Status of Study of Spin Dynamics in Electrostatic Rings to Search Electric Dipole Moment

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Contents

Our goal

Track particle with initial horizontal spin polarization for a very large number of orbits, say 10^9, to detect the appearance of a vertical spin component that will indicate the presence of an electric dipole moment. EDM of proton $d_p < 10^{-26} e \cdot \text{cm}$.

So we need a storage ring which will conserve horizontal spin polarization for a long time.

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Spin coherence time

SCT

Spin coherence time (SCT) — time when RMS spin orientation of the bunch particles reaches one radian.

Requirements of planned SrEDM experiment: the SCT should be more than 1000 seconds.
During this time each particle performs about 10^9 turns in storage ring moving on different trajectories through the optics elements.

T-BMT equation

$$\frac{d\vec{S}}{dt} = \mu\vec{S} \times \left(\vec{B} - c\beta \times \vec{E}\right) + d\vec{S} \times \left(\vec{E} + \vec{\beta}c \times \vec{B}\right)$$

$\mu,\ d$ — magnetic and electric dipole moments, c is the speed of light, β is the relative velocity and $E,\ B$ — the electric and the magnetic field vectors.

$$\begin{aligned} \frac{d\vec{S}}{dt} &= \vec{\omega}_G \times \vec{S} \\ \vec{\omega}_G &= -\frac{e}{m_0\gamma c}\left\{Gc\vec{B} - \left(G - \frac{1}{\gamma^2 - 1}\right)\left(\vec{\beta} \times \vec{E}\right)\right\} \\ G &= \frac{g-2}{2}, \end{aligned}$$

G — the anomalous magnetic moment, g is the gyromagnetic ratio and ω_G is the spin precession frequency.

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Frozen Spin Method

Consider $\gamma = \gamma_{mag}$:

$$\frac{1}{\gamma_{mag}^2-1} - G = 0,$$

and $\omega_G = 0$. Proton "magic" energy is 232 MeV.

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Spin oscillation of non-magic particle

If $\gamma \neq \gamma_{mag}$:

$$\vec{\omega}_G = -\frac{e}{m_0 \gamma c}\left\{-2G\frac{\Delta p}{p}\left(\vec{\beta} \times \vec{E}\right)\right\}.$$

Spin oscillation tune ν_{sz} satisfies:

$$S_z = S_{z_0} \cos 2\pi\nu_{sz} n,\ \nu_{sz} = \frac{e\overline{E}_x L_{cir}}{2\pi m_0 c^2 \gamma} G \frac{\Delta p}{p},$$

where $\overline{E}_x$ is the average value of the deflecting electric field.
If $(\Delta p/p)_{\max} = 10^{-4}$, then $\nu_{sz} = 1.588 \cdot 10^{-4}$, or
SCT $= 6300$ turns $\approx$ 1msec.

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SCT $= 6300$ turns ≈ 1msec.

Using of RF cavity to increase SCT

With $(\Delta p/p) = (\Delta p/p)_{\max} \cos(\nu_z \varphi)$ equation describing the oscillation of the spin:

$$\frac{d^2 S_z}{d\varphi^2} + \left\{ \frac{e\overline{E}_x L_{cir}}{2\pi m_0 c^2 \gamma} 2G \left(\frac{\Delta p}{p}\right)_{\max} \cos(\nu_z \varphi) \right\}^2 S_z = 0.$$

The spin vibrates within a very narrow angle $\Phi_{\max}$ with longitudinal tune $\Phi \sim \Phi_{\max} \sin(\nu_z \varphi)$. The value $\Phi_{\max} \sim (\nu_{sz}/\nu_z)^2$ depends on the frequency
ratio.

Equilibrium energy level modulation

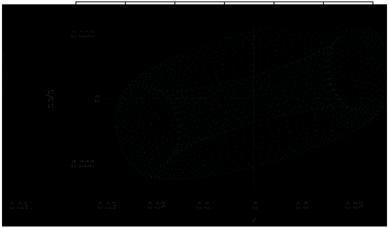

Figure: Phase trajectory in longtitudinal plane for initial coordinate $x = 3\text{mm}, y = 0$.

If $(\Delta p/p)_{max} = 10^{-4}$ and the beam emmitance $2\text{mm} \cdot \text{mrad}$, SCT will be ~ 500 sec.

The second order influence on spin

The spin tune in the second approach versus momentum:

$$\frac{d^2 S_z}{d\varphi^2} + \left\{ \frac{e\overline{E}_x L_{cir}}{2\pi m_0 c^2 \gamma} \left[-2G\frac{\Delta p}{p} + \frac{1+3\gamma^2}{\gamma^2} G \left(\frac{\Delta p}{p} \right)^2 \right] \right\}^2 S_z = 0$$

and

$$\nu_{sz} = \frac{e\overline{E}_x L_{cir}}{2\pi m_0 c^2 \gamma} \left\langle -2G \left(\frac{\Delta p}{p} \right)_m \cos(\nu_z \varphi) + \frac{1+3\gamma^2}{\gamma^2} G \left(\frac{\Delta p}{p} \right)_m^2 \cdot \cos^2(\nu_z \varphi) \right\rangle = \frac{e\overline{E}_x L_{cir}}{2\pi m_0 c^2 \gamma} \frac{1+3\gamma^2}{\gamma^2} \frac{G}{2} \left(\frac{\Delta p}{p} \right)_m^2$$

Variation of spin tune

Assuming "magic" condition we define a variation of the spin tune through the finite differences up to second order:

$$\delta\nu_S = \frac{e}{2\pi m_0 c^2}\delta\left(\frac{1}{\gamma^2-1} - G\right) L_{\text{orb}} E_x \frac{1}{\gamma}\left[1 + \frac{\delta L_{\text{orb}}}{L_{\text{orb}}} + \frac{\delta E_x}{E_x} + \gamma\delta\left(\frac{1}{\gamma}\right)\right],$$

$$\delta\left(\frac{1}{\gamma^2-1} - G\right) = -2G\frac{\Delta p}{p} + \frac{1+3\gamma^2}{\gamma^2}G\left(\frac{\Delta p}{p}\right)^2 + \dots$$

$$\frac{\delta L_{\text{orb}}}{L_{\text{orb}}} = \alpha_1\frac{\Delta p}{p} + \alpha_2\left(\frac{\Delta p}{p}\right)^2 + \dots$$

$$\frac{\delta E_x}{E_x} = -k_1\frac{x}{R} + k_2\left(\frac{x}{R}\right)^2 + \dots$$

$$\gamma\delta\left(\frac{1}{\gamma}\right) = -\frac{\gamma^2-1}{\gamma^3}\left(\frac{\Delta p}{p}\right) + \frac{(\gamma^2-1)^2}{2\gamma^5}\left(\frac{\Delta p}{p}\right)^2 + \dots$$

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Grouping the coefficients we have:

$$\begin{aligned}\delta\nu_S \quad = \quad & \frac{eL_{\text{orb}}E_x}{2\pi m_0\gamma c^2}G\left\{-2\frac{\Delta p}{p}+\left(\frac{\Delta p}{p}\right)^2\left[\frac{5\gamma^2-1}{\gamma^2}-2\alpha_1-\right.\right.\\ & \left.-\frac{1+3\gamma^2}{\gamma^2}k_1\frac{x}{R}+\frac{1+3\gamma^2}{\gamma^2}k_2\left(\frac{x}{R}\right)^2\right]+\\ & \left.+2\frac{\Delta p}{p}\left[k_1\frac{x}{R}-k_2\left(\frac{x}{R}\right)^2\right]\right\}\end{aligned}$$

Variation of spin tune

Finally we have

$$\delta\nu_S = \frac{eL_{\text{orb}}E_x}{2\pi m_0\gamma c^2} G \left[F_2\left(\alpha_1, k_1, k_2, \frac{x}{R}\right)\left(\frac{\Delta p}{p}\right)^2 + 2F_1\left(k_1, k_2, \frac{x}{R}\right)\frac{\Delta p}{p}\right]$$

Or the spin tune equation can be represented in another form:

$$\delta\nu_S = \frac{eL_{\text{orb}}E_x}{2\pi m_0\gamma c^2} G \left[\tilde{F}_2\left(k_2, \frac{\Delta p}{p}\right)\left(\frac{x}{R}\right)^2 + 2\tilde{F}_1\left(k_1, \frac{\Delta p}{p}\right)\frac{x}{R} + \tilde{F}_0\left(\alpha_1, \frac{\Delta p}{p}\right)\right]$$

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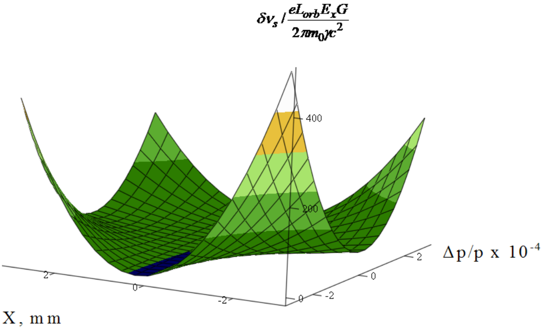

Figure: Two dimensional parabolic dependence of spin tune aberration on $\left(\frac{\Delta p}{p}\right)^2$ and $\left(\frac{x}{R}\right)^2$

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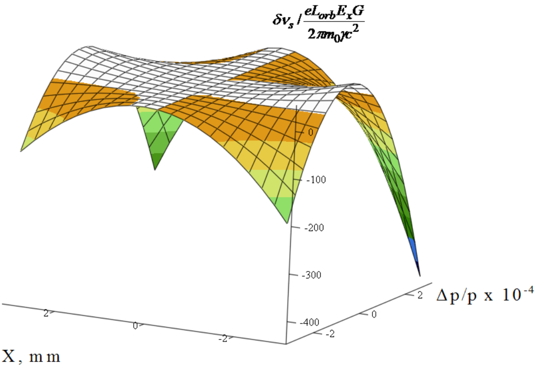

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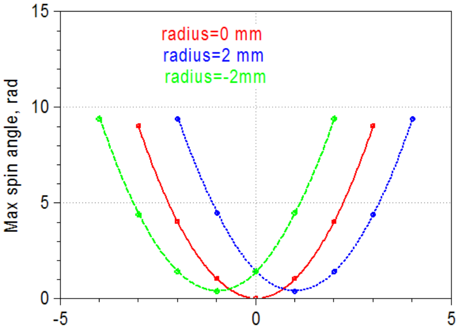
Cylindrical deflectors

We studied how the SCT depends on initial conditions in a structure without optimization with $k_1 = 1$, $k_2 = -1$. Figure shows the maximum spin deflection angle after 10^9 turns for different initial horizontal deviations.



Cylindrical deflectors

The maximum angle for spin particles with initial $x = 2$mm can be up to 10 radians even at the momentum deviation $\Delta p/p = 10^{-4}$. It means that spin of the particle will make ≈ 1.5 turnovers during this time, or RMS spin deviation is about 2 radians, which corresponds to a SCT about 500 seconds.

Methods to increase SCT

Our goal is to achieve maximum flatness in the working range of the beam parameters.

- The first method is to fit the parameters of electrical deflector and ring lattice in order to reduce this dependence that is to choose the lattice with a compensation of the mutual influence of all parameters.
  In other words, we need to make the surface maximally flat in the workspace of $(\Delta p/p)^2$ and $(x/R)^2$.
- The second method is to alternately change the deflector parameters and thereby to alternate spin rotation.

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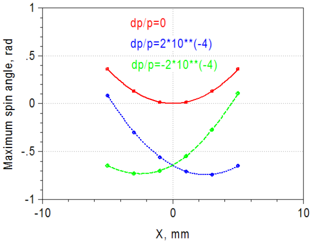

Figure: Maximum spin deflection angle after 10^9 turns versus x deviation

Maximum spread spin angle for the entire beam is less than 1 radian, or RMS deviation is about 0.2 radians, which corresponds to SCT about 5000 seconds.

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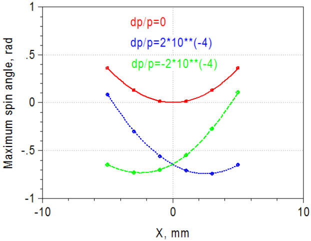

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Alternating spin aberration lattice

The ring is equipped with two types of deflector with $k_1 = \text{const}$, and $k_2 = k_{av} \pm \delta k$ changing from one deflector to another.

- In such optics is easier to achieve minimum spin aberration
- Raising the field strength between the plates in even deflectors and reducing in the odd deflectors it effectively adjust the required coefficients k_1 and k_2. It allows to adjust the spin of aberration to minimum.

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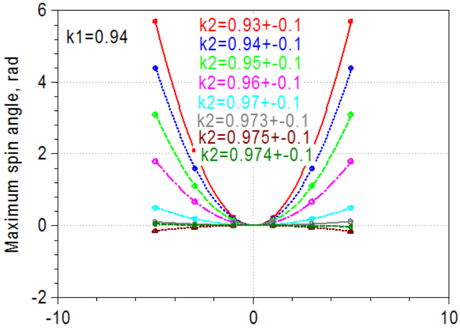

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Thank you for your attention.