Electron model lattice with added edge focusing

- Introduction:
  - Rick Baartman: “Spiral focusing slides”
  - Parallels to the my ‘TRIPLET’ design

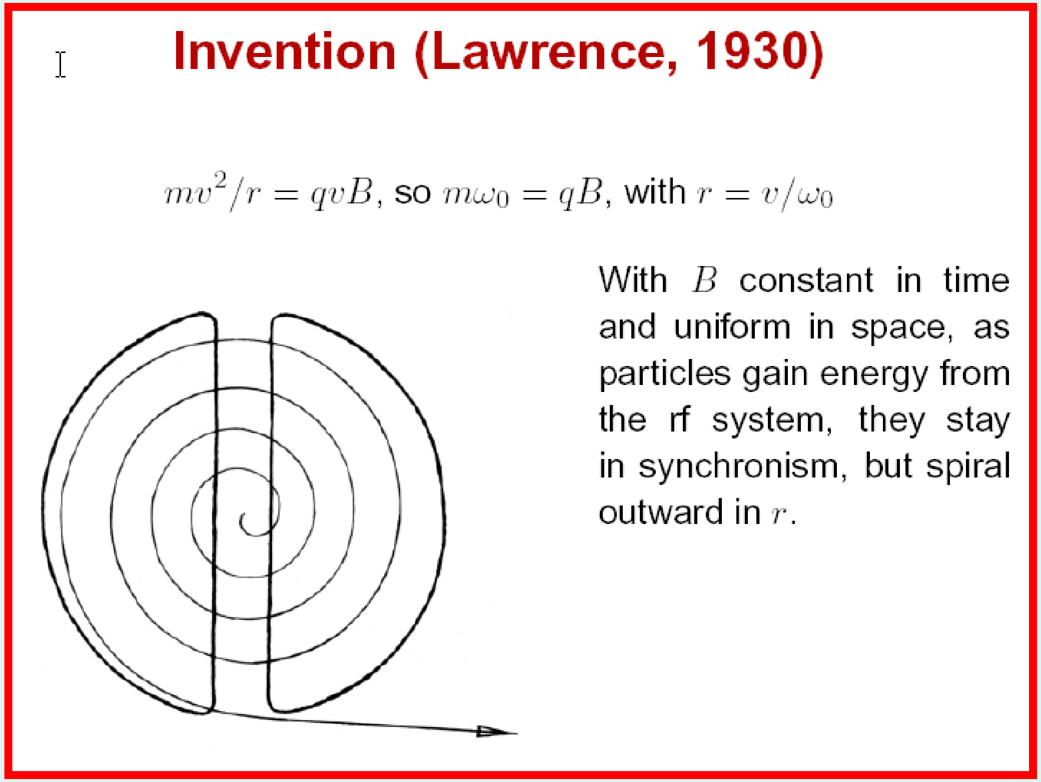
- Our previous electron demonstration rings results presented in Vancouver:
  - Simulation of the muon acceleration (10 GeV – 20 GeV) required:
    - Small path length – acceleration at the top of the RF wave
    - Small orbit offsets
    - Betatron tunes in the basic cell within the a range $0.1 < \nu_{x,y} < 0.4$
    - $-0.333 < dp/p < +0.333$

- A little bit about Polymorphic Tracking Code (PTC) edge effect calculation

- Results from C=16 m example:
  - Where is “additional” edge focusing introduced.
  - Betatron tunes dependence on momentum comparison with and without “edge”
  - Orbits through the basic cell
  - Courant-Snyder amplitude functions dependence on momentum

- Summary

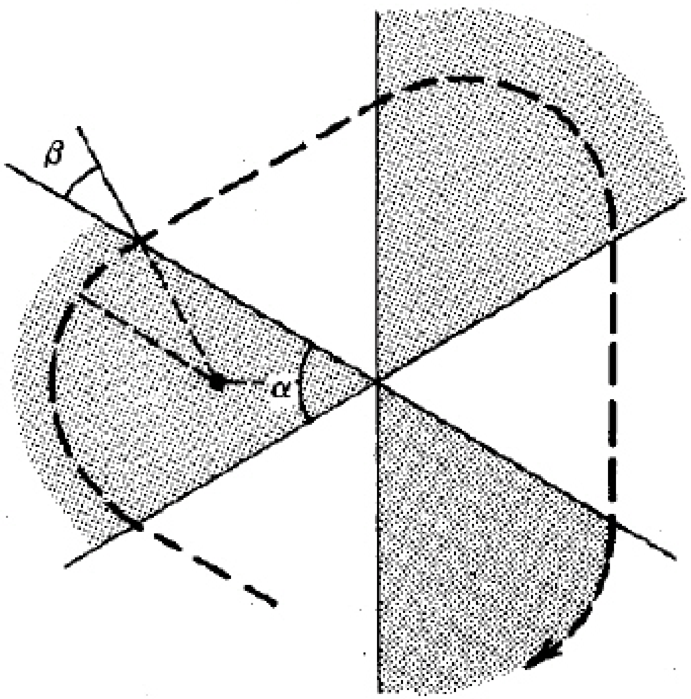
Slides from the Rick Baartman presentation: 'Cyclotrons: Classic to FFAG' - 2002



Invention (Lawrence, 1930)

$mv^2/r = qvB$, so $m\omega_0 = qB$, with $r = v/\omega_0$

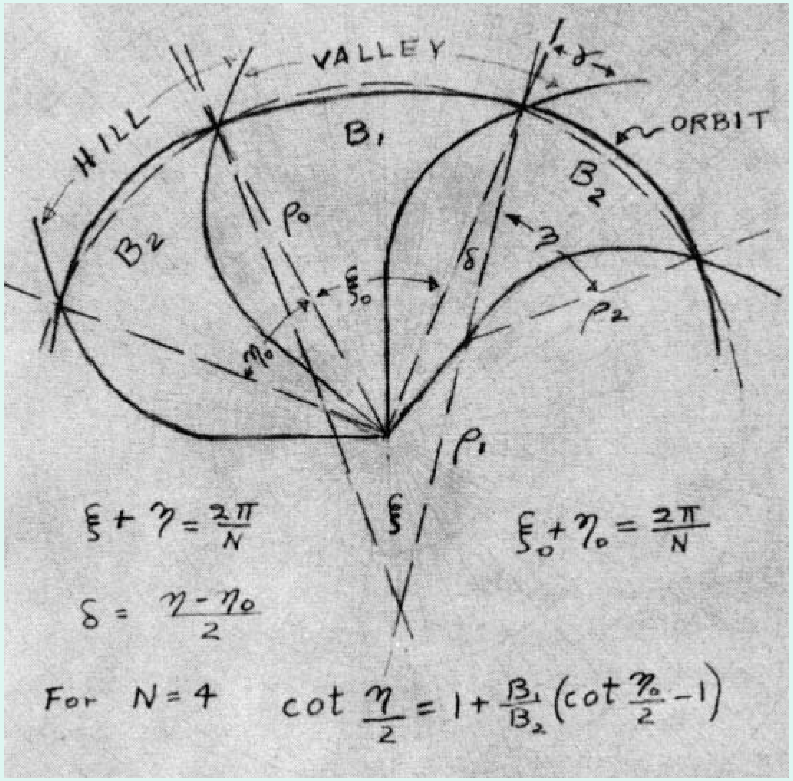
With B constant in time and uniform in space, as particles gain energy from the rf system, they stay in synchronism, but spiral outward in r.



Thomas focusing and later the Okhawa-Symon-Kolomenski FFAG

Slides from the Rick Baartman presentation:

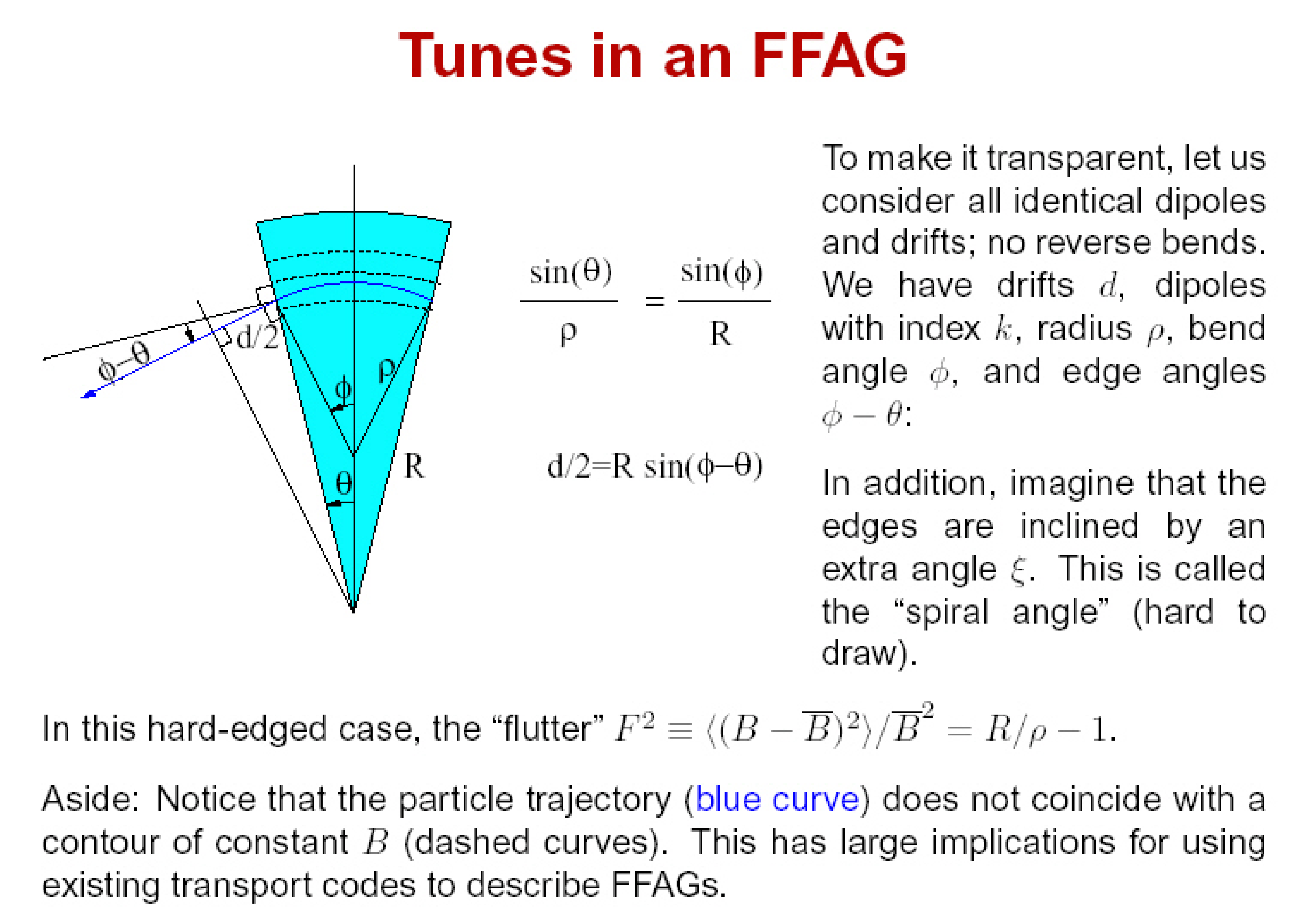
In 1954, Kerst realized that the sectors need not be symmetric.By tilting the edges, the one edge became more focusing and theother edge less. But by the strong focusing principle (larger betatron amplitudes in focusing, smaller in defocusing), one could gain nevertheless. This had the important advantage that reverse bends would not be needed (reverse bends made the machine excessively large). (Figure is from J.R. Richardson notes.) The resulting machines no longer had *alternating gradients*, but Kerst and Symon called them FFAGs anyway.



VALLEY
HILL
ORBIT
γ
B_1
B_2
B_2
ρ_0
ξ_0
η_0
δ
η
ρ_2
ρ_1
ξ
$\xi + \eta = \frac{2\pi}{N}$
$\xi_0 + \eta_0 = \frac{2\pi}{N}$
$\delta = \frac{\eta - \eta_0}{2}$
For $N = 4$
$\cot \frac{\eta}{2} = 1 + \frac{B_1}{B_2}\left(\cot \frac{\eta_0}{2} - 1\right)$

$$M_z := \left(\begin{pmatrix} \text{Cosh}[k_y\, s] & \text{Sinh}[k_y\, s]\,/\,k_y \\ k_y\, \text{Sinh}[k_y\, s] & \text{Cosh}[k_y\, s] \end{pmatrix} . \begin{pmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{pmatrix} . \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{pmatrix} . \begin{pmatrix} \text{Cosh}[k_y\, s] & \text{Sinh}[k_y\, s]\,/\,k_y \\ k_y\, \text{Sinh}[k_y\, s] & \text{Cosh}[k_y\, s] \end{pmatrix}\right)$$

Slides from the Rick Baartman presentation:



Tunes in an FFAG

$$\frac{\sin(\theta)}{\rho} = \frac{\sin(\phi)}{R}$$

$$d/2 = R\sin(\phi - \theta)$$

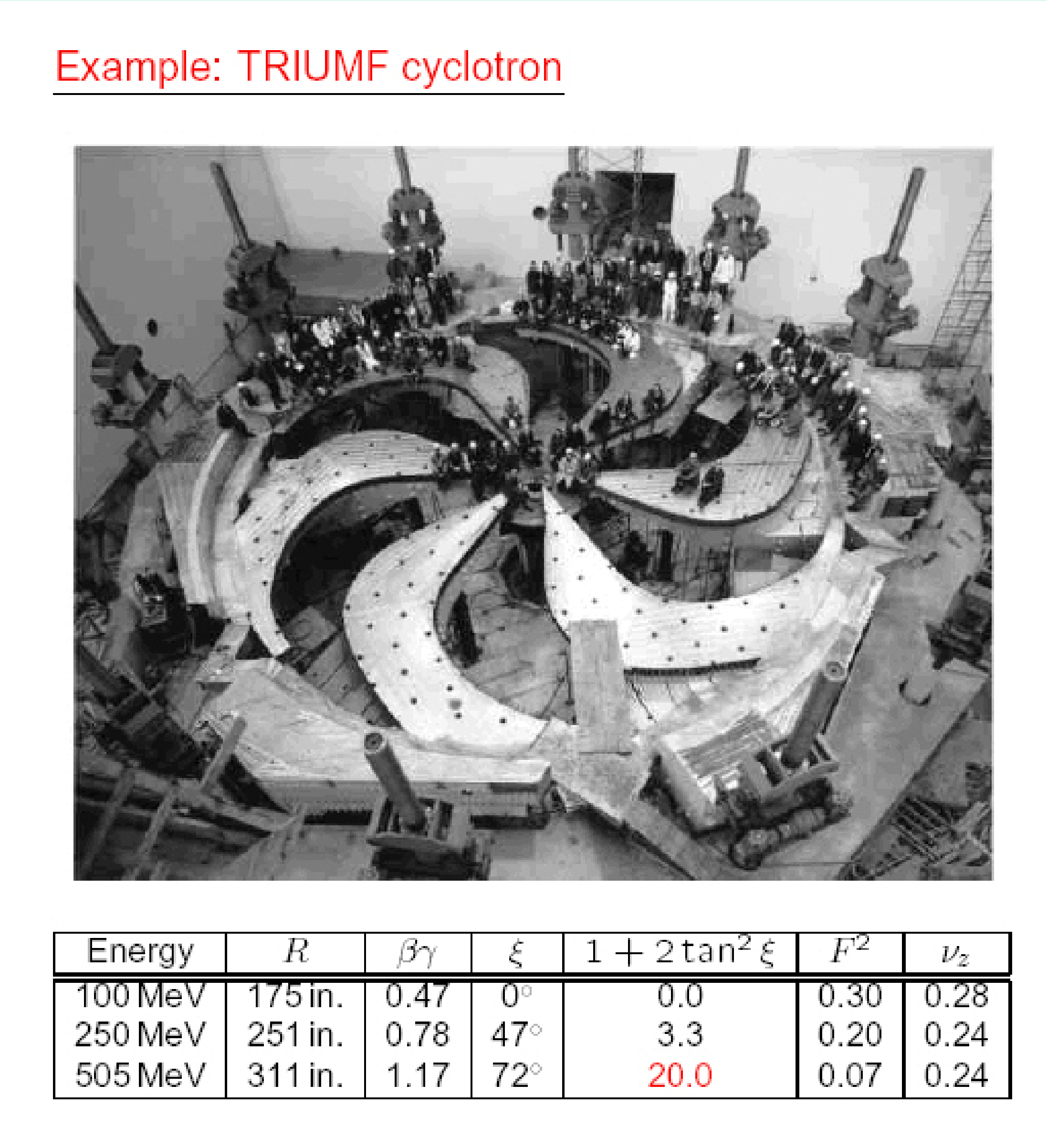
To make it transparent, let us consider all identical dipoles and drifts; no reverse bends. We have drifts d, dipoles with index k, radius ρ, bend angle ϕ, and edge angles $\phi - \theta$:

In addition, imagine that the edges are inclined by an extra angle ξ. This is called the "spiral angle" (hard to draw).

In this hard-edged case, the "flutter" $F^2 \equiv \langle (B - \overline{B})^2 \rangle / \overline{B}^2 = R/\rho - 1$.

Aside: Notice that the particle trajectory (blue curve) does not coincide with a contour of constant B (dashed curves). This has large implications for using existing transport codes to describe FFAGs.

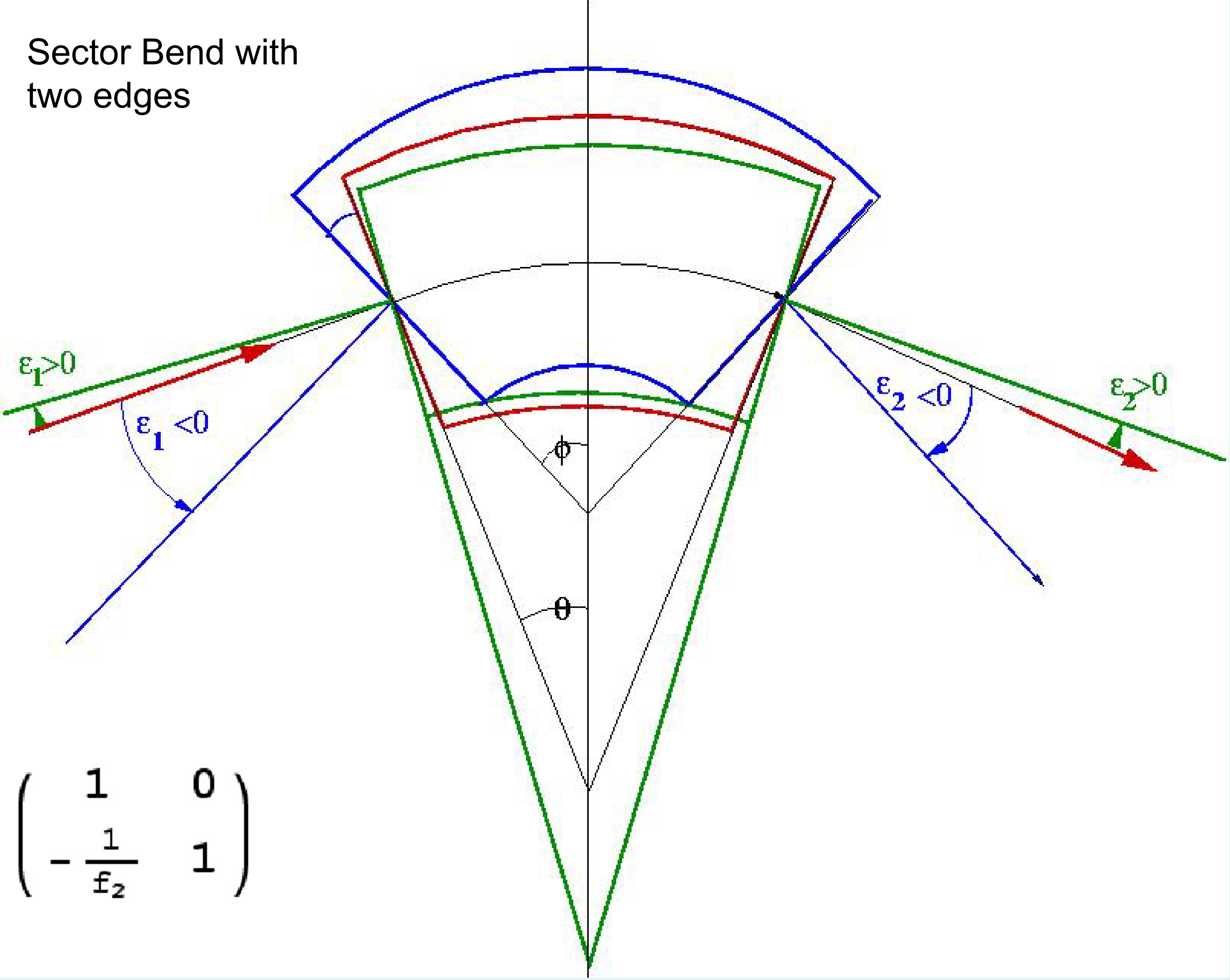
Slides from the Rick Baartman presentation:



Example: TRIUMF cyclotron

Energy	R	$\beta\gamma$	ξ	$1 + 2\tan^2\xi$	F^2	ν_z
100 MeV	175 in.	0.47	0°	0.0	0.30	0.28
250 MeV	251 in.	0.78	47°	3.3	0.20	0.24
505 MeV	311 in.	1.17	72°	20.0	0.07	0.24

Sector Bend with two edges

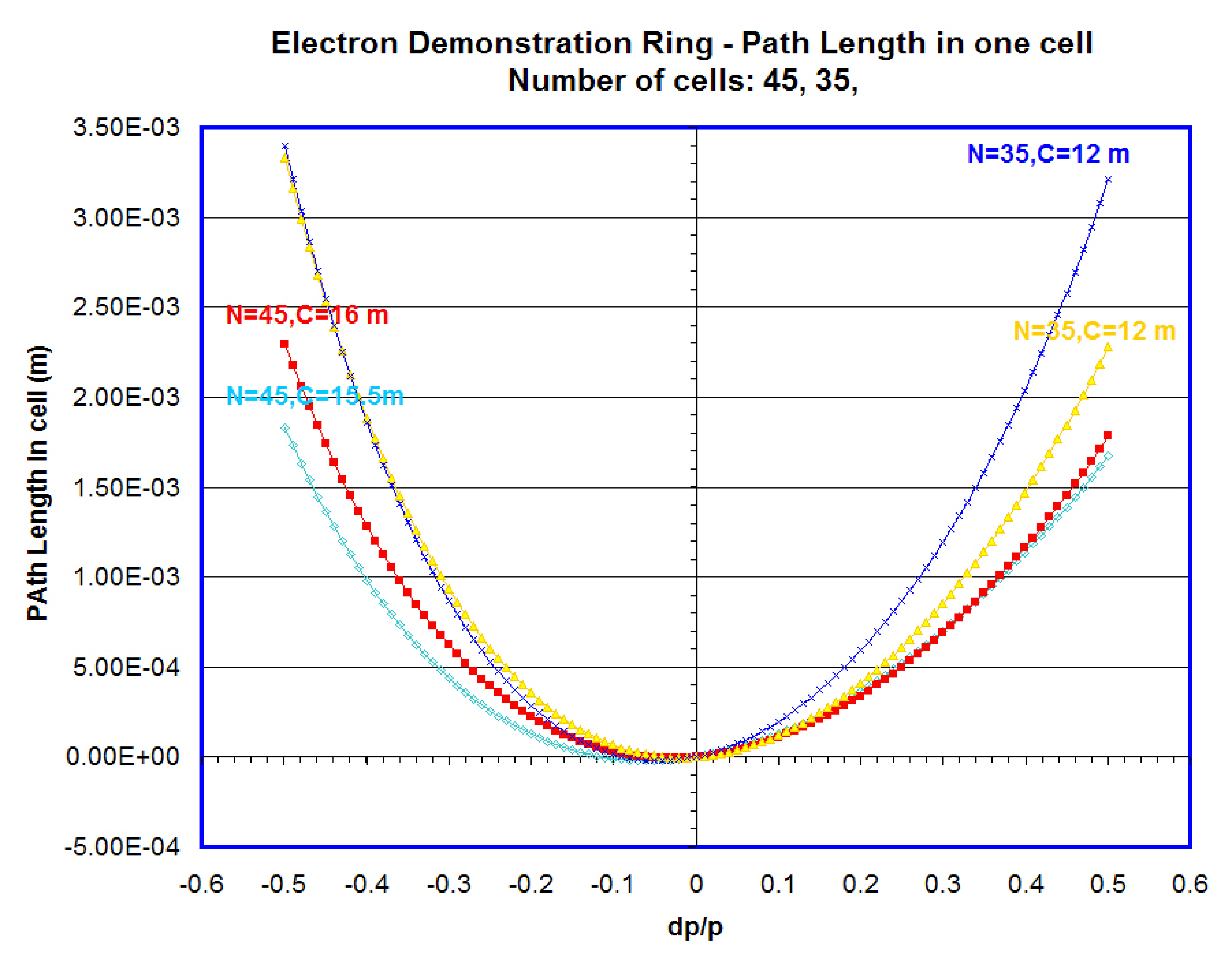


$$\begin{pmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{pmatrix}$$

2. Our previous electron demonstration rings results presented in Vancouver

E. D. Courant and D. Trbojevic

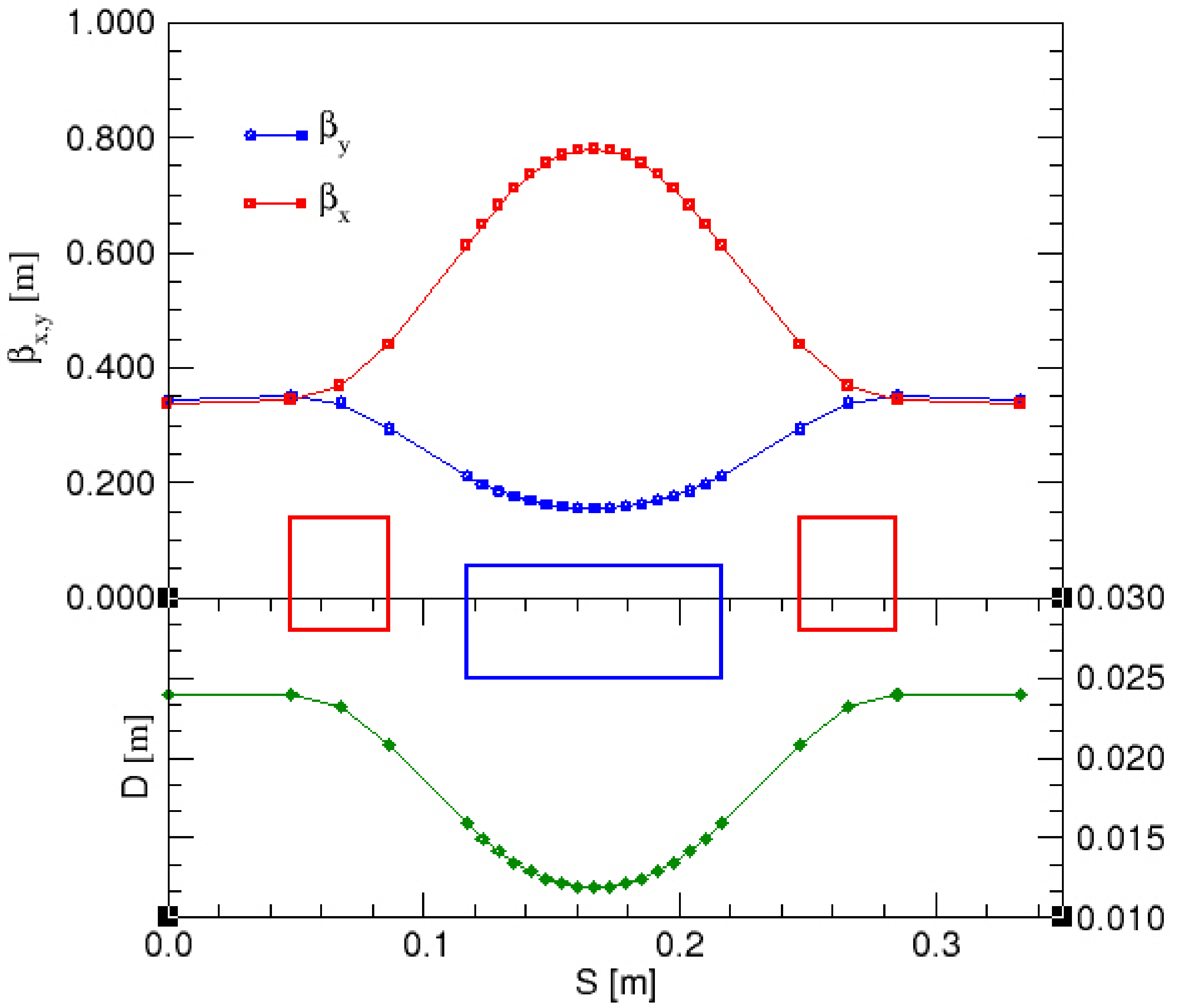
- Lattice Properties:
  - Dimensions and lattice functions for the central energy E_o=15 MeV:
    - Circumference C and number of periods N:
      - C = 13 m with N = 32 ,
      - C = 13 m with N = 34,
      - C = 16 m with N = 45,
      - C = 17 m with N = 45.
  - Gradients and bending fields/angles.
  - Momentum Dependence:
    - Path length per unit cell and for a total length vs. momentum.
    - Orbit offsets vs. momentum.
    - tunes vs. momentum.
    - Amplitude and dispersion functions vs. momentum.



FFAG Workshop 2005

Dejan Trbojevic
April 3, 2005

Electron Demonstration Ring C=15m N=45



GRADIENTS: GF = 7.06035 T/m

GD = - 4.59590 T/m

Dimensions:

L_{BD} = 10 cm

L_{QF} = 3.8 cm

CAV = 10.6 cm

Drift = 3.12 cm

Bending Fields:

By_{QD} = 0.1093 T

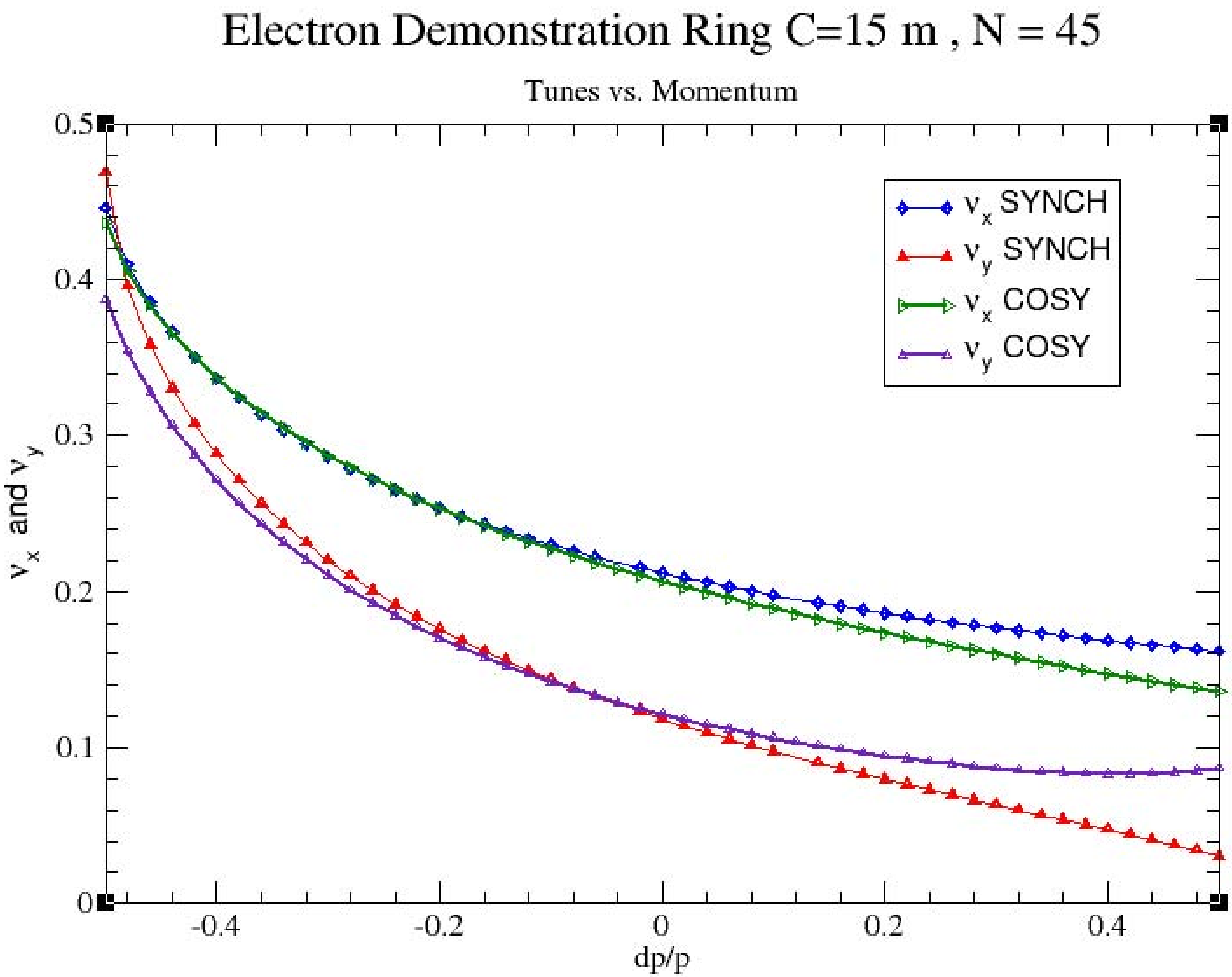
By_{QF} = – 0.0520 T

Bending angles:

ANG_{QD} = 0.2186575

ANG_{QF} = -0.039516

$\theta_2/\theta_1 = 0.180$


Electron Demonstration Ring C=15 m , N = 45
Tunes vs. Momentum
ν_x and ν_y
dp/p
0
0.1
0.2
0.3
0.4
0.5
-0.4
-0.2
0
0.2
0.4
ν_x SYNCH
ν_y SYNCH
ν_x COSY
ν_y COSY

A little bit about Polymorphic Tracking Code (PTC) edge effect calculation

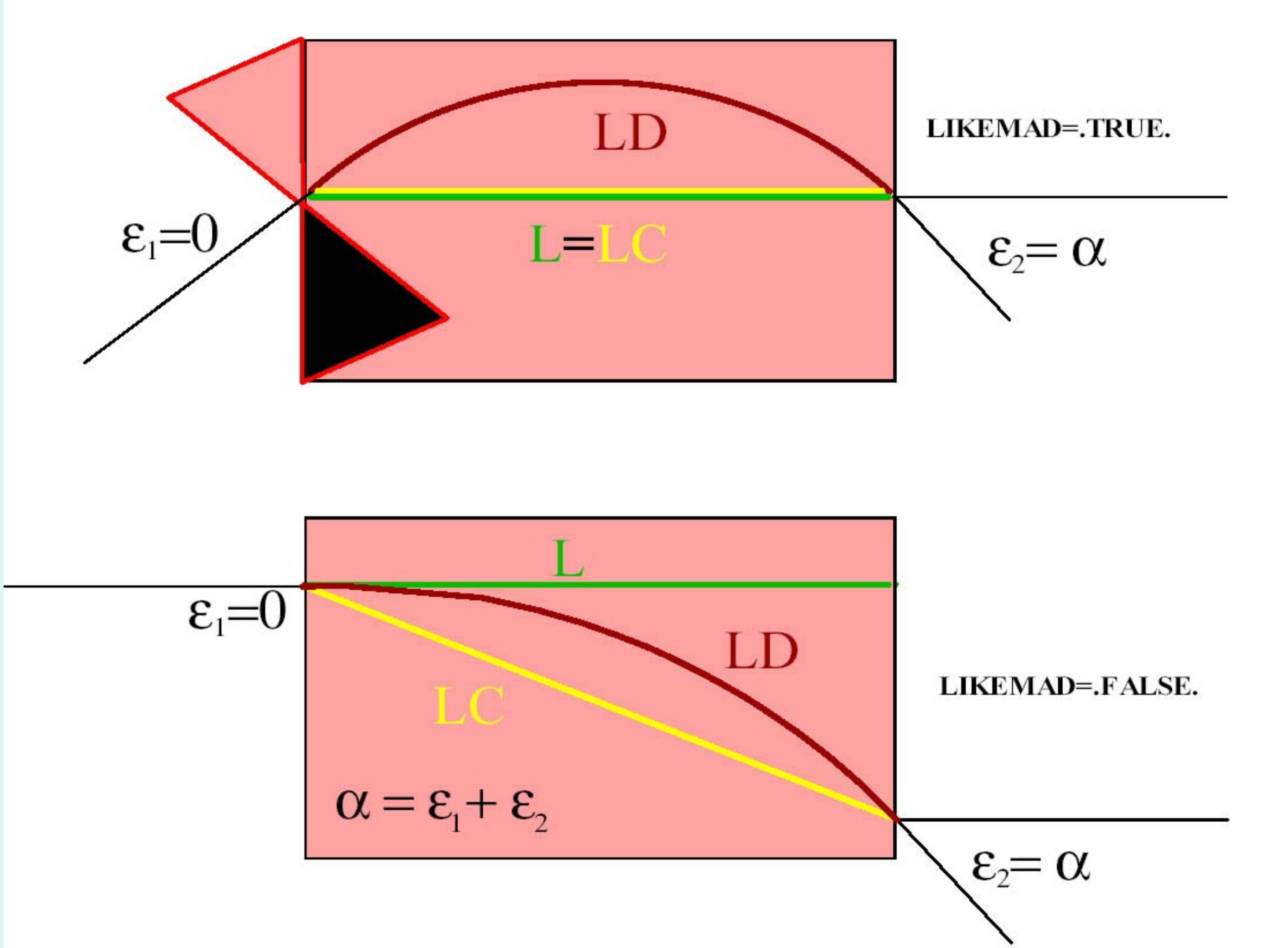
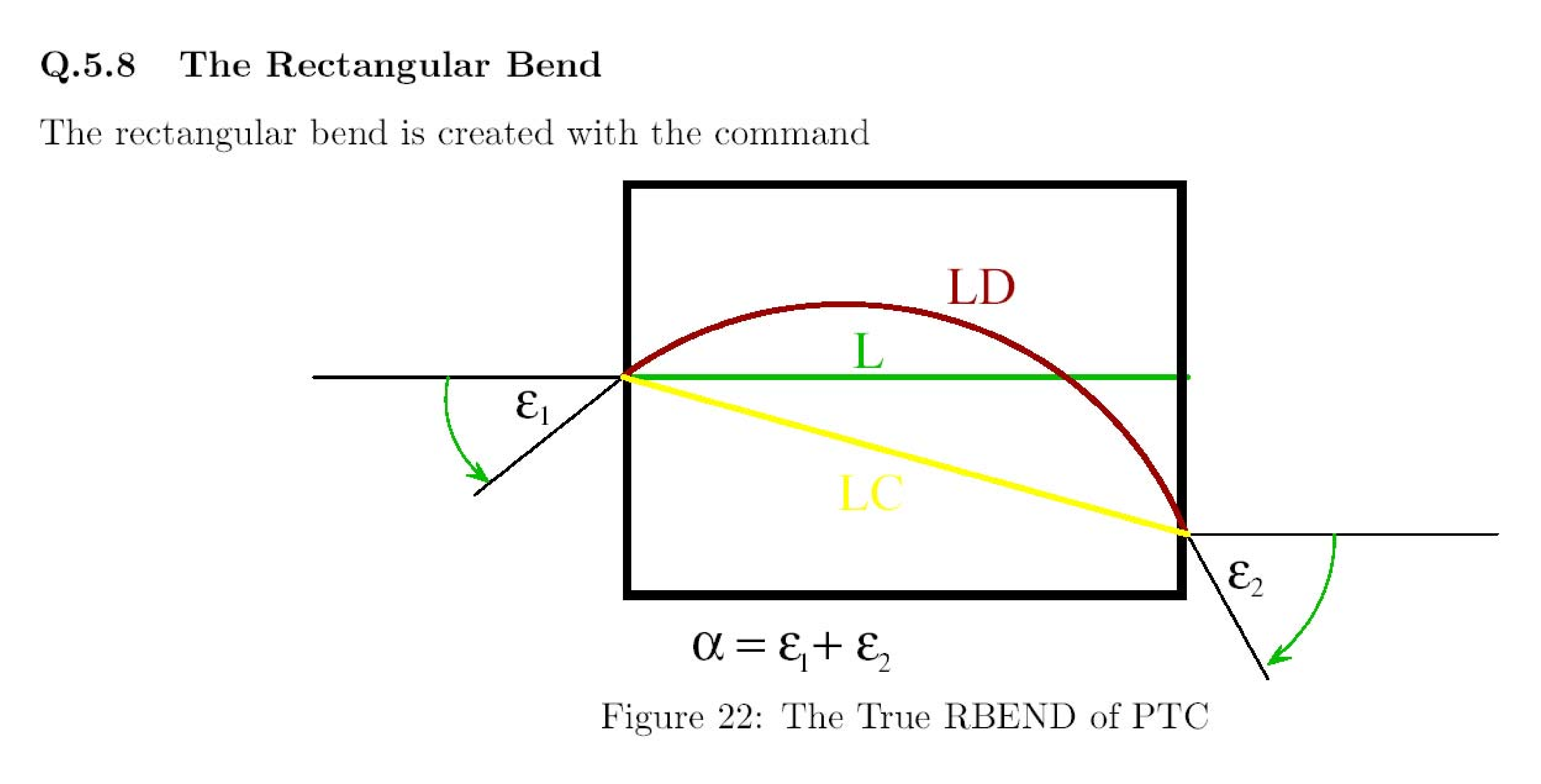


Figure 19: Two Different Bends constructed from the RBEND by PTC



Q.5.8 The Rectangular Bend

The rectangular bend is created with the command

$$\alpha = \varepsilon_1 + \varepsilon_2$$

Figure 22: The True RBEND of PTC

```
B = RBEND("B",L,ANGLE)! L and ANGLE are keywords of the optional construct.
       or
B = RBEND("B",L,ANGLE,TILT)
```

It is also possible to call RBEND with the syntax

```
B = RBEND("B",L,ANGLE,E1)! L, ANGLE, and E1 are keywords of the optional construct.
       or
B = RBEND("B",L,ANGLE,E1,TILT)

B = RBEND("B",L,ANGLE,E2)! L, ANGLE, and E1 are keywords of the optional construct.
       or B = RBEND("B",L,ANGLE,E2,TILT)
```

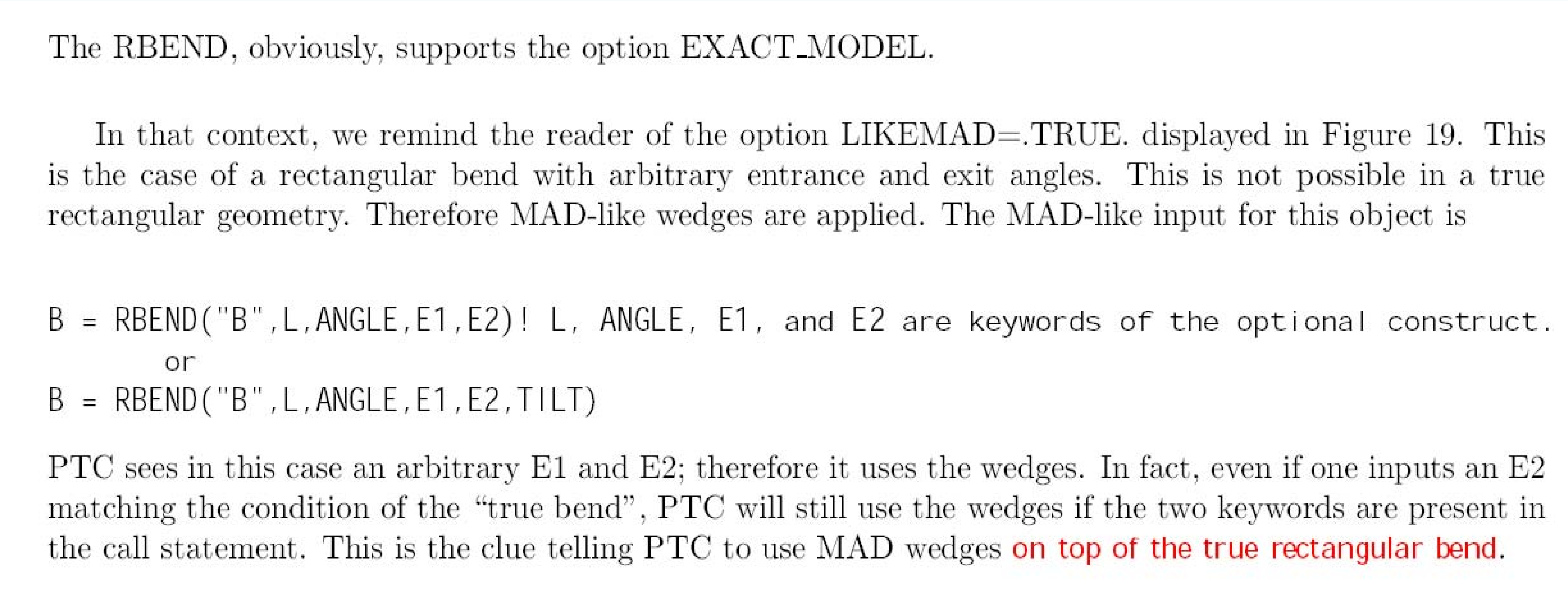
In this case the entrance angle E1 of the bend is following the MAD convention that E1 is zero for ideal parallel face bend, that is to say:

$$\varepsilon_1 = \alpha + E1 \text{ and } \varepsilon_2 = \alpha + E2 \tag{62}$$

where α is the total angle and ε is defined in Figure 22.

The total bending angle is ANGLE and the exit angle is ANGLE-ε_1. In PTC, unlike MAD, RBEND really means a rectangular Cartesian bend in the EXACT_MODEL option. Therefore ε_1, the exit angle, must be ANGLE-ε_1. This bend is very interesting because it is the only element of PTC for which EL%L, EL%P%LD, and EL%P%LC are all different!

As seen in Figure 22, this is a perfect example of the generality needed to cover an arbitrary magnet. Here the variable EL%L is tied, as we said, to the inner field details of the magnet. Since the magnet is a rectangular object, it must be integrated in Cartesian coordinates. This has nothing to do with the end purpose of the magnet. This length L is really a "private" variable of the magnet similar to a bizarre B-field or other strange properties that arbitrary complex magnets might have. On the other hand, EL%P%LD and EL%P%LC are once more the variables indicating the coordinate needed to patch to the outside world.

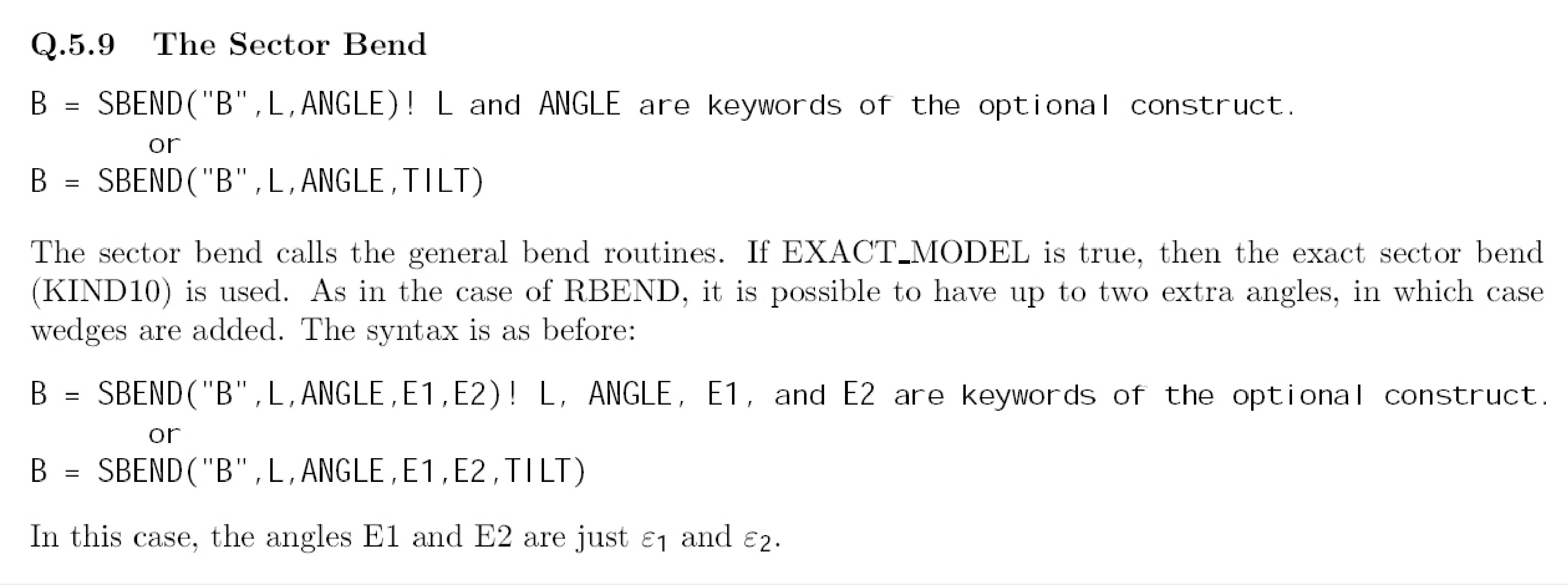


The RBEND, obviously, supports the option EXACT_MODEL.

In that context, we remind the reader of the option LIKEMAD=.TRUE. displayed in Figure 19. This is the case of a rectangular bend with arbitrary entrance and exit angles. This is not possible in a true rectangular geometry. Therefore MAD-like wedges are applied. The MAD-like input for this object is

```
B = RBEND("B",L,ANGLE,E1,E2)! L, ANGLE, E1, and E2 are keywords of the optional construct.
      or
B = RBEND("B",L,ANGLE,E1,E2,TILT)
```

PTC sees in this case an arbitrary E1 and E2; therefore it uses the wedges. In fact, even if one inputs an E2 matching the condition of the “true bend”, PTC will still use the wedges if the two keywords are present in the call statement. This is the clue telling PTC to use MAD wedges on top of the true rectangular bend.



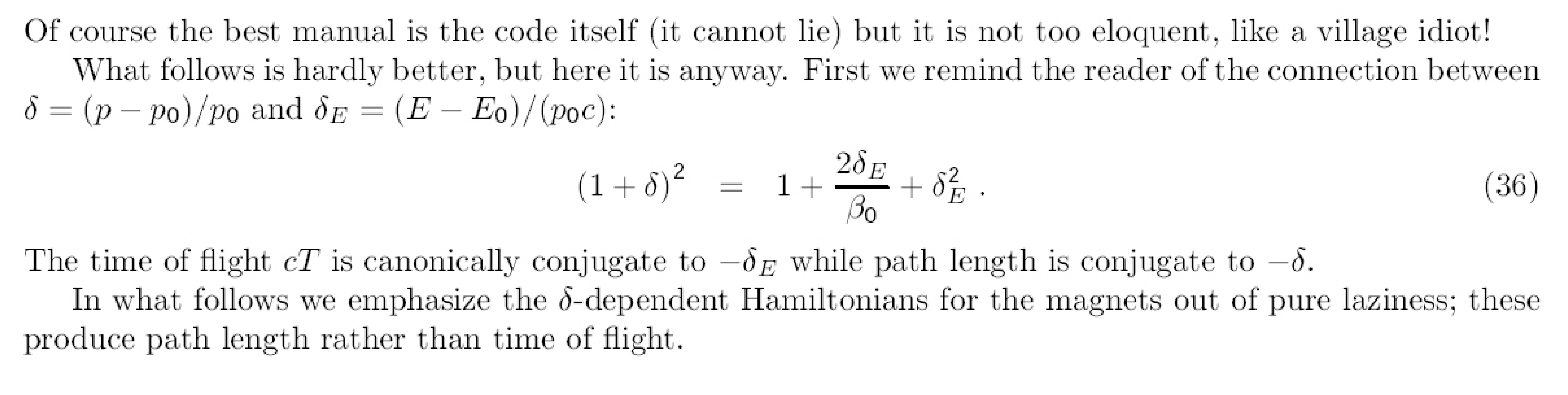
Q.5.9 The Sector Bend

```
B = SBEND("B",L,ANGLE)! L and ANGLE are keywords of the optional construct.
       or
B = SBEND("B",L,ANGLE,TILT)
```

The sector bend calls the general bend routines. If EXACT_MODEL is true, then the exact sector bend (KIND10) is used. As in the case of RBEND, it is possible to have up to two extra angles, in which case wedges are added. The syntax is as before:

```
B = SBEND("B",L,ANGLE,E1,E2)! L, ANGLE, E1, and E2 are keywords of the optional construct.
       or
B = SBEND("B",L,ANGLE,E1,E2,TILT)
```

In this case, the angles E1 and E2 are just ε_1 and ε_2.



Of course the best manual is the code itself (it cannot lie) but it is not too eloquent, like a village idiot!

What follows is hardly better, but here it is anyway. First we remind the reader of the connection between $\delta = (p - p_0)/p_0$ and $\delta_E = (E - E_0)/(p_0 c)$:

$$(1+\delta)^2 \quad = \quad 1 + \frac{2\delta_E}{\beta_0} + \delta_E^2 \ . \tag{36}$$

The time of flight cT is canonically conjugate to $-\delta_E$ while path length is conjugate to $-\delta$.

In what follows we emphasize the δ-dependent Hamiltonians for the magnets out of pure laziness; these produce path length rather than time of flight.

K.4.1 DRIFT1: Drift

Well this is a drift! Drifts come in two flavors: exact and expanded. The exact Hamiltonian is given by the formula:

$$H = -\sqrt{(1+\delta)^2 - p_x^2 - p_y^2}. \tag{37}$$

The expanded Hamiltonian is

$$H = \frac{p_x^2 + p_y^2}{2(1+\delta)} - \delta \; . \tag{38}$$

Incidentally, the expression in Equation (38) gives us

$$(x', y') = \frac{1}{(1+\delta)} (p_x, p_y) \quad , \tag{39}$$

which, in turn, leads to a multipole kick of the variables (x', y') proportional to $1/(1+\delta)$: this is the usual non-canonical result.

K.4.2 DKD2: Drift-Kick-Drift Element

This is the "classic" TRACYII-SixTrack Hamiltonian in the non-exact expanded mode. The Hamiltonian of the body of such a magnet is given by

$$H = \underbrace{\frac{p_x^2 + p_y^2}{2(1+\delta)} - \delta}_{H_1} + \underbrace{-\frac{x\delta}{\rho_d} + \frac{x^2}{2\rho_d^2} + V(x,y)}_{H_2}$$

The Teapot code (DRIFTKICK=.TRUE.) split uses the standard ROT_XZ (drift in polar coordinates called SPROT in S_DEF_KIND) and a multipole kick. In PTC the ideal orbit and the computed orbit do not match perfectly if this is used unless BN(1) is adjusted and differs slightly from EL%P%B0. With the Teapot split it is possible to even do a straight element; of course this is not recommended. The other split (DRIFTKICK=.FALSE.) is an "exact bend-multipole" split. The horizontal bend is handled exactly. Notice that it may even differ from the ideal bend since there is no assumption that $b_1 = \rho_d$.

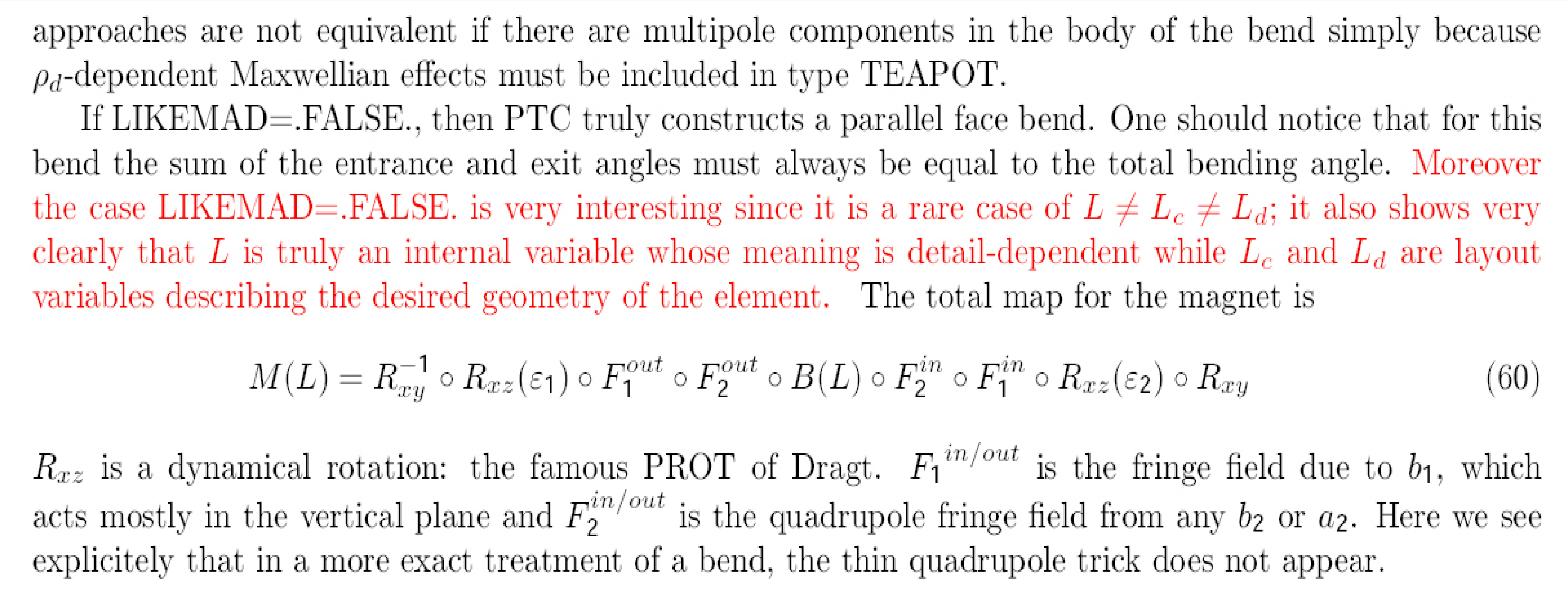
PTC must compute a template from which it extracts the B-field from the multipole components. The potential V in the limit of ρ_d^{-1} going to zero becomes the regular harmonic expansion of Equation (40). However for finite values of ρ_d^{-1}, the potential V obeys the equation:

$$\left\{ (\rho_d + x) \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{\partial}{\partial x} \right) - \frac{\partial}{\partial x} \right\} V = 0 \ . \tag{55}$$

The field, scaled by p_0/q is then given by

$$\begin{aligned} b_x &= -\frac{1}{\rho_d + x} \frac{\partial}{\partial x} V \\ b_y &= \frac{1}{\rho_d + x} \frac{\partial}{\partial y} V \\ b_s &= 0. \end{aligned}$$

Equation (55) is solved in the module ANBN to order SECTOR_NMUL. It is solved by an iterative procedure for $\rho_d^{-1} = 1$. In fact, the beginning of the iteration is the usual harmonic solution for straight elements. The solution for an arbitrary ρ_d can be regained due to the fact that each successive iterate depends on one extra

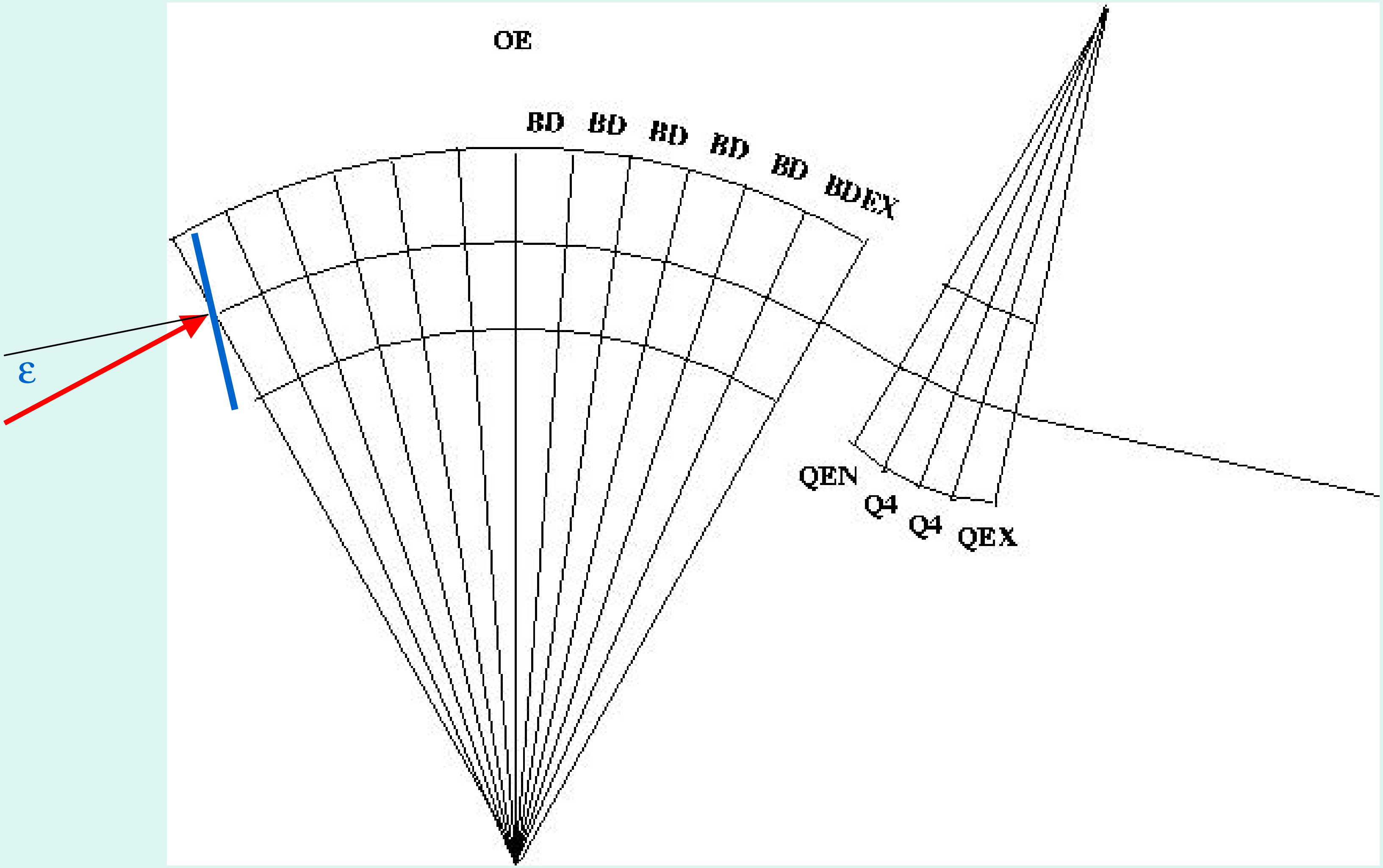


approaches are not equivalent if there are multipole components in the body of the bend simply because ρ_d-dependent Maxwellian effects must be included in type TEAPOT.

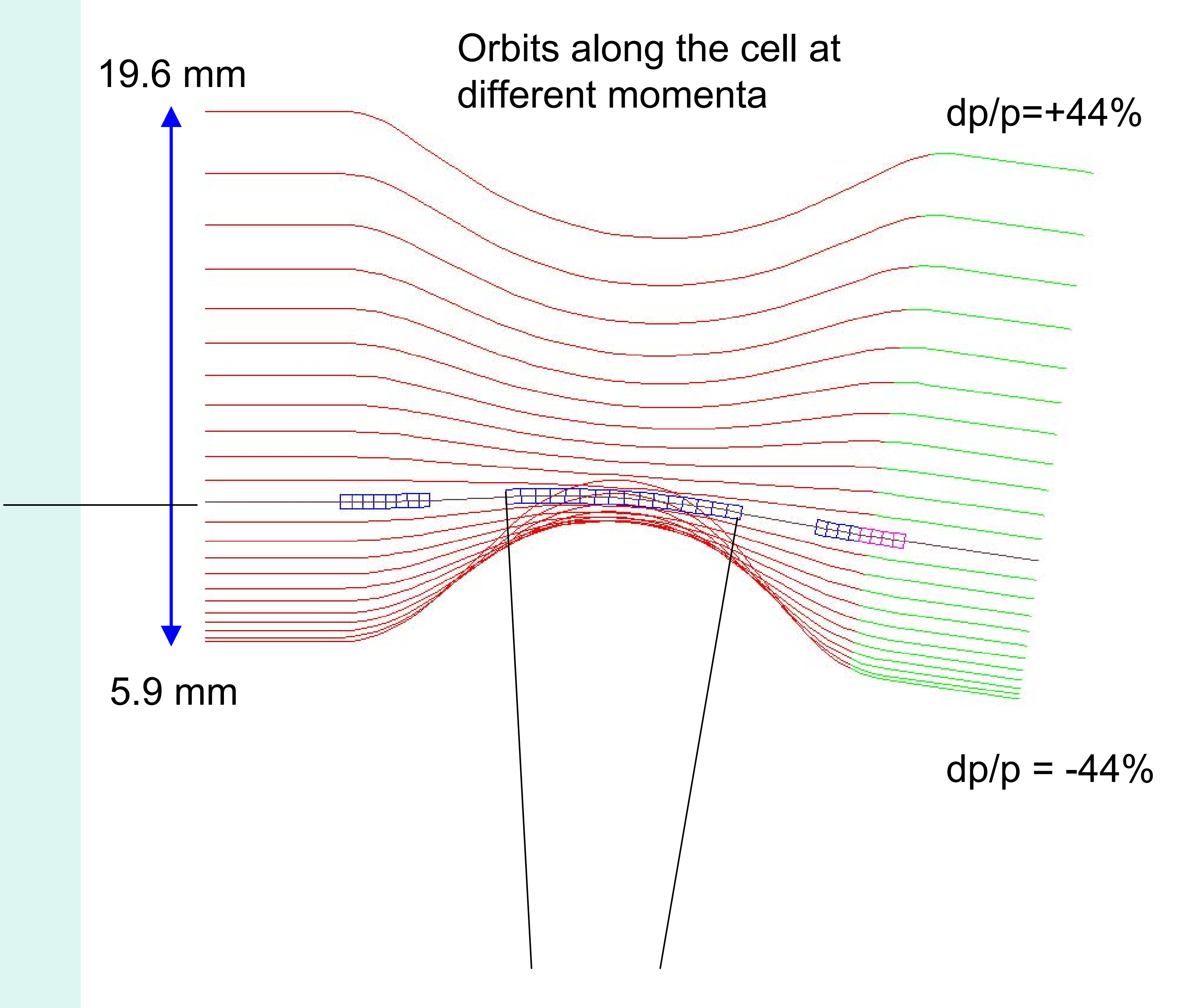
If LIKEMAD=.FALSE., then PTC truly constructs a parallel face bend. One should notice that for this bend the sum of the entrance and exit angles must always be equal to the total bending angle. Moreover the case LIKEMAD=.FALSE. is very interesting since it is a rare case of $L \neq L_c \neq L_d$; it also shows very clearly that L is truly an internal variable whose meaning is detail-dependent while L_c and L_d are layout variables describing the desired geometry of the element. The total map for the magnet is

$$M(L) = R_{xy}^{-1} \circ R_{xz}(\varepsilon_1) \circ F_1^{out} \circ F_2^{out} \circ B(L) \circ F_2^{in} \circ F_1^{in} \circ R_{xz}(\varepsilon_2) \circ R_{xy} \tag{60}$$

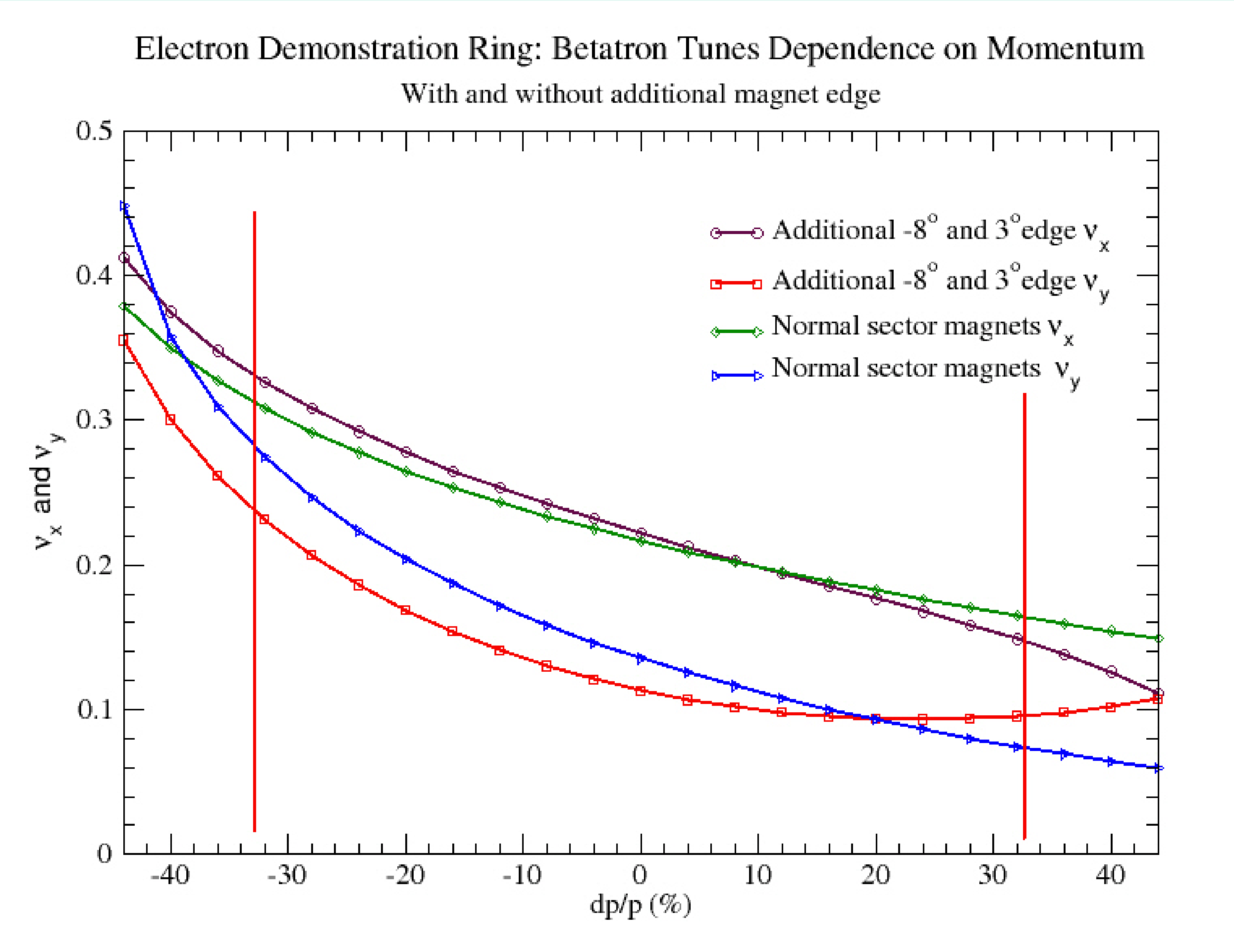
R_{xz} is a dynamical rotation: the famous PROT of Dragt. $F_1{}^{in/out}$ is the fringe field due to b_1, which acts mostly in the vertical plane and $F_2^{in/out}$ is the quadrupole fringe field from any b_2 or a_2. Here we see explicitely that in a more exact treatment of a bend, the thin quadrupole trick does not appear.


OE
BD
BD
BD
BD
BD
BDEX
ε
QEN
Q4
Q4
QEX

Results from C=16 m example:



Results from C=16 m example:



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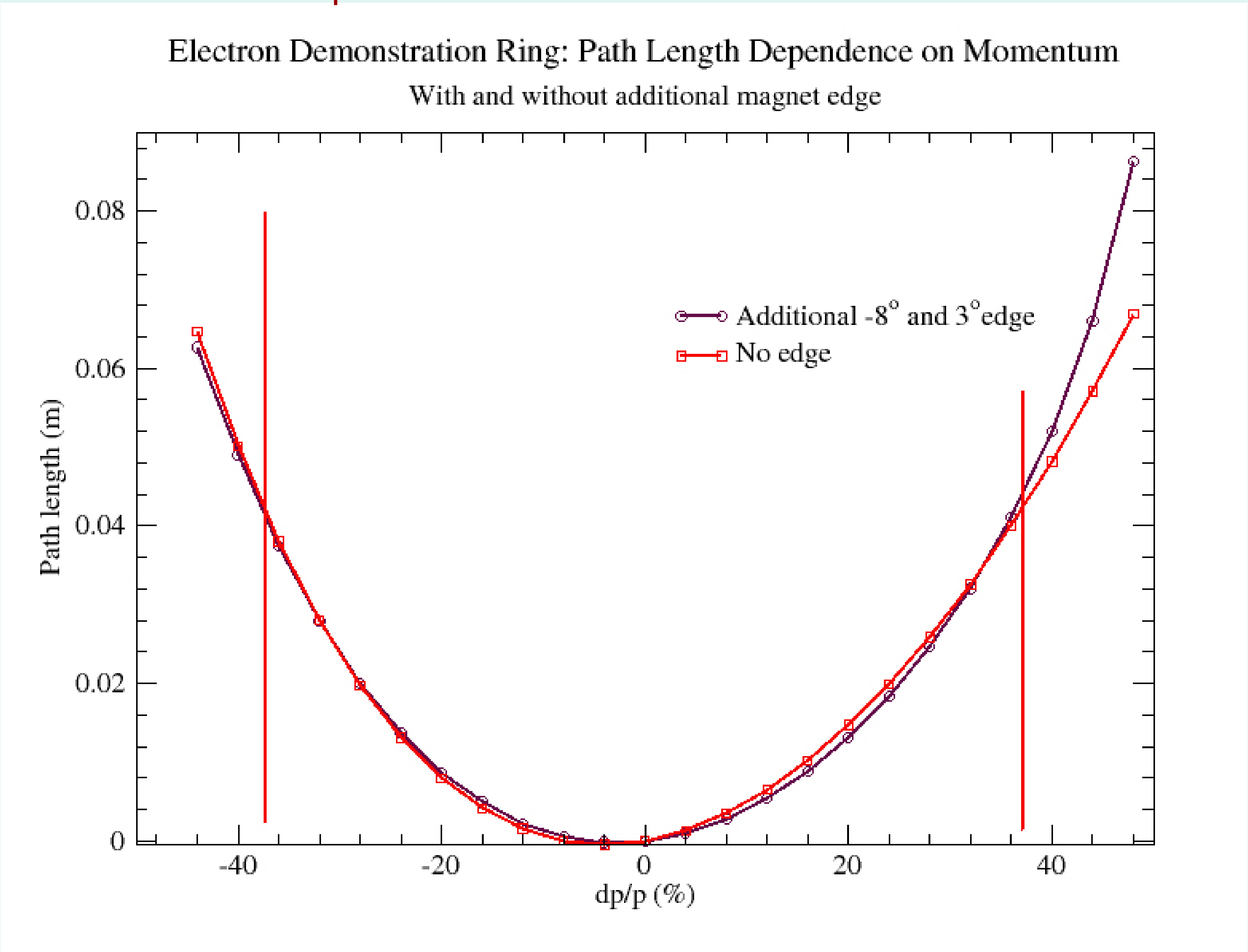
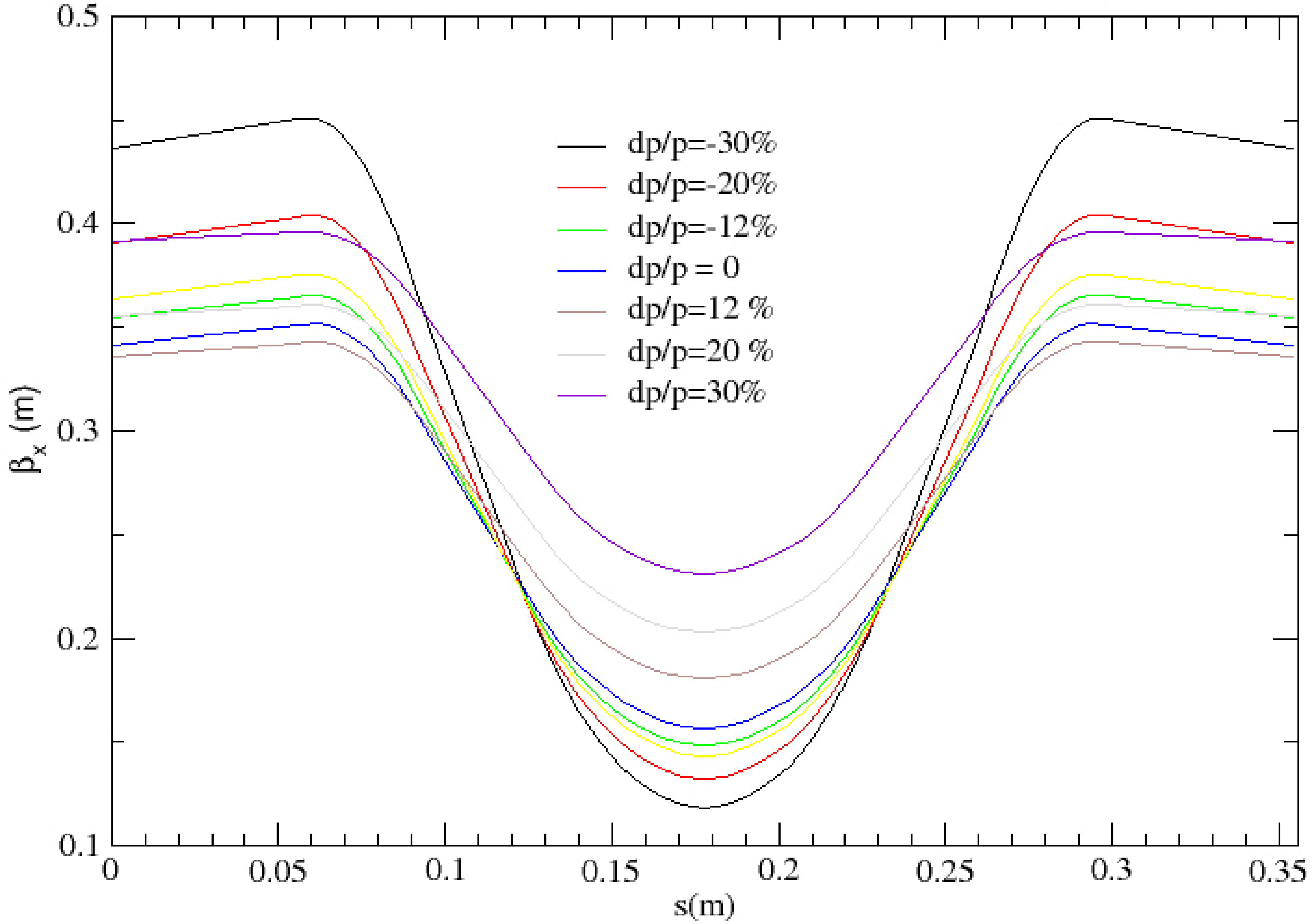


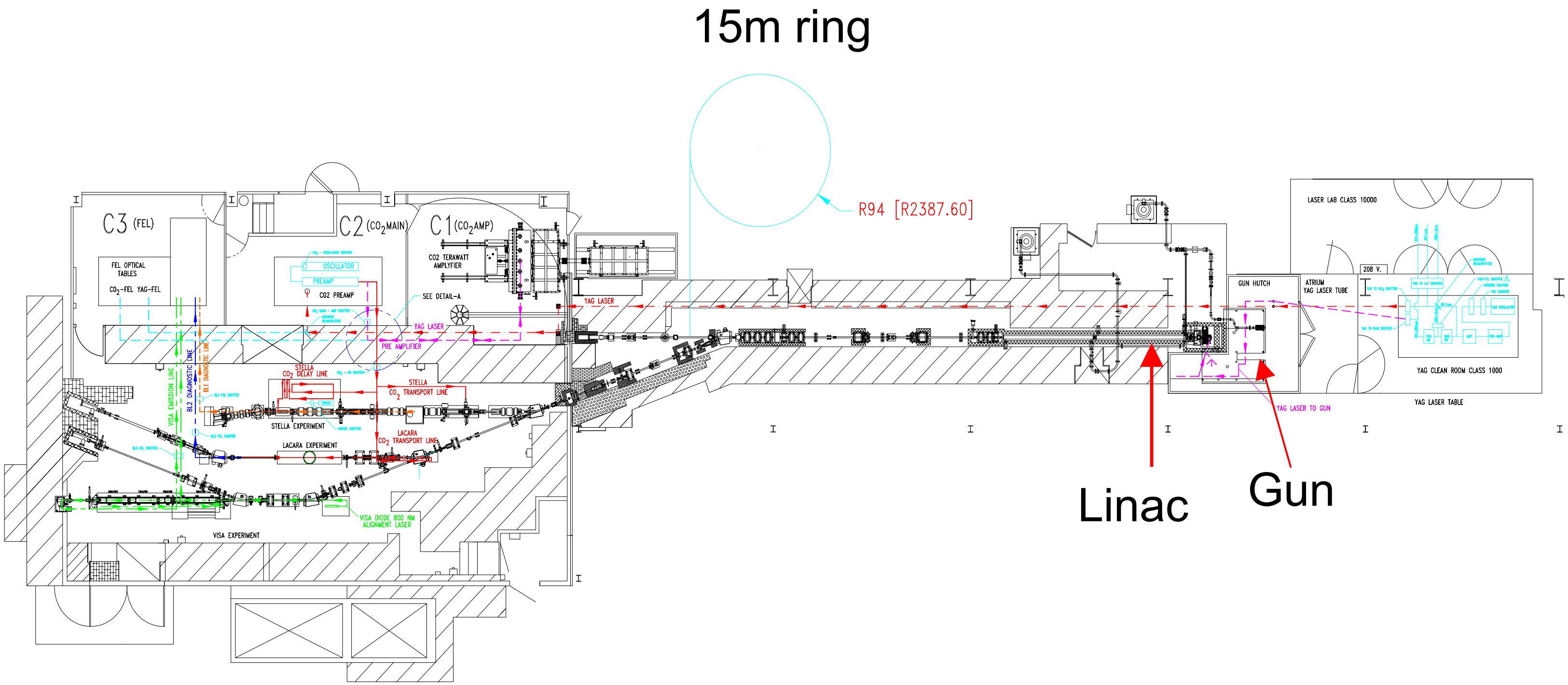
β_x dependence on momentum along the cell

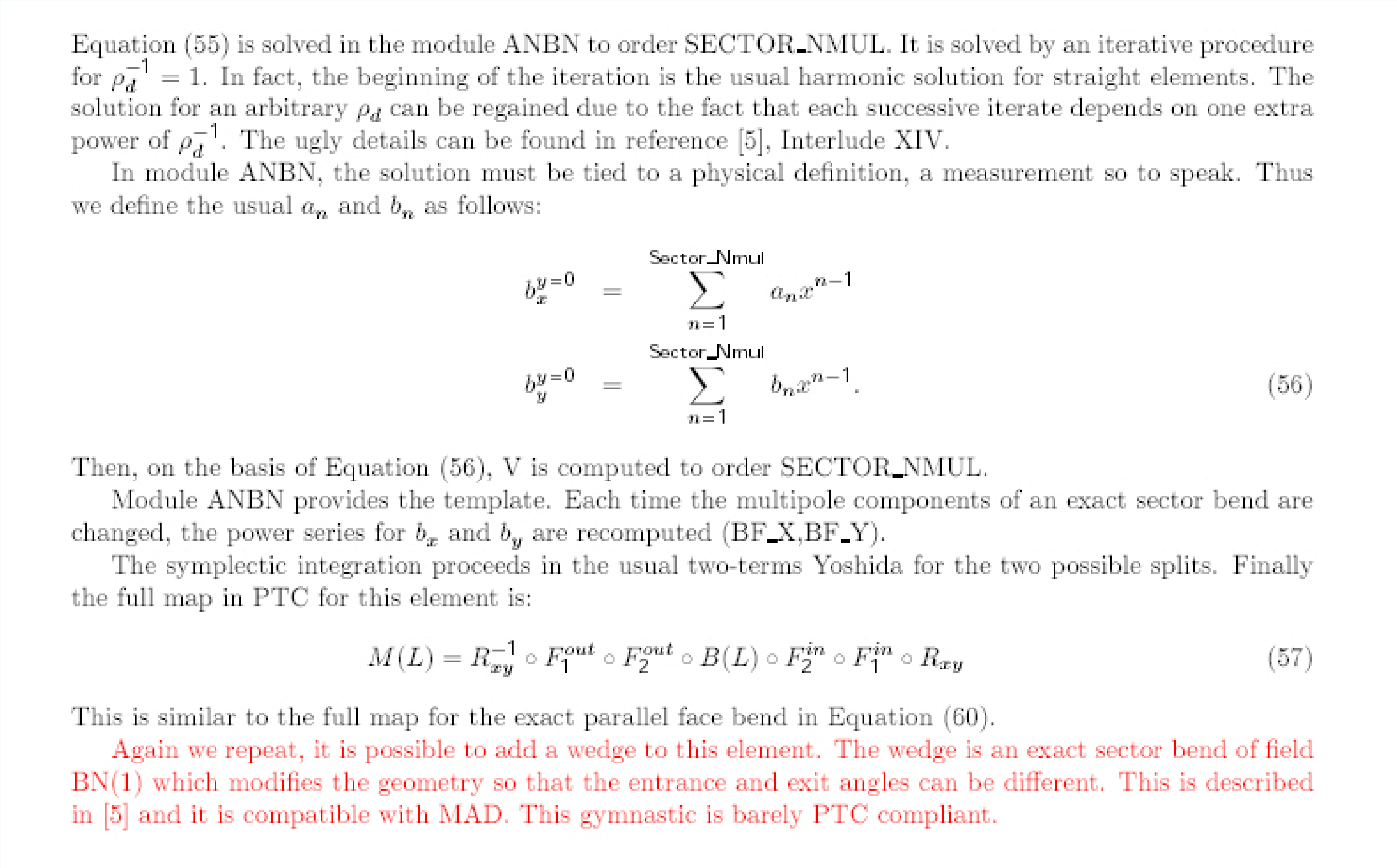


Summary:

A simple straight edge effect application to the ends of the magnets improved the tune dependence on momentum especially in the vertical plane.

Improvements with the curved edges should not be taken seriously because of the very small sizes of the non-scaling FFAG magnets. This approach does improve performance of the scaling FFAG, as Rick Bartmaan had previously explained.


15m ring
R94 [R2387.60]
C3 (FEL)
FEL OPTICAL TABLES
CO_2-FEL YAG-FEL
C2 (CO_2MAIN)
CO_2 - OSCILLATOR SHUTTER
OSCILLATOR
PREAMP
CO2 PREAMP
C1 (CO_2AMP)
CO2 TERAWATT AMPLYFIER
SEE DETAIL-A
YAG LASER
CO_2 MAIN - AMP SHUTTER
DICHROIC BEAMSPLITTER
YAG LASER
PRE AMPLIFIER
VISA EMISSION LINE
BL2 DIAGNOSTIC LINE
BL1 DIAGNOSTIC LINE
STELLA CO_2 DELAY LINE
CO_2 - DH SHUTTER
STELLA CO_2 TRANSPORT LINE
BL1-FEL SHUTTER
CWCO2
CWCO2 SHUTTER
STELLA EXPERIMENT
BL2-FEL SHUTTER
BL3-FEL SHUTTER
LACARA EXPERIMENT
LACARA CO_2 TRANSPORT LINE
VISA DIODE 800 NM ALIGNMENT LASER
VISA EXPERIMENT
LASER LAB CLASS 10000
208 V.
GUN HUTCH
ATRIUM YAG LASER TUBE
YAG TO CO_2 SHUTTER
YAG TO LL1 SHUTTER
DICHROIC BEAMSPLITTER
YAG-FEL SHUTTER
OVERIDE SHUTTER
YAG SHUTTER
YAG OSCILLATOR
YAG TO GUN SHUTTER
FHG 4W
SHG 2W
AMP
PRE-AMP
YAG CLEAN ROOM CLASS 1000
YAG LASER TABLE
YAG LASER TO GUN
Linac
Gun



Equation (55) is solved in the module ANBN to order SECTOR_NMUL. It is solved by an iterative procedure for $\rho_d^{-1} = 1$. In fact, the beginning of the iteration is the usual harmonic solution for straight elements. The solution for an arbitrary ρ_d can be regained due to the fact that each successive iterate depends on one extra power of ρ_d^{-1}. The ugly details can be found in reference [5], Interlude XIV.

In module ANBN, the solution must be tied to a physical definition, a measurement so to speak. Thus we define the usual a_n and b_n as follows:

$$\begin{aligned} b_x^{y=0} &= \sum_{n=1}^{\text{Sector\_Nmul}} a_n x^{n-1} \\ b_y^{y=0} &= \sum_{n=1}^{\text{Sector\_Nmul}} b_n x^{n-1}. \end{aligned} \tag{56}$$

Then, on the basis of Equation (56), V is computed to order SECTOR_NMUL.

Module ANBN provides the template. Each time the multipole components of an exact sector bend are changed, the power series for b_x and b_y are recomputed (BF_X,BF_Y).

The symplectic integration proceeds in the usual two-terms Yoshida for the two possible splits. Finally the full map in PTC for this element is:

$$M(L) = R_{xy}^{-1} \circ F_1^{out} \circ F_2^{out} \circ B(L) \circ F_2^{in} \circ F_1^{in} \circ R_{xy} \tag{57}$$

This is similar to the full map for the exact parallel face bend in Equation (60).

Again we repeat, it is possible to add a wedge to this element. The wedge is an exact sector bend of field BN(1) which modifies the geometry so that the entrance and exit angles can be different. This is described in [5] and it is compatible with MAD. This gymnastic is barely PTC compliant.

To understand the difference between MAD and PTC with regard to true parallel face bends, it is best to look at a simple example. In Figure 19, the two bends displayed have an entrance angle $\varepsilon_1 = 0$ and an exit angle $\varepsilon_2 = \alpha$ where α is the total bending angle. For the first case, LIKEMAD=.TRUE., PTC constructs this bend using a wedge glued to a true Cartesian bend. In PTC this can be done with type TEAPOT, in which case the glueing is done on a sector bend or on type STREX, as it is the case here. These two approaches are not equivalent if there are multipole components in the body of the bend simply because ρ_d-dependent Maxwellian effects must be included in type TEAPOT.

If LIKEMAD=.FALSE., then PTC truly constructs a parallel face bend. One should notice that for this bend the sum of the entrance and exit angles must always be equal to the total bending angle. Moreover the case LIKEMAD=.FALSE. is very interesting since it is a rare case of $L \neq L_c \neq L_d$; it also shows very clearly that L is truly an internal variable whose meaning is detail-dependent while L_c and L_d are layout variables describing the desired geometry of the element. The total map for the magnet is

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