY is not based on divided differences, the necessary in-plane derivatives can actually be calculated to any order desired with an accuracy that is always close to machine precision.[33]

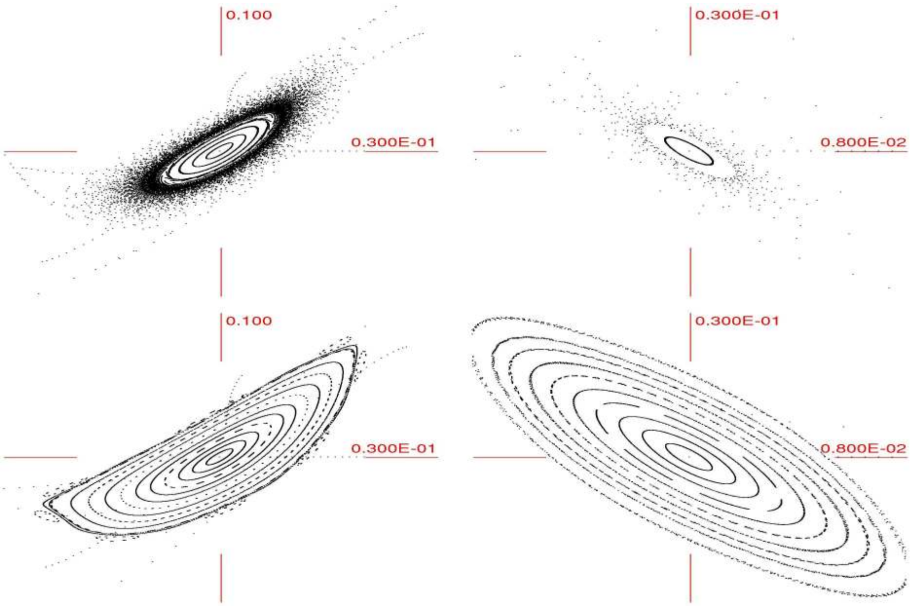

Fig. 4. Tracking in a model non-scaling FFAG with third-order out of plane expansion, without symplectification (top) and with symplectification (bottom).

The results of tracking without symplectification and with Expo symplectification are shown in Figs. 4–6. The Expo symplectification scheme is known to minimize the alterations to the non-symplectic tracking results compared to other symplectification methods. Still, symplectification greatly affects the inferred dynamic aperture of the system.

However, Fig. 6, which is based on order eleven out-of-plane expansion, shows significant additional effects and a different dynamic aperture, suggesting that the low order methods for out-of-plane expansion and dynamics are not sufficient to capture the details of the dynamics. It would in fact lead to an incorrect prediction of dynamic aperture, underestimating it in the horizontal direction and overestimating it in the vertical. Further increases in order beyond eleven do not significantly affect the details of the symplectic motion shown, but continue to influence the non-symplectic motion. A rough estimate reveals that in this particular case, the dynamics as seen in non-symplectic tracking seems to begin to stabilize around order 17, which is still rather easily obtained within the power of a modern workstation.

6.2. *FFAG and cyclotron design*

A powerful new methodology has been pioneered for all fixed-field accelerator optics design (FFAGs and cyclotrons), using control theory and optimizers to develop executable design scripts. These procedures allowed global exploration of all important machine parameters in a simplified lattice. With this methodology, the stable

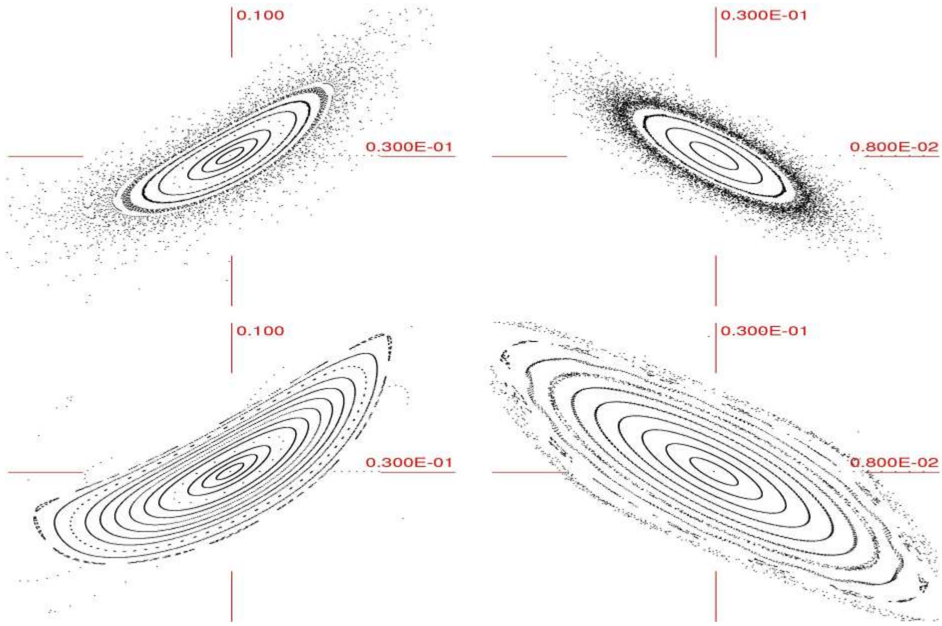

Fig. 5. Tracking in a model non-scaling FFAG with fifth-order out of plane expansion, without symplectification (top) and with symplectification (bottom).

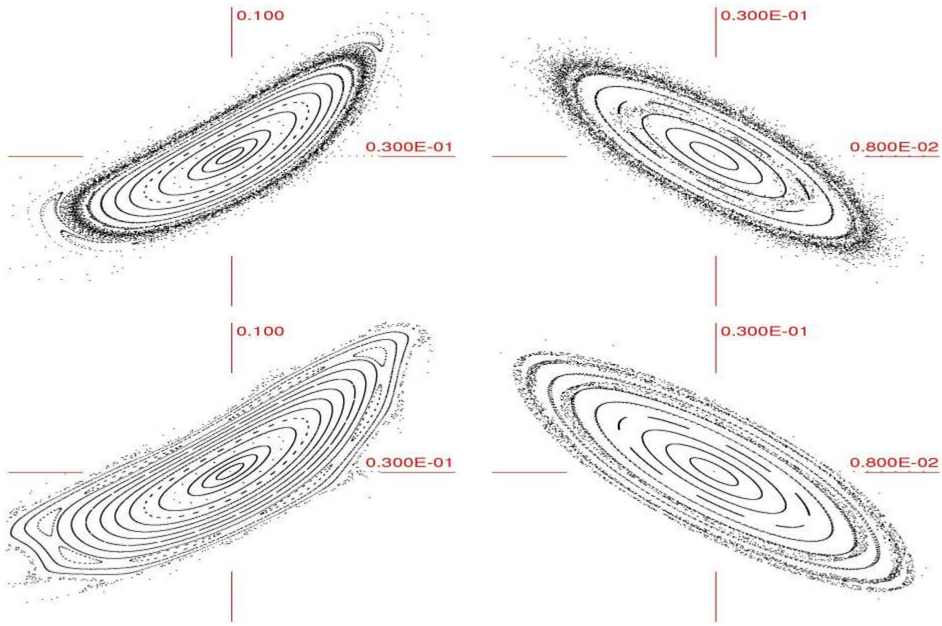

Fig. 6. Tracking in a model non-scaling FFAG with eleventh-order out of plane expansion, without symplectification (left) and with symplectification (right).

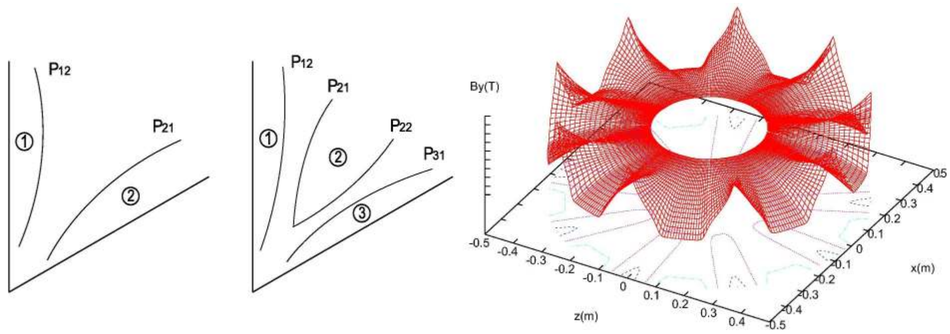

Fig. 7. Complex edge profiles supported in COSY (left) and realistic 3D field expansion output (right) by COSY in polar coordinates derived from simple starting design lattices.

machine tune for FFAGs, for example, was expanded over an acceleration range of 3 up to 6 in momentum with linear fields and a factor of 44 with nonlinear fields and included optimization of complex edge contours, footprint, and components. Full evaluation of the starting lattice, however, required the specific, new advanced simulation tools not existing in current accelerator codes. Such tools have been developed and implemented as an add-on to COSY INFINITY; the FACT (FFAG And Cyclotron Tools) accurately predict and optimize machine performance (Fig. 7). One output format from this add-on software is a 3D field expansion in polar coordinates which can be used by other codes (Fig. 7). In practice, the starting points describing the lattice as output from the design scripts are directly imported into and modeled in COSY INFINITY using FACT software.

Using this powerful methodology, scripts were implemented to design and optimize a FFAG and also for an equivalent cyclotron (low-energy, 4 MeV designs) shown in Fig. 8. The isochronous sector cyclotron employs a 5 kG field. The FFAG initiates injection at 5 kG, but the field rises to 1 T at extraction which allows longer straight sections between magnets thus improved extraction efficiency.

6.3. *High-energy isochronous FFAG example*

As discussed above, the concept of isochronous orbits has been invented for non-scaling FFAGs. This concept has been tested on a preliminary 0.25–1 GeV non-scaling FFAG designed using the new methodologies and optimizers described above. Two options are available to extend this initial effort to a complete accelerator system:

a) a two-ring system, both isochronous, with the lower energy one H^- or
b) a single ring with a high-order field profile which reaches 5T at extraction to increase compactness and energy range.

For the two-ring system use of H^- in the lower energy ring permits CW injection into the higher-energy ring through charge-changing or stripping methods.

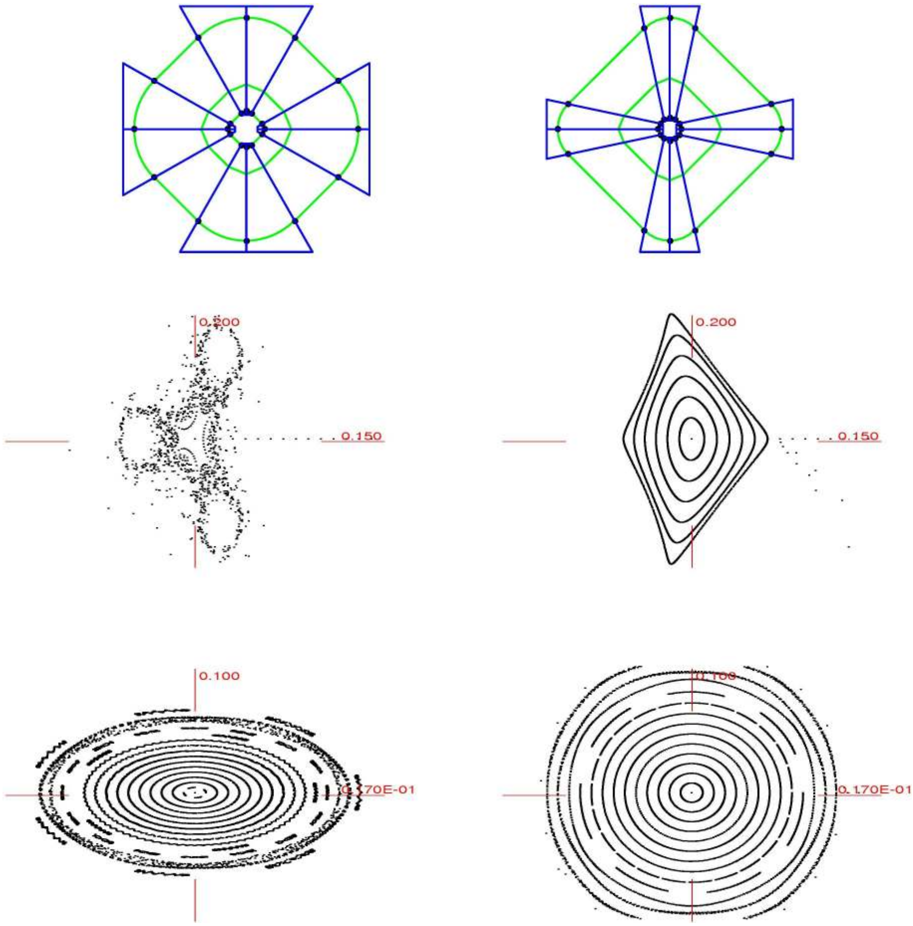

Fig. 8. Subtleties in the transverse dynamics of a cyclotron and an equivalent FFAG at 100 keV: horizontal cyclotron and FFAG (middle row) and vertical cyclotron and FFAG (bottom row) as observed in advanced tracking simulations in COSY.

Extraction can either be resonant, as in the cyclotron or synchrotron, or more likely through outer orbit field shaping as often used in the cyclotron. The advantage of the fixed-field accelerators is the orbit separation in energy. In a cyclotron (and a scaling FFAG), the higher the energy, the closer the orbit spacing as a function of energy, but this is not necessarily true for non-scaling FFAGs. A larger orbit separation can be maintained through appropriate field gradients in the FFAG, much more so than can be achieved with a strictly dipole field. Another advantage of the FFAG is that strong focusing in both planes permits insertion of long straight sections into the periodic cell structure, as in a synchrotron. Long straights promote low-loss extraction; there is room for an extended septum magnet. At high intensities, beam loss is a serious issue.

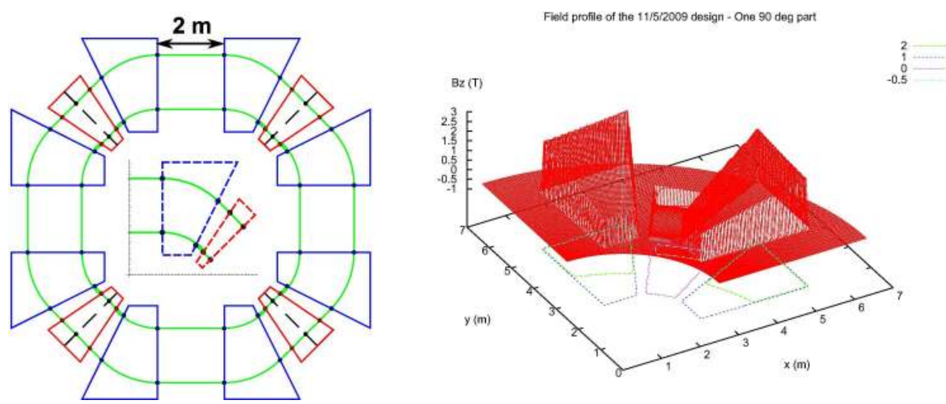

Fig. 9. Ring layout and 3D field profile from COSY. The 3D field profile of a quarter of the ring generated by the new tools in COSY INFINITY expanded from a simple hard-edge, radial field profile and azimuthal distribution.

As discussed above, the design is initiated and machine parameters explored using sophisticated scripts and powerful new methodologies in FFAG accelerator design (pioneered by Particle Accelerator Corporation). The approximate starting machine parameters generated by the scripts are then imported into the advanced accelerator simulation code, COSY INFINITY, which now has a full complement of sophisticated simulation tools (FACT) to fully and accurately describe the FFAG's complex electromagnetic fields—including realistic edge-field effects and high-order dynamics.[1,33] Performance and the isochronous condition can be accurately confirmed. Using this newly-developed design approach and advanced FFAG tools, a preliminary lattice has been discovered, and is discussed next.

6.4. *Lattice details*

As in the muon non-scaling FFAG, the ring must be completely periodic and a triplet cell structure containing a vertically defocusing D magnet positioned between two F magnets was chosen as the optimal base lattice unit. A minimum 0.3–0.5 m length has been imposed between magnets to prevent end-field overlap and cross talk between magnets. The long straight is 2 m to accommodate injection, extraction and the acceleration cavities, but may be increased in future designs. A 4-cell ring periodicity was found to be a strong initial starting point. The ring layout is shown in Fig. 9 derived from the simplistic design scripts. A simulation was then initiated in the code COSY INFINITY, with fully-described dynamics and realistic fringe fields, as the first test of the basic premise in a realistic computation. The 3D field profile generated by COSY INFINITY and subsequently tracked is also plotted in Fig. 9. General parameters of the ring are given in Table 1, the tracking results in Fig. 10 and the radius versus momentum in Fig. 11. The dynamic aperture is enormous—over ± 20 cm in the horizontal and ± 1 cm in the vertical.

Table 1. General Parameters of an initial 0.25–1 GeV non-scaling, isochronous FFAG lattice design.

Parameter	250 MeV	585 MeV	1000 MeV
Avg. Rad. (m)	3.419	4.307	5.030
Cell ν_x/ν_y (2π rad)	0.380/0.237	0.400/0.149	0.383/0.242
Ring ν_x/ν_y (2π rad)	1.520/0.948	1.600/0.596	1.532/0.968
Field F/D (T)	1.62/-0.14	2.06/-0.31	2.35/-0.42
Mag. length F/D (m)	1.17/0.38	1.59/0.79	1.94/1.14

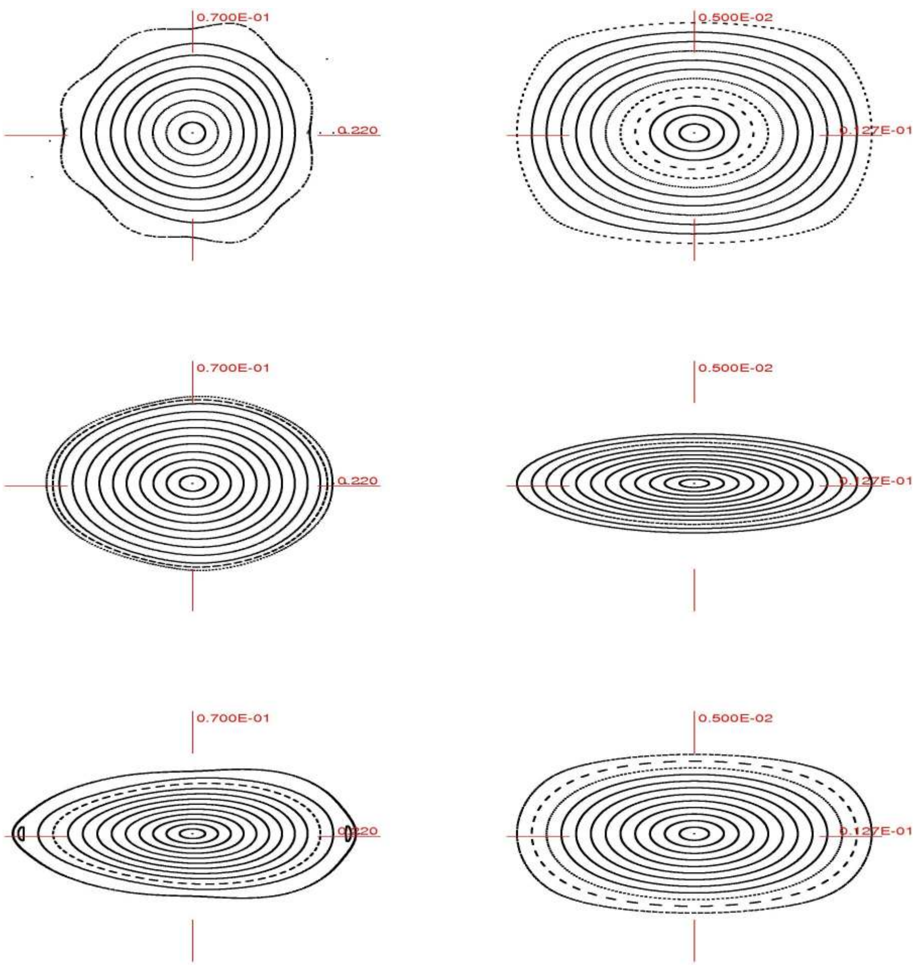

Fig. 10. Dynamic aperture at 250, 585, and 1000 MeV—step size is 15 mm in the horizontal (left) and 1 mm in the vertical (right).

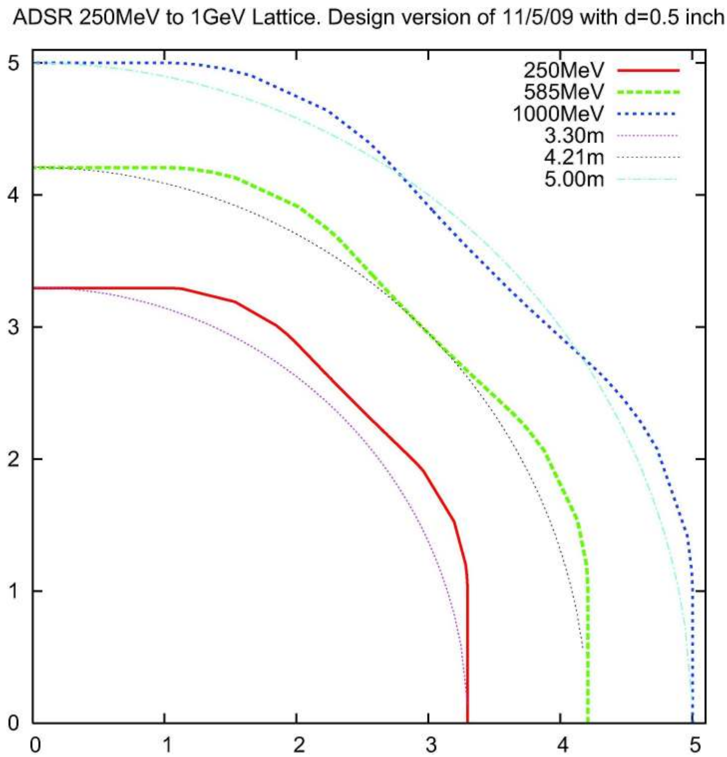

Fig. 11. Details of orbits at the 3 tracked energies over 1/4 of the ring.

Minor adjustment of the lattice parameters provides changes in the tunes in the horizontal and vertical, which can be set independently through relative changes in the F and D gradients. As stated above, this is the first iteration of the lattice design.

The first isochronous FFAG lattice has elicited international interest and further benchmarking is proceeding in alternative advanced codes by international accelerator experts. Fig. 12 and 13 show the corresponding results achieved by Craddock, et al.[35] using the cyclotron code, CYCLOPS using the 3D field map in Figure 9. The level of isochronous behavior is $\pm 3\%$ in this preliminary design.

The preliminary results of these initial studies indicate stable tunes and large dynamic apertures—additional optimization will establish desired machine tunes (tune shifts from the hard edge depend strongly on the vertical aperture due to fringe-field effects) and, more importantly, the results indicate a strong degree of isochronous operation. This lattice proves to be a viable starting point for development of an isochronous FFAG with either a fixed, or a rapidly modulated, RF system. It is anticipated this residual variation can be further reduced with optimization and extended development of the concept using more advanced optimizers available in COSY.

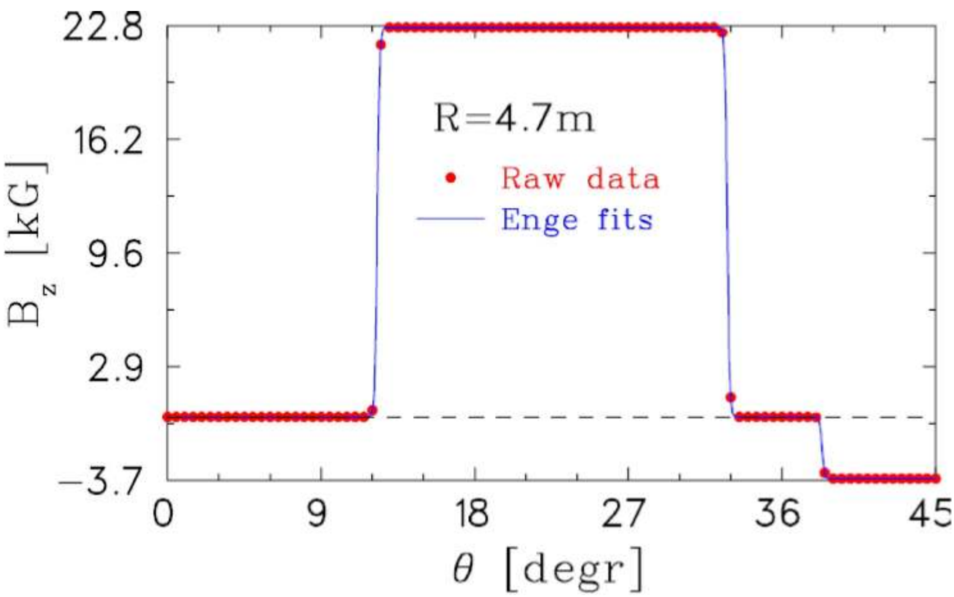

Fig. 12. Details of the B field implemented in CYCLOPS with Enge function fall off (fringe field) and a fine mesh.

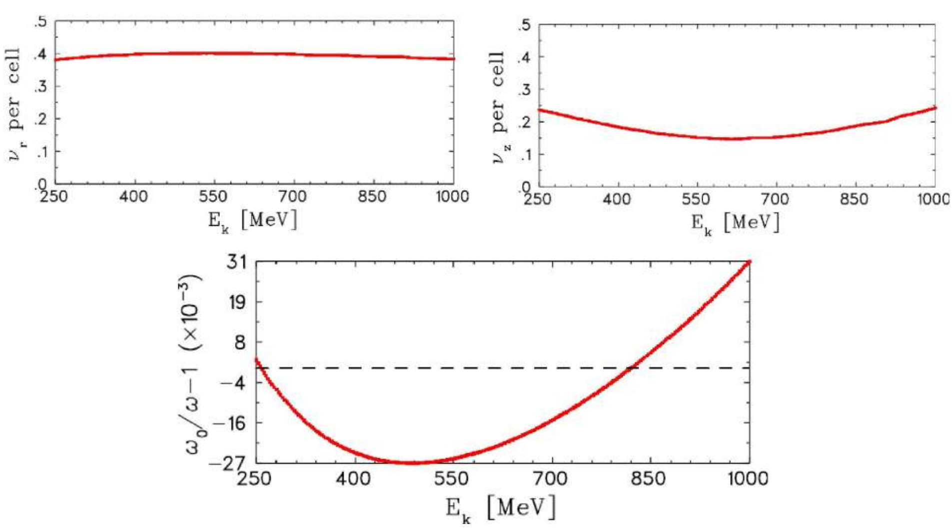

Fig. 13. Results using the cyclotron code CYCLOPs showing radial tune per cell (top, left), azimuthal tune per cell (top, right), and frequency change in percent (bottom).

7. Summary

Powerful new advanced accelerator design scripts have been developed using control theory and optimizers the results of which are directly imported into the advanced accelerator code COSY INFINITY. Various methods of describing complex fields and components are now supported in COSY and include representation in radius-dependent Fourier modes, complex magnet edge contours, as well as the capability to interject calculated or measured field data from a magnet design code or actual components. With these new tools, a high-energy isochronous FFAG has been designed and the performance verified.

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