

Beam transmission in isochronous FFAG lattices

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This follows from FFAG tracking tools setting up (scaling as well as non-sc.), as presented at FFAG04/KEK.

Substantial progress about transmission in non-isochronous lattice since then.

Main topics addressed here :

- **3-D simulation of the field in a non scaling FFAG, non-linear - isochronous, optics**
- **Application to the study of beam transmission by multiturn tracking, in**
 - . **muon ring**
 - . **e-model**

1 The ray-tracing method - Ingredients necessary for magnet simulation

- *Position* : $\vec{R}(M_1) \approx \vec{R}(M_0) + \vec{u}(M_0) \Delta s + \vec{u}'(M_0) \frac{\Delta s^2}{2!} + \dots + \vec{u}''''(M_0) \frac{\Delta s^6}{6!} [+ \dots]$ (1)

- *Velocity* : $\vec{u}(M_1) \approx \vec{u}(M_0) + \vec{u}'(M_0) \Delta s + \vec{u}''(M_0) \frac{\Delta s^2}{2!} + \dots + \vec{u}''''(M_0) \frac{\Delta s^5}{5!} [+ \dots]$

using $\vec{u}' = \vec{u} \times \vec{B}$ (Lorentz equation), $\vec{u}'' = \vec{u}' \times \vec{B} + \vec{u} \times \vec{B}'$, $\vec{u}''' = \dots$ etc.

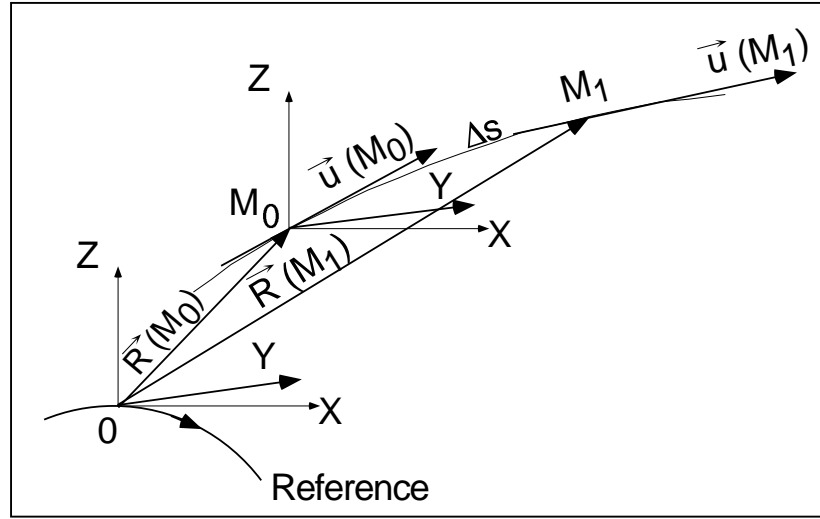


Figure 1: Position and velocity of a particle in the reference frame.

A field model yields the field and its derivatives in the Zgoubi reference frame

$$\vec{B}(X, Y, Z), \quad \partial^{i+j+k} \vec{B} / \partial X^i \partial Y^j \partial Z^k$$

Eventually, the derivatives $d^n \vec{B} / ds^n$ needed in Eqs. 1 are derived from the above using

$$\vec{B}'(s) = \sum_i \frac{\partial \vec{B}(X, Y, Z)}{\partial X_i} u_i(s), \quad \vec{B}''(s) = \sum_{ij} \frac{\partial^2 \vec{B}(X, Y, Z)}{\partial X_i \partial X_j} u_i(s) u_j(s) + \sum_i \frac{\partial \vec{B}(X, Y, Z)}{\partial X_i} u_i'(s) \text{ etc.}$$

($X_{i,j,\dots}$, $i, j, \dots = 1, 3$ stand for X, Y or Z).

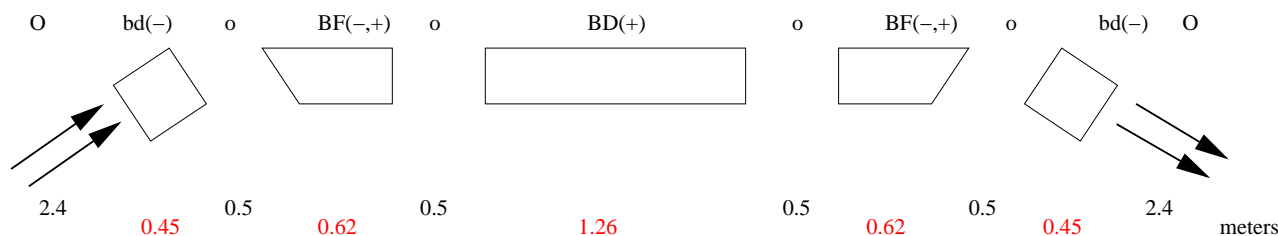
Design of the isochronous cell [Ref. G. Rees]

Isochronism improves even further the gain wrt. RLA, compared with linear FFAG, by more efficient use of RF

Use of a negative bend in the cell allows $\gamma = \gamma_{tr}, \forall \gamma$. Optimization accounts for minimizing transverse aperture

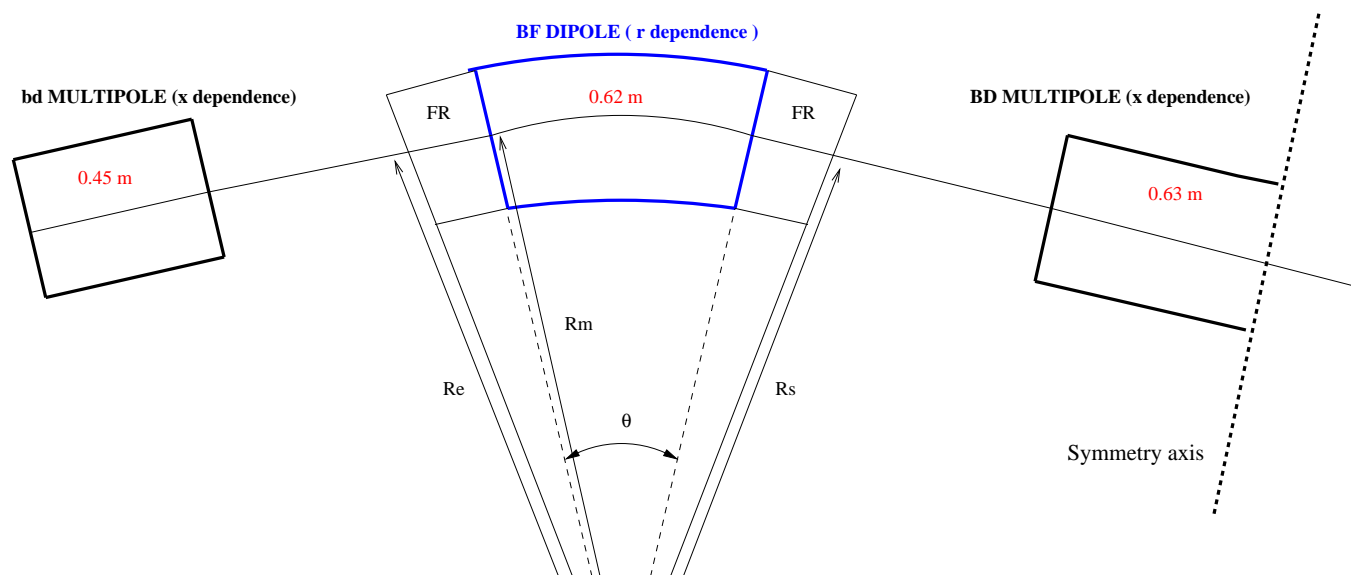
The ring comprises 123 such cells ($C = 1254.6$ m), one (201.20 MHz) cavity every three cells

Acceleration from 8 GeV (dFBFd type cell) to 20 GeV (bFDFb type cell) in 16 turns



In the reference design, BF is a multipole, gradient is dB/dx , whereas in the Zgoubi model :

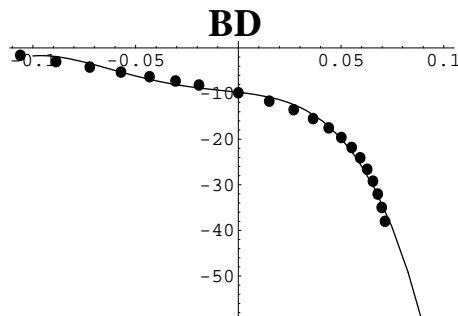
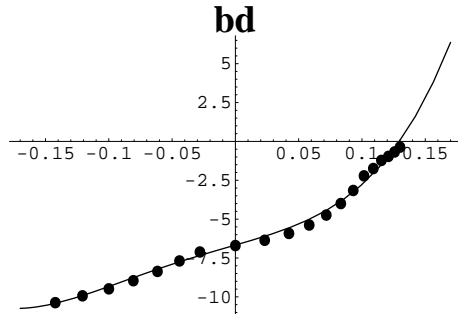
- **BF is a sector magnet,**
 - with gradient dB/dr presumed not to sensibly differ from dB/dx due to the large geometrical radius $R_m \approx 24$ m,
 - with a sector angle value of half the cell deviation $\theta = \frac{1}{2}360/123 = 1.463$ deg.
- In cases where sharp edge magnets are considered, the first order effect of fringe field extent $FR = 0.4$ m on vertical focusing is accounted for via a correction by a vertical kick $\Delta z' = (-\tan(\alpha)/\rho + FR/(6 * \rho^2 * \cos(\alpha)))z$ with ρ being the curvature and α being the angle of the trajectory wrt. the normal to the magnet face.



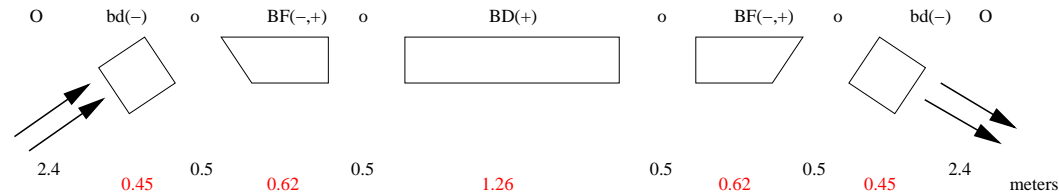
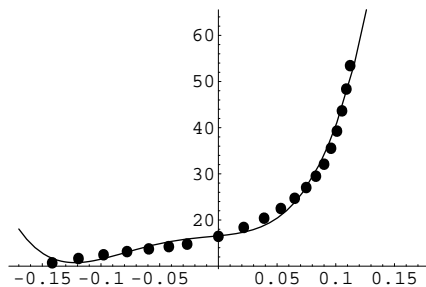
Field modelling

- **bd and BD multipoles :**

Gradient profiles K (m^{-2}) vs. x (m)



- **BF sector magnet :**



The magnets' gradients are constitutive of the design data, they are approximated using 4th degree polynomials.

$$B_{bd}(x) = -6.66771 + 23.5565r x + 11.9699 x^2 + 926.188 x^3 + 4952.98 x^4$$

$$B_{BD}(x) = -9.723 - 51.5803 x - 697.091 x^2 - 33956.1 x^3 - 241808 x^4$$

The gradients are integrated to get the multipole coefficients of the field.

$$B_{bd}(x) = b_{bd0} - 6.66771 x + 11.7783 x^2 + 3.98996 x^3 + 231.547 x^4 + 990.6 x^5 \quad (2)$$

$$B_{BD}(x) = b_{BD0} - 9.723 x - 25.7902 x^2 - 232.364 x^3 - 8489.03 x^4 - 48361.5 x^5 \quad (3)$$

After finding the dipole coefficients b_{bd0} , b_{BD0} with the matching procedure (next slide), the field and its derivatives are derived from the classical multipole modelling of the form.

$$\vec{B} = \text{grad} V_n \text{ with } V_n(s, x, z) = (n!)^2 \left(\sum_{q=0}^{\infty} \frac{(-)^q G^{(2q)}(s)(x^2 + z^2)^q}{4^q q!(n+q)!} \right) \left(\sum_{m=0}^n \frac{\sin(m\frac{\pi}{2}) x^{n-m} z^m}{m!(n-m)!} \right) \quad (4)$$

with $G^{(2q)}(center)$ representing the (derivatives of) the fringe field form factor.

The same gradient matching procedure is applied to obtain

$$B_{BF}(r) = b_{BF0} + 16.5655 r + 12.612 r^2 + 86.4359 r^3 + 2987.43 r^4 + 13647.1 r^5$$

Transform from BF cylindrical frame into Zgoubi Cartesian frame, using

$$\partial B_z / \partial X = (1/r) \partial B_z / \partial \theta, \quad \partial B_z / \partial Y = \partial B_z / \partial r, \quad \partial^2 B_z / \partial X^2 = (1/r^2) \partial^2 B_z / \partial \theta^2 + (1/r) \partial B_z / \partial r, \text{ etc.}$$

Z-derivatives and extrapolation off mid-plane yield the 3-D \vec{B} model

$$\vec{B}(X, Y, Z) \quad , \quad \partial^{i+j+k} \vec{B} / \partial X^i \partial Y^j \partial Z^k$$

Typical Zgoubi input data

```
'MULTIPOL'      bd
  00 02 000
  45 100.00 -3.45374050E+01  -66.6771 117.783 39.8996 2315.47 9905.97 0. 0.0 0.0 0.0
0. 0. 9. 4.  1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4  .1455  2.2670  -.6395 1.1558 0. 0. 0.
0. 0. 9. 4.  1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4  .1455  2.2670  -.6395 1.1558 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
.5 step bd
1 0. 0. 0.
'DRIFT'
50.
'DIPOLES'      BF
  00 02 000
1  1.463414634  24.274311920375e2          nbmag AT/deg, RM/cm
0.731707317  0. -2.25637704 5 -1782.12892 -32935.7018 -5479274.31 -4.59698831E+09 -5.09757397E+11
0. 0.          EFB 1
4  .1455  2.2670  -.6395 1.1558 0. 0. 0.
0.731707317  0. 1.E6 -1.E6 1.E6 1.E6
0. 0.          EFB 2
4  .1455  2.2670  -.6395 1.1558 0. 0. 0.
-0.731707317  0. 1.E6 -1.E6 1.E6 1.E6
0. 0.          EFB 3
0 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. 0.
0  2 64.
.5 step BF          step
2  2.42294098E+03  0. 2.43432058E+03 0.  24.228e2  24.3458e2
'DRIFT'
50.
'MULTIPOL'      BD
  00 02 000
  63. 100.00 4.21503506E+01  -97.23 -257.902 -2323.64 -84890.3 -483615 0. 0. 0. 0.
0. 0. 9. 4.  1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4  .1455  2.2670  -.6395 1.1558 0. 0. 0.
0. 0. 9. 4.  1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4  .1455  2.2670  -.6395 1.1558 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
.5 step BD
1 0. 0. 0.
```

Parameter adjustment

In order to reach desired constraints, prior to tracking simulations, like for instance

- closed orbit angles in the drifts between the magnets as close as possible to the design ones,
- tunes

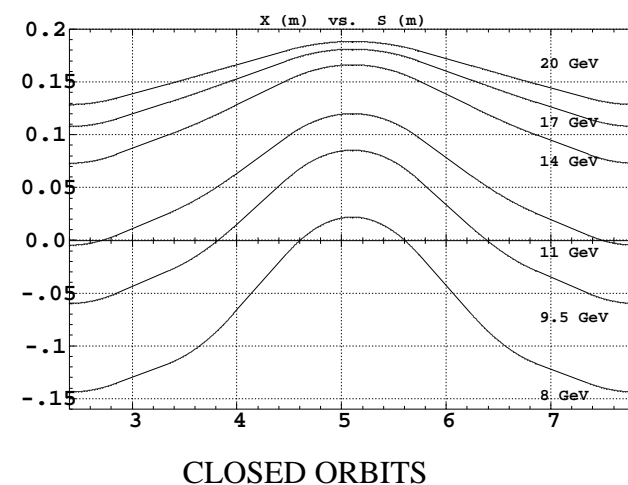
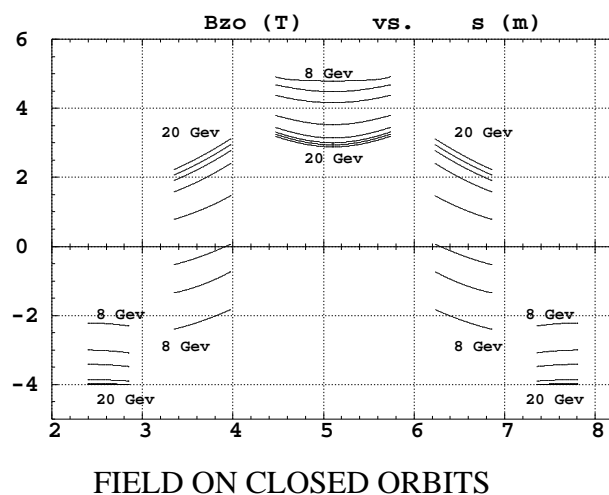
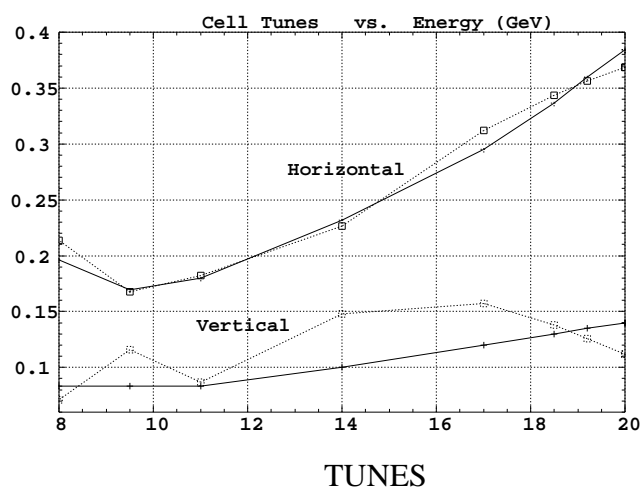
various parameters can be adjusted :

- x_{co} closed orbit coordinates, e.g. at the middle of the drift,
- Dipole coefficients : b_{bd0} , b_{BF0} and b_{BD0} of the polynomial expansion of bd, BF and BD.
- Positioning of bd and BD wrt. BF

Results of these preliminary matchings :

a sample of 3 particles, representative of the 8, 11 and 20 GeV closed orbits, is used for that :

- c.o. angles at all energies take a value in agreement with design data at the level of percent and better
- T.O.F in a cell is 34.1048 ns with isochronicity better than a ps
- the agreement for the tunes is not so tight, yet if tunes need better adjustment they can be constrained harder



New means to the "FIT" procedure

Allows to fit tunes, closed orbits, etc., for sets of given reference energies (up to 5).
 Would also allow precise adjustment of isochronism.

```

STATUS OF VARIABLES (Iteration # 24)
LMNT  VAR  PARAM  MINIMUM  INITIAL  FINAL  MAXIMUM  STEP
1 1 30 9.65 12.3 1.22737818E+01 14.5 3.101E-03
1 2 40 -4.99 -0.471 -4.59424691E-01 4.08 5.725E-03
1 3 50 -17.0 -14.1 -1.41008080E+01 -11.4 3.629E-03
'FIT'
21 21 VARIABLES 4 4 4 -48.4 -34.6 -3.45623000E+01 -20.7 1.719E-02
1 30 0 .2 x_co, energy sample #1 (20 GeV) 4 5 5 -93.6 -67.0 -6.69915923E+01 -40.1 3.316E-02
1 40 0 10. #2 (11 GeV) 4 6 6 56.4 88.4 8.83516654E+01 132. 4.656E-02
1 50 0 .2 #3 ( 8 GeV) 4 7 7 -107. -95.2 -9.52352930E+01 -45.7 3.777E-02
4 4 013.04 .4 bd multipole coefficients 4 8 8 2.162E+03 2.920E+03 2.91679565E+03 5.045E+03 1.78
4 5 013.05 .4 4 9 9 1.007E+04 1.293E+04 1.29085242E+04 2.349E+04 8.29
4 6 013.06 .4 6 10 7 -3.61 -2.58 -2.57760000E+00 -1.55 1.396E-03
4 7 013.07 .4 6 11 9 -2.191E+03 -1.565E+03 -1.56521730E+03 -939. 0.773
4 8 013.08 .4 6 12 10 -9.518E+04 -6.799E+04 -6.79856787E+04 -4.079E+04 33.6
4 9 013.09 .4 6 13 11 -1.233E+07 -8.648E+06 -8.64760350E+06 -5.282E+06 4.348E+03
6 7 011.07 .4 BF multipole coefficients 6 14 12 -3.417E+09 -2.437E+09 -2.43740957E+09 -1.465E+09 1.205E+06
6 9 011.09 .4 6 15 13 -3.511E+11 -2.504E+11 -2.50405569E+11 -1.505E+11 1.238E+08
6 10 011.10 .4 8 16 4 25.3 42.2 4.22034000E+01 59.1 2.097E-02
6 11 011.11 .4 8 17 5 -135. -96.6 -9.65952000E+01 -58.0 4.783E-02
6 12 011.12 .4 8 18 6 -637. -462. -4.62137154E+02 -273. 0.225
6 13 011.13 .4 8 19 7 -4.510E+03 -3.345E+03 -3.34850236E+03 -1.933E+03 1.59
8 4 09.04 .4 BD multipole coefficients 8 20 8 -7.510E+04 -5.484E+04 -5.48881826E+04 -3.219E+04 26.5
8 5 09.05 .4 8 21 9 -3.851E+05 -2.775E+05 -2.77821369E+05 -1.651E+05 136.
8 6 09.06 .4
8 7 09.07 .4
8 8 09.08 .4
8 9 09.09 .4
STATUS OF CONSTRAINTS
TYPE I J LMNT# DESIRED WEIGHT REACHED KI2 [Parameters]
11 11 constraints
3 12 3 4 41.99 1.000 41.87843 3.2838E-02 //
3 12 3 6 71.83 0.5000 71.64159 0.3866 //
3 1 3 8 0.000 5.0000E-02 3.6392768E-03 1.3975E-02 //
3 12 3 8 0.000 5.0000E-02 4.4248091E-03 2.0660E-02 //
3 1 3 8 0. .05 0 exit angle BF 0 7 0 14 0.3600 0.2000 0.3314014 5.3939E-02 //
3 12 3 8 0. .05 0 '' 0 8 0 14 0.1350 2.0000E-02 0.1265732 0.4683 //
3 23 3 8 0. .05 0 '' 0 7 0 14 0.1800 0.2000 0.1840663 1.0904E-03 //
0.1 7 0 14 .36 .1 0 Tunes 0 8 0 14 8.3000E-02 0.2000 8.7017285E-02 1.0643E-03 //
0.1 8 0 14 .135 .02 0 '' 0 7 0 14 0.1960 0.2000 0.2127596 1.8524E-02 //
0.2 7 0 14 .18 .2 0 '' 0 8 0 14 8.3000E-02 0.2000 7.6809106E-02 2.5277E-03 //
0.2 8 0 14 .083 .2 0 ''
0.3 7 0 14 .196 .2 0 ''
0.3 8 0 14 .083 .2 0 ''
Function called 1186 times
'END' Xi2 = 0.379016 Busy...
    
```

Stability limits :

Two goals :

1. Check symplecticity of the motion over the all energy span.
2. Find the maximum stable amplitudes in both planes, as well as coupled

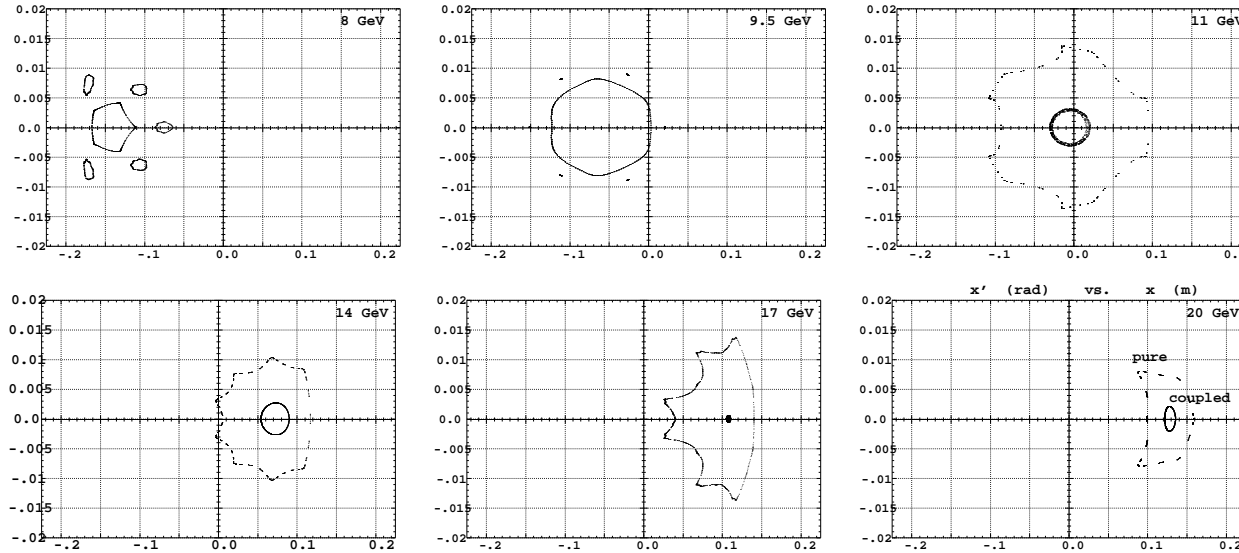


Figure 2: 1000-cell, stability limits of horizontal motion, either pure or with a paraxial z component. Precision is better than 1 mm.

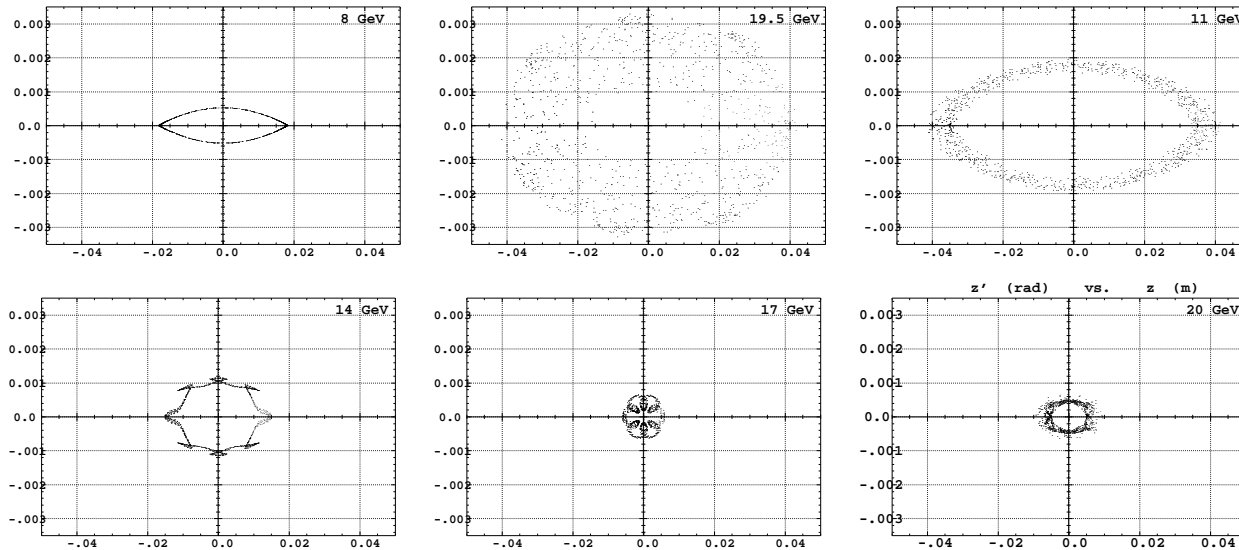


Figure 3: 1000-cell or more, vertical motion stability limits, at better than 1 mm precision.

Amplitude detuning

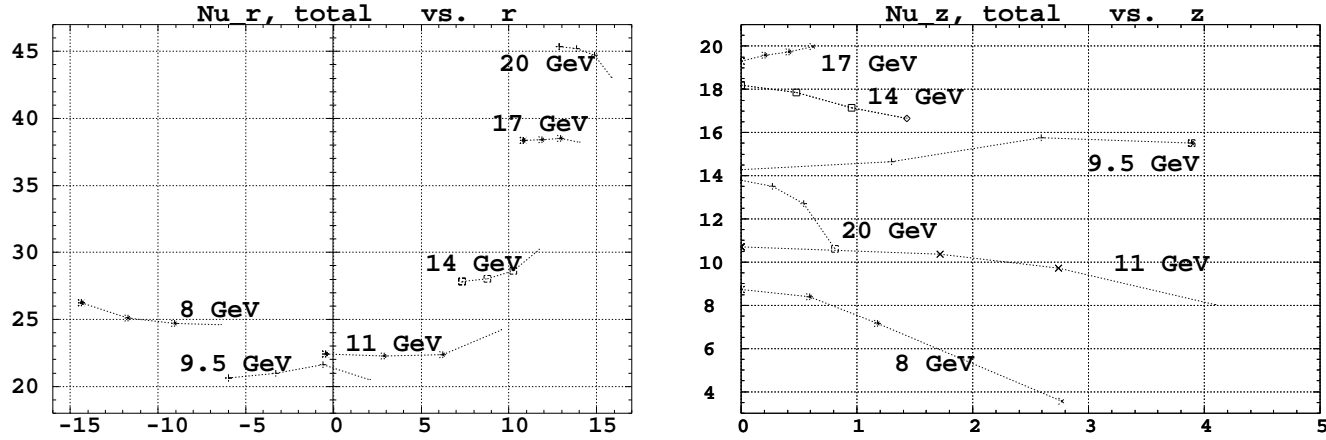


Figure 4: Amplitude detuning, total tunes. Left : pure radial motion, with x_0 varied from closed orbit position to maximum stable amplitude. Right : axial motion, with z_0 varied from zero to maximum stable amplitude while $x \equiv x_{c.o.}$ always.

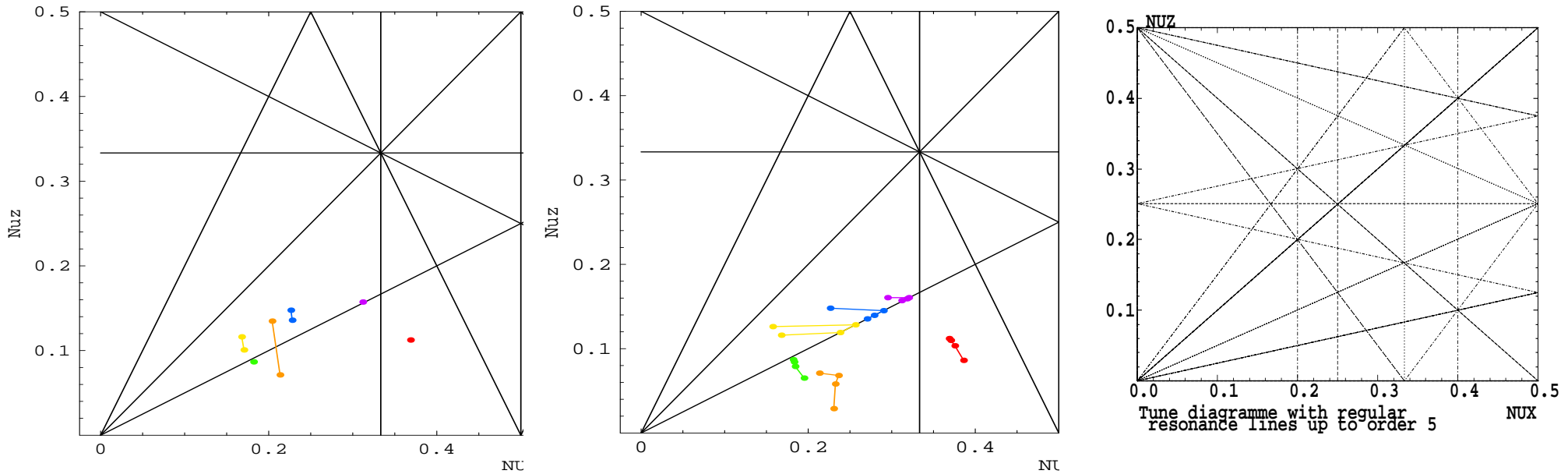


Figure 5: Cell-tune footprint accounting for horizontal (left) or vertical (right) amplitude. E = 8GeV, 9.5GeV, 11GeV, 14GeV, 17GeV, 20GeV.

Beam transmission (1)

A 10000 particles beam is launched for 16 turn acceleration (123 cells/turn), from 8 to 20 GeV - **no decay !**
 18.3 MV per cavity, constant ΔE - no phase effect. Cavities are put every three cells at the center of the drift.

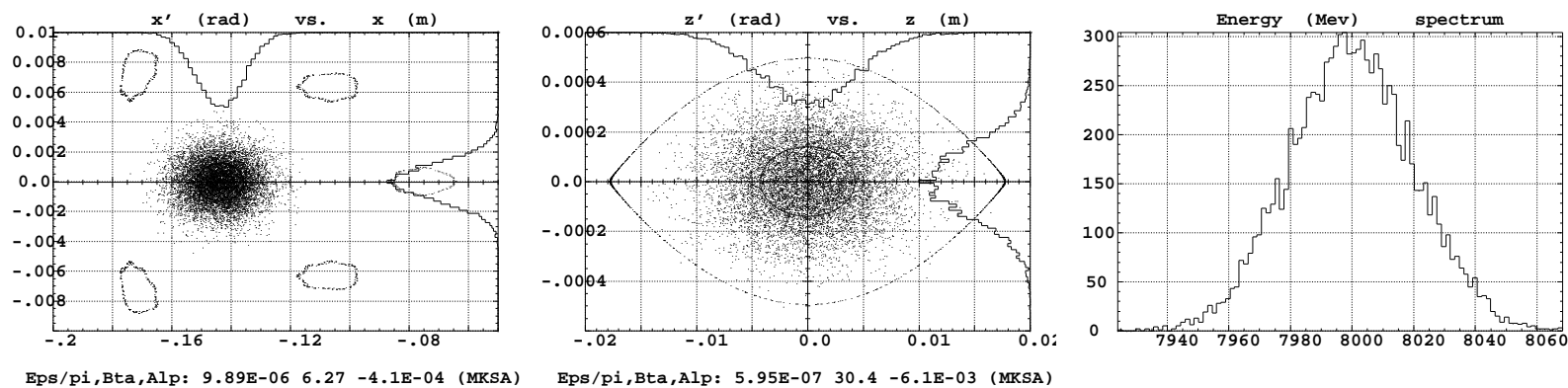


Figure 6: Initial phase spaces (with stability limits shown for comparison), and energy dispersion (right plot).

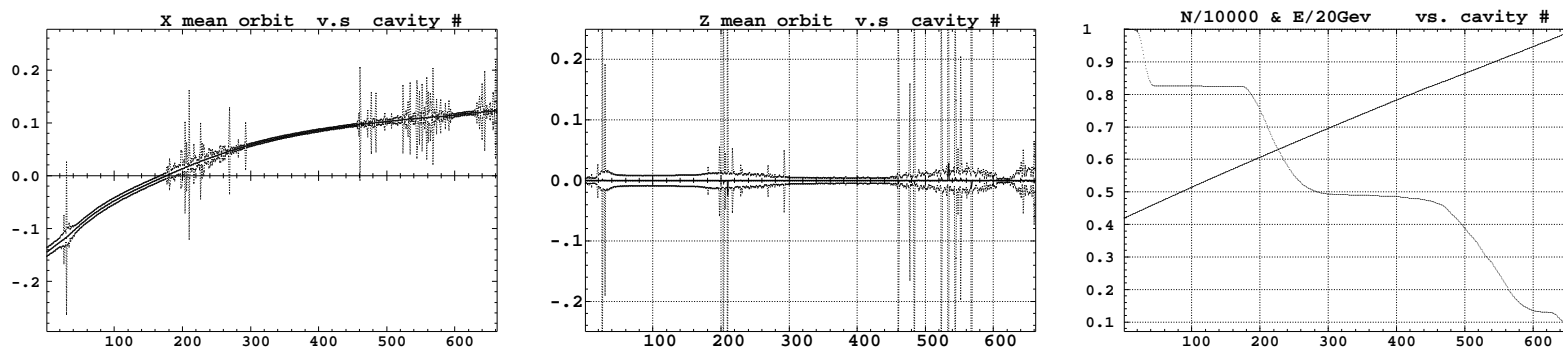


Figure 7: Left two plots : beam centroid position and envelopes, $\bar{y} \pm \sqrt{x^2 - \bar{x}^2}$ (y=x or z). Right : transmission.

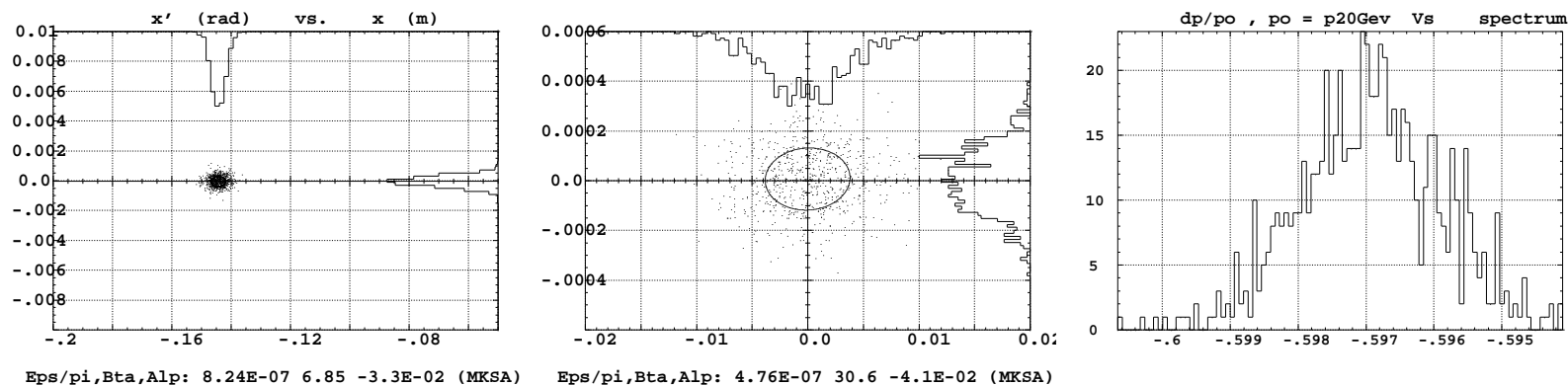


Figure 8: Initial phase spaces of transmitted particles.

Beam transmission (2)

A 5000 particles beam is launched within the acceptances obtained with the previous run, and accelerated from 8 to 20 GeV.

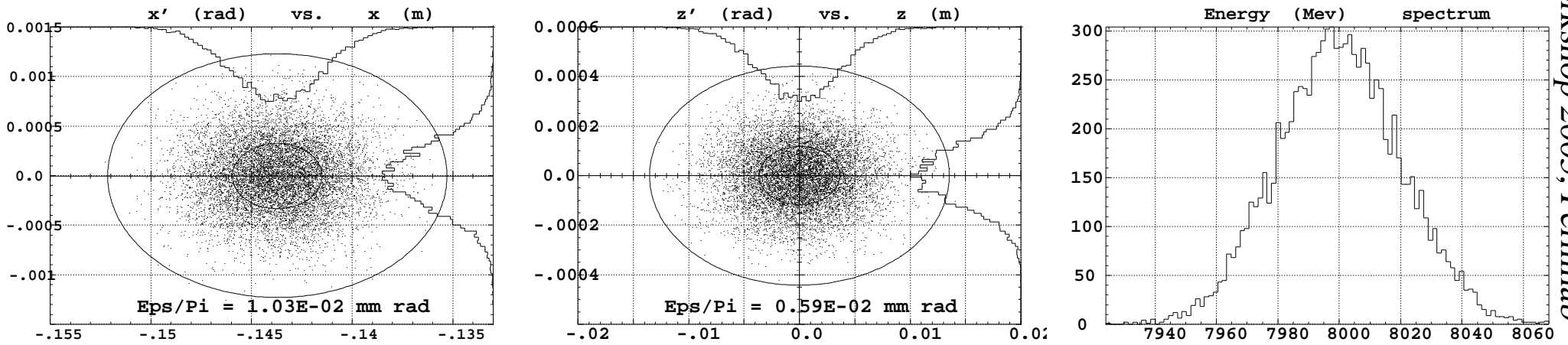


Figure 9: Initial phase spaces and energy dispersion.

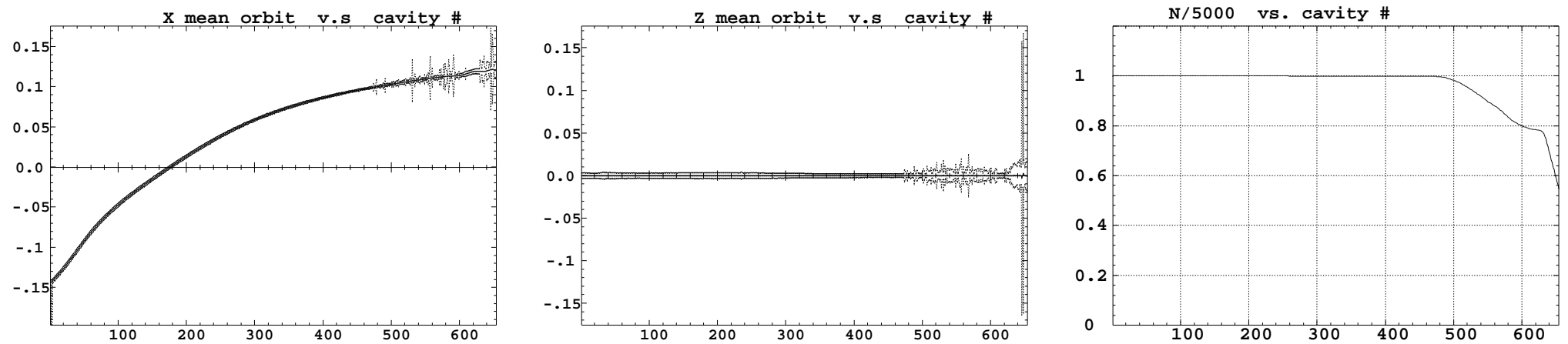


Figure 10: Envelopes and transmission.

Correlation of beam losses and tunes

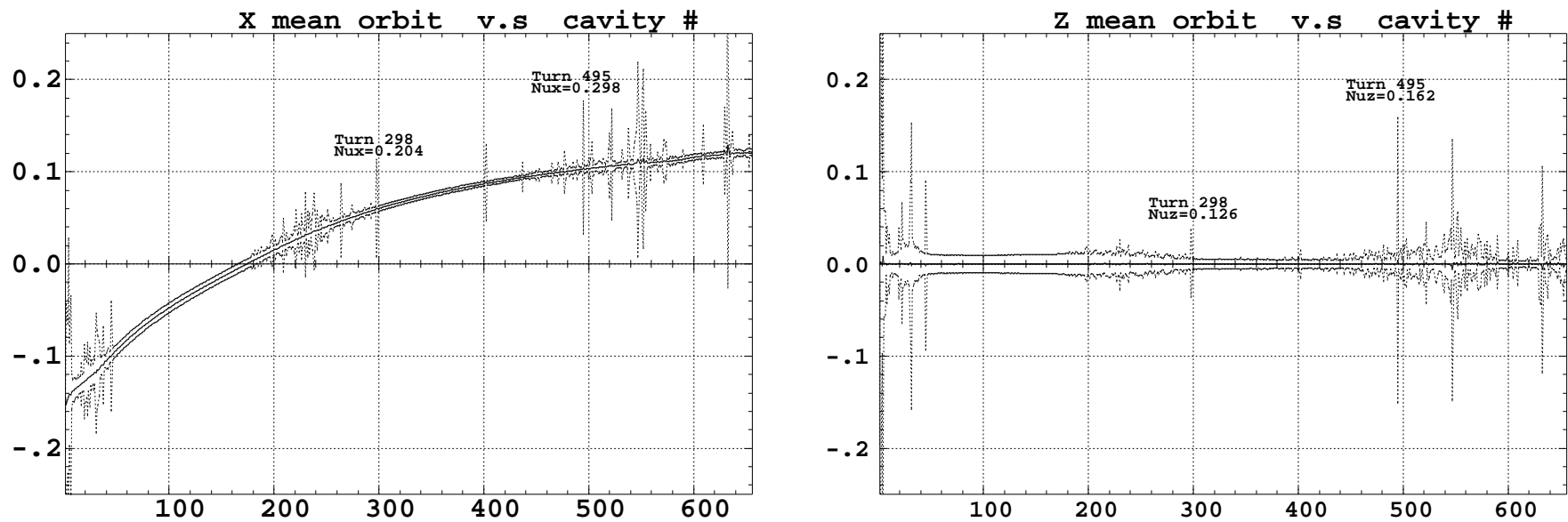
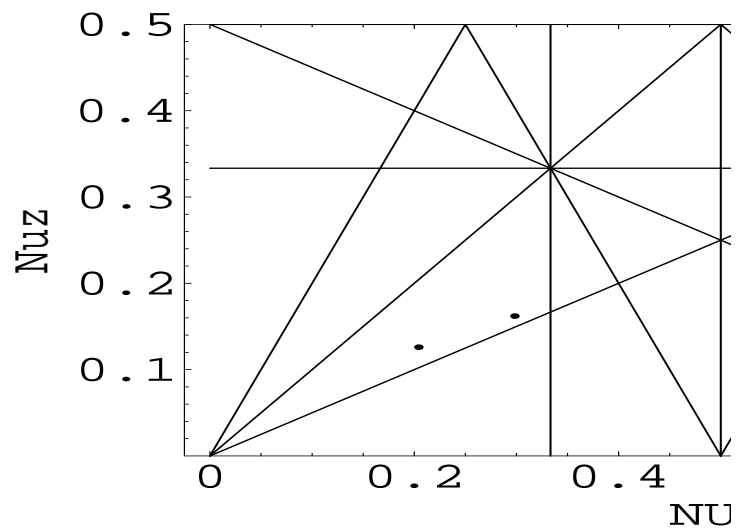
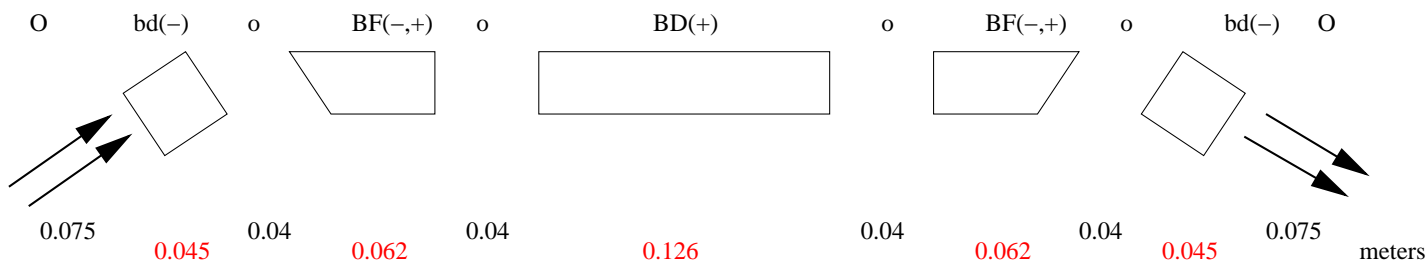


Figure 11:



Electron isochronous ring

45 cells, $\mathcal{C} = 29.25$ m, 11 to 20 MeV in 15 turns. 15 cavities, 3 GHz ($h=293$), 40 keV



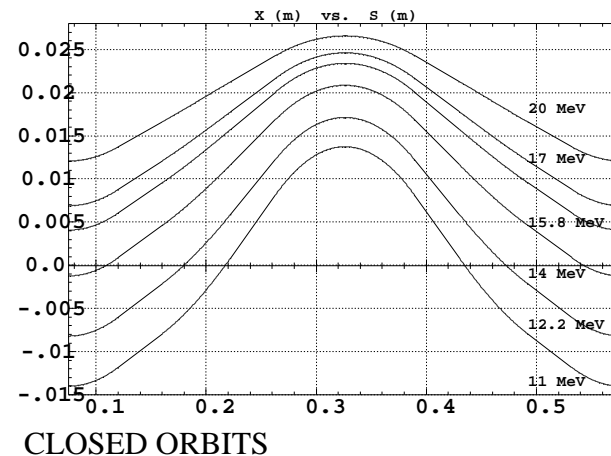
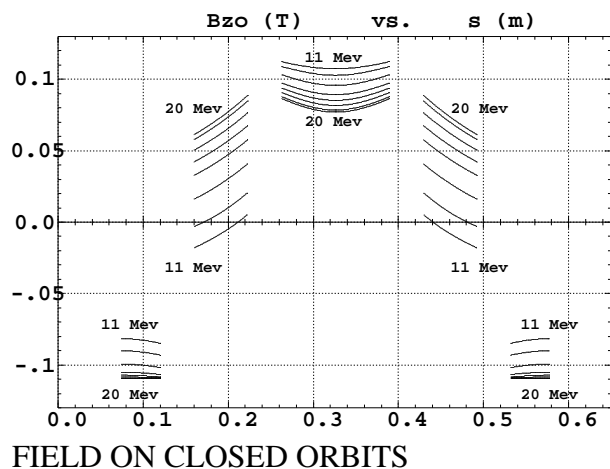
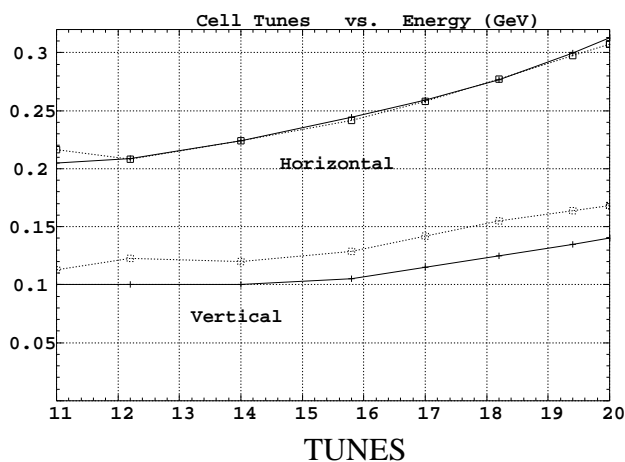
Multipole expansion of the field (obtained as for the muon lattice):

$$B_{bd}(x) = -0.1011 - 1.16096 x + 26.3975 x^2 + 1116.62 x^3 + 6176.79 x^4 - 1.154710^6 x^5$$

$$B_{BF}(r) = 0.028150 + 3.46395 r + 43.2365 r^2 + 508.637 r^3 + 137733 r^4 + 7.895810^6 r^5$$

$$B_{BD}(x) = 0.098782 - 1.92981 x - 84.1762 x^2 - 3352.2 x^3 - 527101. x^4 - 3.4456510^6 x^5$$

Results :



Electron isochronous ring, beam transmission (1)

A 10000 particles beam is launched for 15 turn acceleration (45 cells/turn), from 11 to 20 MeV.
 40 kV per cavity, constant ΔE - no phase effect. Cavities are put every three cells at the center of the long drift.

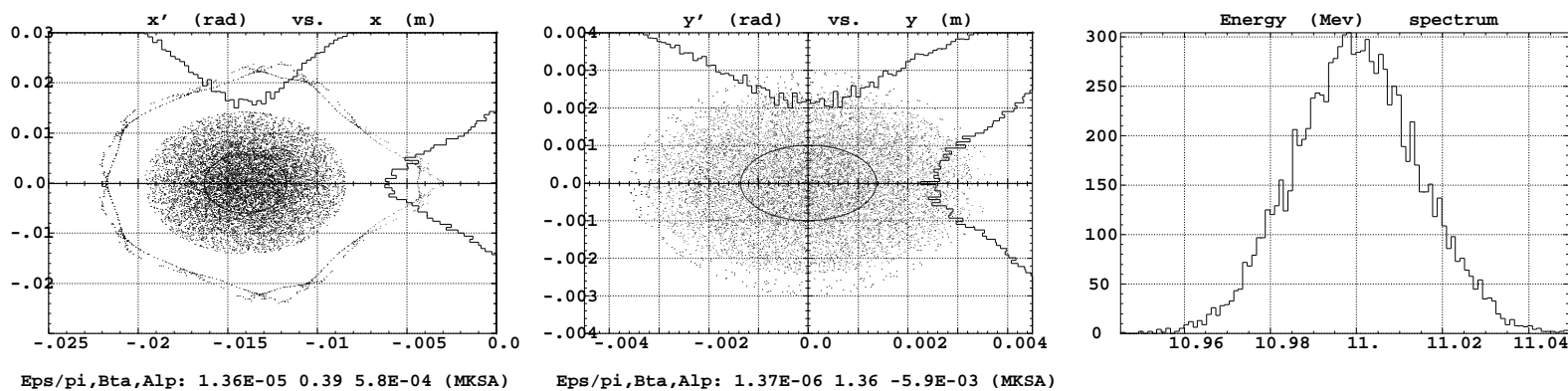


Figure 12: Initial phase spaces and energy dispersion.

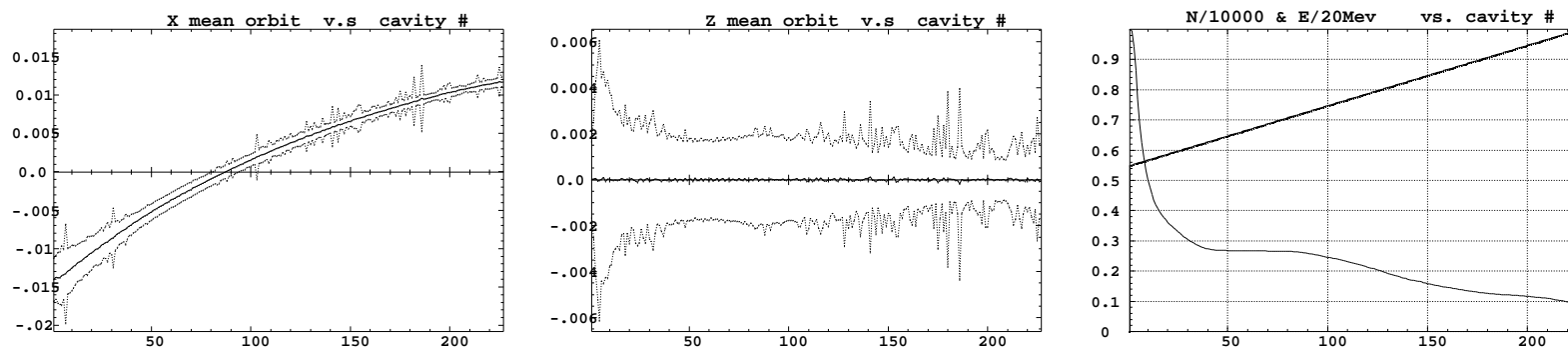


Figure 13: H and V beam centroid and envelopes, and transmission.

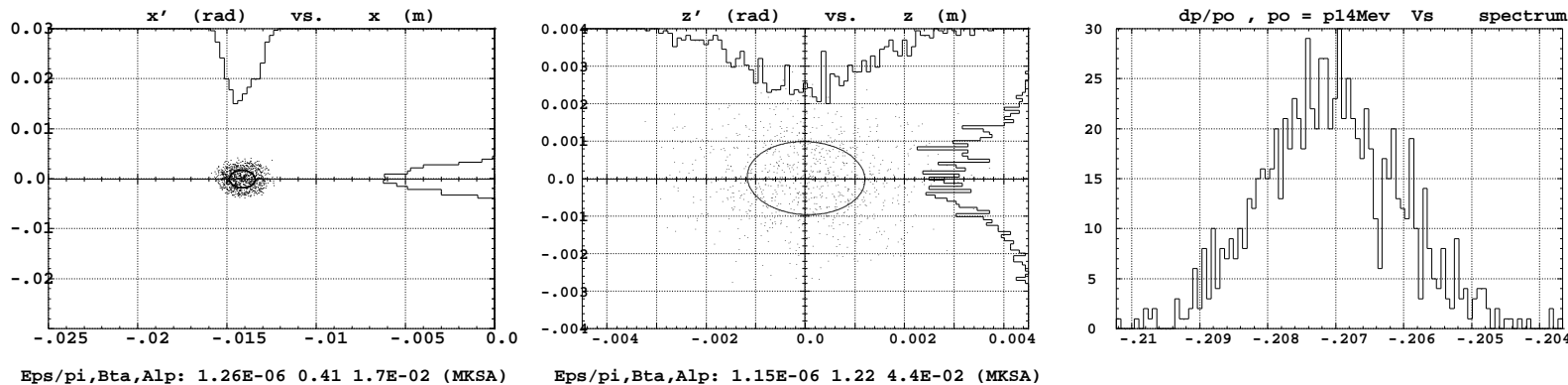


Figure 14: Initial phase spaces of those particles that make it to the end.

Electron isochronous ring, beam transmission (2)

A 10000 particles beam is launched for 15 turn acceleration inside the acceptances obtained with the previous run.

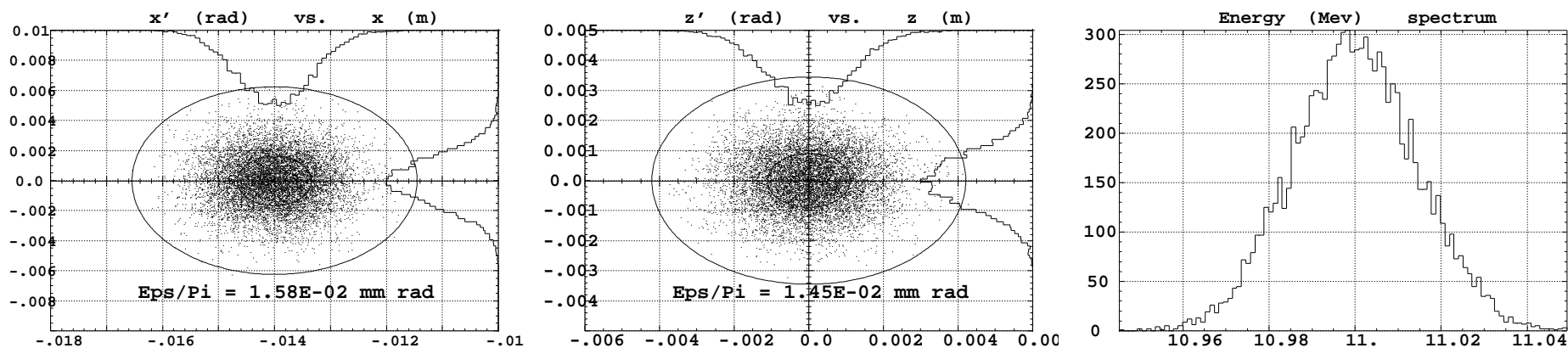


Figure 15: Initial phase spaces

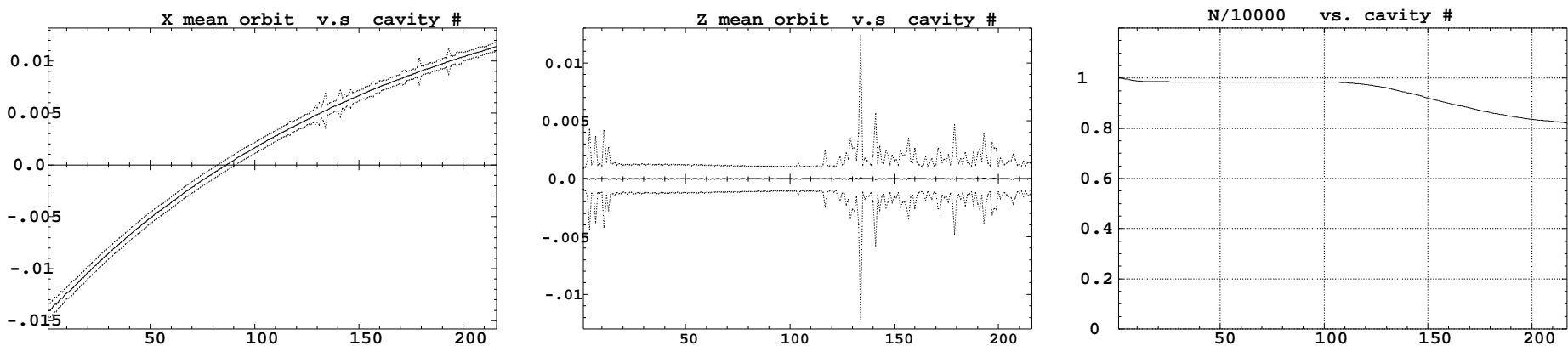


Figure 16: Envelopes.

Future

- Pursue resonance study

- Use 201 MHz acceleration including RF phase

- already done with non-isochronous lattice (Carol's, doublet, FFAG03)

- Introduce fringe fields (some assessments already, no strong effect seen for the moment)

• Field and positioning defect studies, including (again) dynamic aperture and full cycle transmission (this means a lot of runs and of CPU time per run...)

