IEEE 1788 Working Group for the Standardization of Interval Arithmetic a brief overview

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TMW VII - 14-17 December 2011 - Key West

Agenda

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- Interval Arithmetic
  - Important facts that underlie many discussions and decisions
- Standardization of interval arithmetic: IEEE P1788
  - Facts about the working group
  - Motions and topics of discussion
  - Motions adopted
  - Motions not adopted
  - Personal view
  - Exception handling
- Conclusion
- More on decorations

Pros and cons of interval arithmetic

Pros:

- **"thou shalt not lie":** guarantee that the result belongs to the resulting interval;
- **computing in the large:** computing with whole set, global optimization;
- **Brouwer theorem made effective:** if $f(K) \subset K$ then f has a fixed point in K. As this can be checked, existence and uniqueness can be proven.

Cons:

- **implementation** requires a specific algorithm, not only changing *float* into *interval*;
- **overestimation** that can make a computed interval (much) wider than the exact range of the mathematical function on the same input interval.

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The P1788 Project

Belief: interval arithmetic is mature enough to undergo a common definition.

Goal: standardize interval arithmetic.

Creation: Initiated by 15 attenders at Dagstuhl, Jan 2008.
Project authorised as IEEE-WG-P1788, Jun 2008.

How P1788's work is done

- The bulk of work is carried out by email, with electronic voting.
- Motions are proposed, seconded; three weeks discussion period; three weeks voting period.
- IEEE has given us a four year deadline. . . which expires soon, we will ask for a 2-years extension.
- One "in person" meeting per year is planned—last one was be July 25th, 2011, in Tübingen, Germany, during the Arith 20 conference.
- IEEE auspices: 1 report + 1 teleconference quarterly

IEEE-1788 WG: some facts

Since October 2008: **very active mailing list**
over 150 participants, over 20 nationalities, over 4400 messages

Work already done:
adoption of officers, of procedures and policy
roster of (voting or not) members: 88 members, 18 nationalities
31 motions handled.

URL of the working group:
`http://grouper.ieee.org/groups/1788/`
or google **1788 interval arithmetic**.

Motions discussed so far

1	Provisional standard notation for intervals
2-14	Levels structure for standardisation process
3	Standard is based on $\mathbf{R}$ not $\mathbf{R}^*$
4-24	Restrict standard to 754 systems, rounded operations
5-10	Arithmetic operations and elementary functions
6	Multi- & mixed-format interval support
9	Exact dot product
11-12	Reverse Arithmetic Operations
13-14-20	Comparison Relations
21	Overlapping intervals
16-19-23	inf/sup and mid/rad
17	IO
7-8-15-18-22-25-26-27	Exception handling: decorations
28	Containment only

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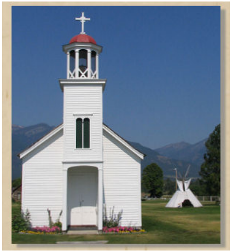
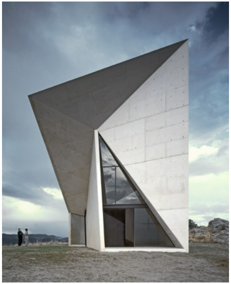
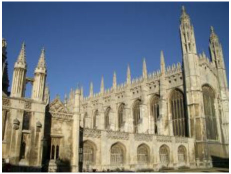
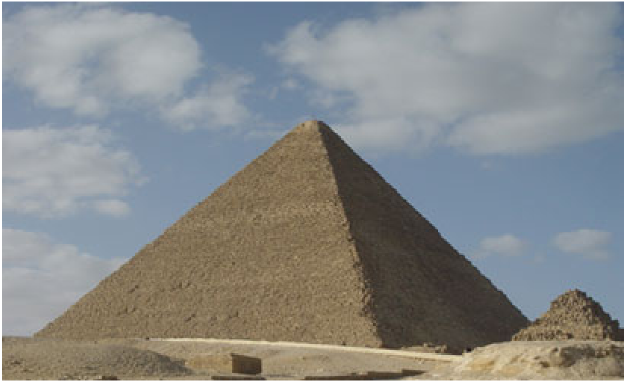
"Quarrels of chapels" - Parochial quarrels







My dream



My feelings last summer

Exception handling

Exceptions must be handled in some way, even if exceptions do look. . . exceptional. (It must have been the same for exception handling in IEEE-754 floating-point arithmetic.)

Best way to handle exceptions? To avoid global flags, flags attached to each interval: decorations.

Decorated intervals 

Discussions about what should be in the decorations (defined and continuous, defined, no-information, nowhere defined).

Exception: arguments outside the domain

How should $f(\boldsymbol{x})$ be handled when $\boldsymbol{x}$ is not included in the domain of f? E.g. $\sqrt{[-1, 2]}$?

- exit?
- return NaI (Not an Interval)? Ie. handle exceptional values such as NaI and infinities?
- return the set of every possible limits $\lim_{y \to x} f(y)$ for every possible x in the domain of f (but not necessarily y)?
- intersect $\boldsymbol{x}$ with the domain of f prior to the computation, silently?
- intersect $\boldsymbol{x}$ with the domain of f prior to the computation and mention it

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To conclude

The IEEE committee will have to

- complete the list of things that have to be part (or not) of the standard, and how they are part of it
  and you can help us!
- discuss every point, its pro and cons (using counterexamples)
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- agree on the most sensible choice. . .
  and then you will vote to tell us if we were right!

See you in 2 (or 4, or 6) years time, to introduce you the new standard!

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Motions 7, 8, 15 and 18: Exceptions

Issue: How to handle exceptions efficiently.

- Typical examples:
  - (a) Invalid interval constructor
    `interval(3,2)`   `interval("[2.4,3;5]")`
    —interface between interval world and numbers or text strings.
  - (b) Elementary function evaluated partly or wholly outside domain
    `sqrt([-1,4])`   `log([-4,-1])`   `[1,2]/[0,0]`
- Type (a) can simply cause nonsense if ignored.
- Type (b) are crucial for applications that depend on fixed-point theorems; but can be ignored by others, e.g. some optimisation algorithms.

Motions 7 and 8: Exceptions, cont.

What to do? A complicated issue.

- Risk that (Level 3) code to handle exceptions will slow down interval applications that don't need it.
- One approach to type (a) is to define an NaI "Not an Interval" datum at level 2, encoded at level 3 within the two FP numbers that represent an interval.

Motion 8: Exceptions by Decorations

- Alternative (Motion 8): An extra tag or decoration field (1 byte?) in level 3 representation.
- Divided into subfields that record different kinds of exceptional behaviour.
- Decoration is optional, can be added and dropped.
  – To compute at full speed, use "bare" intervals and corresponding "bare" elementary function library.
  – "Decorated" library records exceptions separate from numbers, hence code has fewer IFs & runs fast too. (We hope!)

Motions 8, 15 and 18: Decoration issues

Decorations look promising but many Qs exist:

- Bare (double) interval is 16-byte object. Decoration increases this. Can compilers efficiently allocate memory for large arrays of such objects?
- Some proposed decoration-subfields record events in the past; others are properties of the current interval. Can semantic inconsistencies arise?
- Can decoration semantics be specified at Level 2 ...
- ...such that correctness of code can be proven ...
- ...and K.I.S.S. is preserved?

Much work on exceptions remains: list, order...

Remark: arguments outside the domain

Problematic example (Rump, Dagstuhl seminar 09471, 2009).

$$\begin{aligned} f(x) &= |x-1| \\ g(x) &= (\sqrt{x-1})^2 \end{aligned}$$ I know, it is not the best way of writing it...

What happens if $\boldsymbol{x} = [0, 1]$?
With the adopted definitions of operations,

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Without exception handling, **the Thou shalt not lie principle is not valid.**
One has to check whether there has been a possibly undefined operation... Unexperienced programmers will not do it.

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