Taylor Model Methods VII

Casa Marina Hotel

Key West, Florida, December 14-17, 2011

POLITECNICO DI MILANO

Collision risk assessment for perturbed orbits via rigorous global optimization

A. Morselli, R. Armellin, P. Di Lizia, F. Bernelli-Zazzera

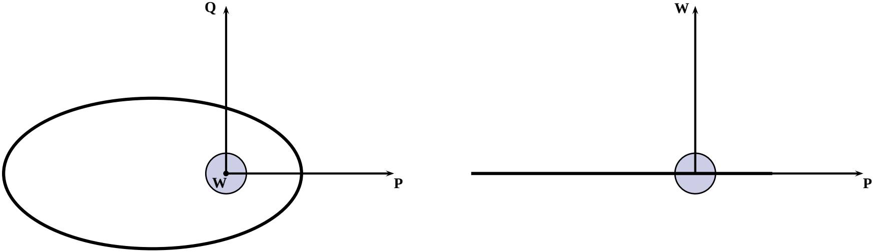
Dipartimento di Ingegneria Aerospaziale, Politecnico di Milano, Milano, Italy

Motivation

- High number of threatening objects orbit the Earth
- In-orbit collisions increase debris amount
- **Goal:** identify intersecting orbits
- Intersection criteria: Minimum Orbit Intersection Distance (**MOID**)
- MOID computation methods: analytical, geometrical, d^2 minimization
- **State of the art:** MOID is computed only for Keplerian orbits
- **Problem:** perturbations acts on debris and satellites
  - Atmospheric drag
  - Gravitational field zonal harmonics
  - Luni-solar perturbations
- **Objective:** include perturbation effects into MOID computation
  - **Method:** global minimization of d^2
  - **Optimizer**: COSY-GO, based on Taylor Models and Differential Algebra

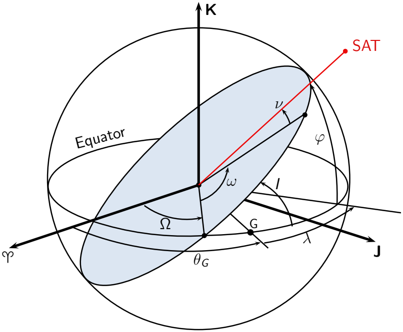
Outline

Perifocal reference frame



$$r = \frac{a\left(1 - e^2\right)}{1 + e\cos(\nu)}$$

Earth Centered Intertial (ECI) frame



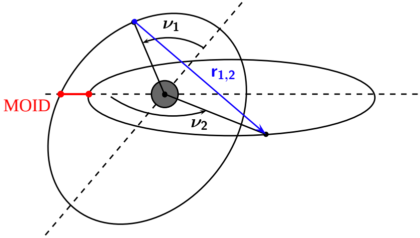
$$\mathbf{r}_i = \begin{Bmatrix} r_{I_i} \\ r_{J_i} \\ r_{K_i} \end{Bmatrix} = r_i \begin{Bmatrix} \cos(\Omega)\cos(\ell) - \sin(\Omega)\cos(I)\sin(\ell) \\ \sin(\Omega)\cos(\ell) + \cos(\Omega)\cos(I)\sin(\ell) \\ \sin(I)\sin(\ell) \end{Bmatrix}, \qquad \ell = \omega + \nu$$

Square distance computation

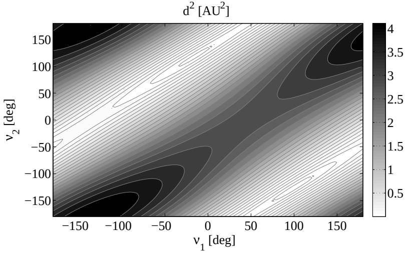
Keplerian orbits distance computation:

- Two sets of five fixed keplerian elements $\mathcal{K}_i = \{a_i, e_i, I_i, \Omega_i, \omega_i\}$, $i = 1, 2$
- Position of body i depends on true anomaly ν_i
- **True anomaly domain:** $\nu_i \in [-\pi, \pi]$ rad

Square distance: $$d^2 = f(\nu_1, \nu_2) = (r_{I_1} - r_{I_2})^2 + (r_{J_1} - r_{J_2})^2 + (r_{K_1} - r_{K_2})^2$$



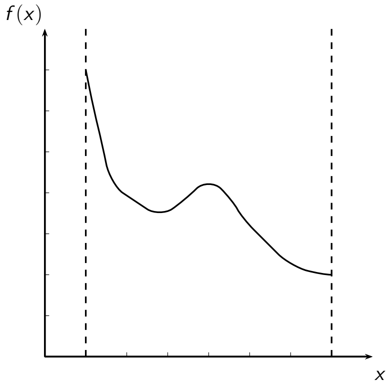
Landscape plot of d^2



Global optimizer COSY-GO (1)

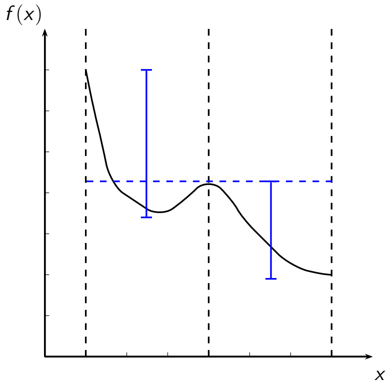
Branch & Bound algorithm

- Branch: based on 1st e 2nd order information of the objective function
- Bound: objective function computed as TM and appropriate bounder used (Interval bounder, Linear Dominated Bounder (LDB), Quadratic Fast Bounder (QFB))



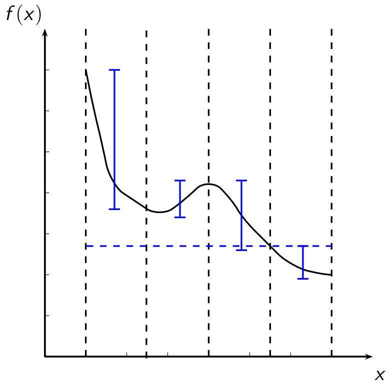
Branch & Bound algorithm

- Branch: based on 1st e 2nd order information of the objective function
- Bound: objective function computed as TM and appropriate bounder used (Interval bounder, Linear Dominated Bounder (LDB), Quadratic Fast Bounder (QFB))



Branch & Bound algorithm

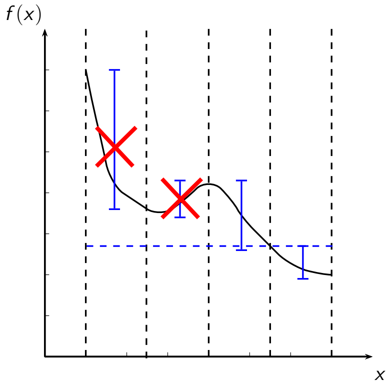
- Branch: based on 1st e 2nd order information of the objective function
- Bound: objective function computed as TM and appropriate bounder used (Interval bounder, Linear Dominated Bounder (LDB), Quadratic Fast Bounder (QFB))



Global optimizer COSY-GO (1)

Branch & Bound algorithm

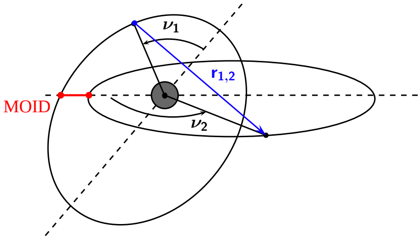
- Branch: based on 1st e 2nd order information of the objective function
- Bound: objective function computed as TM and appropriate bounder used (Interval bounder, Linear Dominated Bounder (LDB), Quadratic Fast Bounder (QFB))



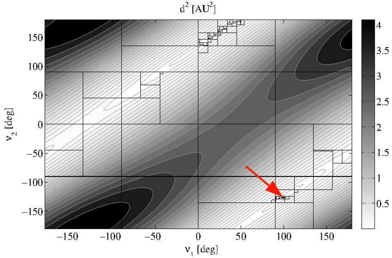
Keplerian orbits MOID

Objective function: square distance $d^2 = f(\nu_1, \nu_2)$

Search domain: $\nu_1, \nu_2 \in [-\pi, \pi]$



COSY-GO optimization example



$\nu_1 \in$[99.8444266, 99.8444279]; $\nu_2 \in$[-126.554010, -126.554007]

COSY-GO characteristics

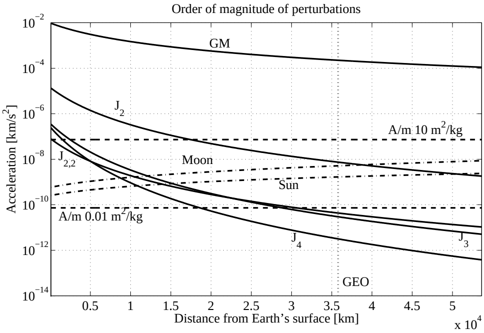
- Returns validated bounds of the global minimum
- Computes **all** the global minima

Orbital perturbations

Main sources of perturbation

- Earth's gravitational fields harmonics
  - Modify orbital plane and orbit orientation (Ω, ω)
- Atmospheric drag
  - Reduces orbit altitude (a, e)
- Solar radiation pressure
  - Affects mainly e and other orbital parameters in complicate way
- Third body attraction (Moon, Sun)
  - Acts mainly on Ω, ω, I, M

Orbital perturbations (2)

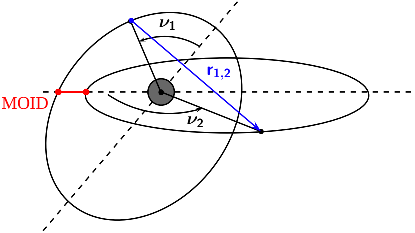


Type of perturbations

- Secular perturbations
- Long period perturbation ($T > T_{\text{rev}}$)
- Short period perturbations ($T < T_{\text{rev}}$)

MOID of perturbed orbits

- **Orbital perturbations** act on debris and satellites
- Orbital parameters $(a,\ e,\ I,\ \Omega,\ \omega)$ become functions of time t



$$r_i = \frac{a_i\left(1 - e_i^2\right)}{1 + e_i\cos(\nu_i)}, \qquad \ell_i = \omega_i + \nu_i$$

$$\mathbf{r}_i = \begin{Bmatrix} r_{I_i} \\ r_{J_i} \\ r_{K_i} \end{Bmatrix} =$$

$$= r_i \begin{Bmatrix} \cos(\Omega_i)\cos(\ell_i) - \sin(\Omega_i)\cos(I_i)\sin(\ell_i) \\ \sin(\Omega_i)\cos(\ell_i) + \cos(\Omega_i)\cos(I_i)\sin(\ell_i) \\ \sin(I_i)\sin(\ell_i) \end{Bmatrix}$$

$$d^2 = \left(r_{I_1} - r_{I_2}\right)^2 + \left(r_{J_1} - r_{J_2}\right)^2 + \left(r_{K_1} - r_{K_2}\right)^2$$

Objective function: $\boxed{d^2 = f\left(\nu_1,\ \nu_2,\ t\right)}$

Search domain: $\nu_1,\ \nu_2 \in [-\pi,\ \pi]$ rad; $\boxed{t \in [0,\ 365] \text{ days}}$

Analytical models

HANDE (LEO) 1987

- Zonal harmonics J_2, J_3, and J_4
- Atmospheric drag (arbitrary density model)
- Canonical variables: no singularities when $I = 0$ and/or $e = 0$

Aksnes' solution 1972

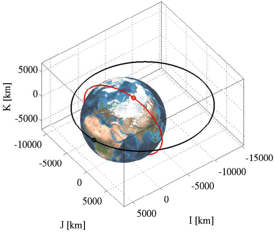
- Zonal harmonics J_2, J_3, J_4, and J_5
- Hill's variables: no singularities for $I = 0$ and $e = 0$

SGP4 (GEO) 1988

- Zonal harmonics J_2, J_3, and J_4
- Luni-solar perturbations (secular, long period)
- Resonance 1:1 ($\lambda_{2,2}$, $\lambda_{3,1}$ and $\lambda_{3,3}$)

Initial osculating elements

Orbit #	Orbit type	Dynamical Model	a	e	I	Ω	ω
			km	–	deg	deg	deg
1	Sun-syncr.	Aksnes	6878.136	0.0	97.0	110.0	70.0
2	MEO	Kepler	11130.227	0.4	6.5	300.0	73.0

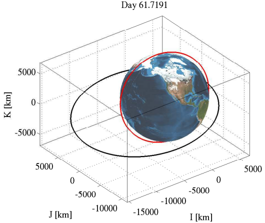


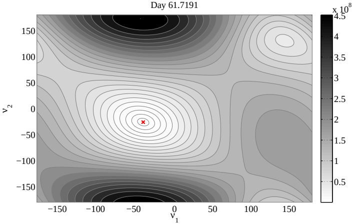
- At initial time MOID is 1880.083 km
- **NB:** this correspond to the MOID with keplerian approximation
- J_2 perturbations rotates orbital plane of orbit 1 (red)
- Orbit 2 (black) is Keplerian (satellite)
- 4 intersections are possible in 1 year

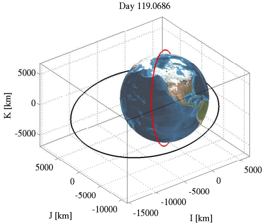
Computed minimum enclosure: $d^2 = [-0.22250739E-307,\ 0.68795338E-014]$

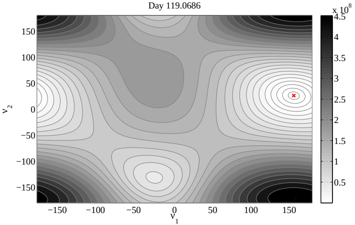
Test case #1: intersections

ν_1 [deg]	ν_2 [deg]	Δt [days]
[-40.9823329, -40.9823327]	[-26.0738536, -26.0738534]	[61.7190874, 61.7190876]
[155.990229, 155.990231]	[26.0738534, 26.0738536]	[119.068571, 119.068573]

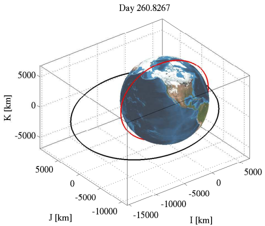


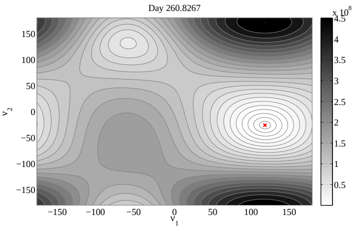


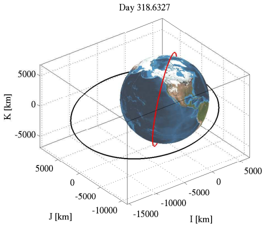


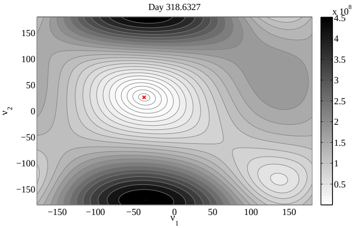


ν_1 [deg]	ν_2 [deg]	Δt [days]
[118.290326, 118.290328]	[-26.0738536, -26.0738534]	[260.826736, 260.826738]
[-39.7805915, -39.7805913]	[26.0738534, 26.0738536]	[318.632662, 318.632664]





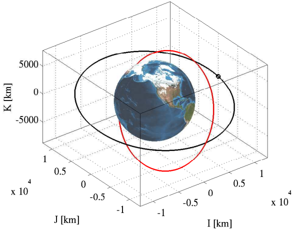




Test case #2 and #3: atmospheric drag

Orbit #	Orbit type	Dynamical Model	a	e	I	Ω	ω	B
			km	–	deg	deg	deg	
1	Molnyia-like	Aksnes/HANDE	9825.909	0.3	63.43	276.6	168.7	0.04
2	MEO	Aksnes	12559.681	0.0	10.0	0.0	0.0	-

Initial osculating elements

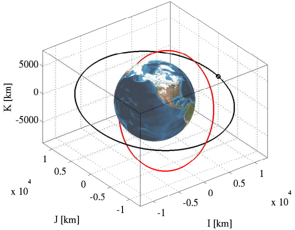


- Atmospheric drag effects on MOID
- Orbit 1 (red) has low perigee
- Test case #2, orbit 1 modelled with zonal harmonics
- Test case #3, orbit 1 modelled with zonal harmonics and atmospheric drag
- Orbit 2 (black) is modelled with zonal harmonics in test cases #2 and #3

Test case #2 and #3: atmospheric drag

Initial osculating elements

Orbit #	Orbit type	Dynamical Model	a	e	I	Ω	ω	B
			km	–	deg	deg	deg	
1	Molnyia-like	Aksnes/HANDE	9825.909	0.3	63.43	276.6	168.7	0.04
2	MEO	Aksnes	12559.681	0.0	10.0	0.0	0.0	-



- Atmospheric drag effects on MOID
- Orbit 1 (red) has low perigee
- Test case #2, orbit 1 modelled with zonal harmonics
- Test case #3, orbit 1 modelled with zonal harmonics and atmospheric drag
- Orbit 2 (black) is modelled with zonal harmonics in test cases #2 and #3

Test case#2 minimum enclosure: $d^2 = [294.108777,\ 294.111215]$

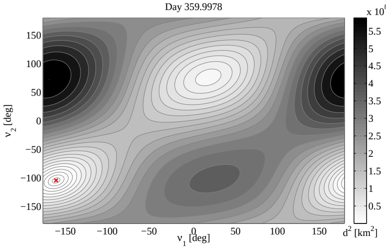
Test case#3 minimum enclosure: $d^2 = [-0.22250739E-307,\ 0.25157065E-018]$

Semi-major axis a reduction due to atmospheric drag causes orbits intersection

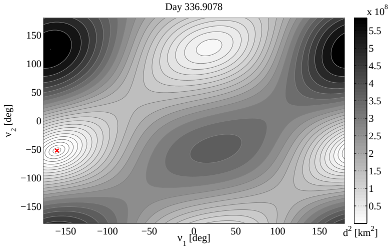
Test case #2 and #3

Test case #	ν_1 [deg]	ν_2 [deg]	t [days]
2	[-164.464846, -164.460773]	[-104.832006, -104.824949]	[359.995569, 360.000001]
3	[-163.805136, -163.805134]	[-52.4528862, -52.4528823]	[336.907751, 336.907755]

No atmospheric drag



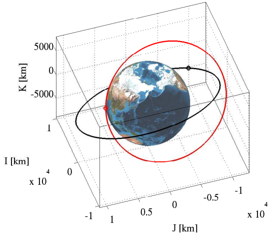
Atmospheric drag



Obs. Similar contour plots between test cases #2 and #3

Initial osculating elements

Orbit #	Orbit type	Dynamical Model	a	e	I	Ω	ω	B
			km	–	deg	deg	deg	
1	Molnyia-like	HANDE	9825.909	0.3	63.4	276.6	171.5	0.04
2	MEO	Aksnes	11278.136	0.0	25.0	110.0	200.0	-



- Atmospheric drag and zonal harmonics act on orbit 1 (red)
- Orbit 2 (black) is perturbed by zonal harmonics
- $d^2 = [1566860.77, 1566861.40]$ km^2
- **No intersections** occur in 1 year
- MOID: [1251.7431, 1251.7433] km

Computational time analysis

Cut-off and expansion order

- Cut-off value definition reduces computational time
- Optimization time increases with expansion order

Test case #3 computational time

Cut-off [km²]	Box size	Exp. order	Elapsed Time [s]
-	0.1	6	57.60
100	0.1	6	49.58
-	0.1	4	31.74
100	0.1	4	27.38
-	0.1	2	31.39
100	0.1	2	26.79

Box size and expansion order

- Expansion order more effective than box size
- Large initial d^2 $\Rightarrow$ significant time saving with cut-off

Test case #4 computational time

Cut-off [km²]	Box size	Exp. order	Elapsed Time [s]
-	0.01	6	216.07
100	0.01	6	15.88
100	0.1	6	15.55
100	0.1	4	10.74
100	0.1	2	8.02

*Processor: Intel® Pentium® M 1.73GHz; RAM: 1.0 GB; OS: Linux 2.6.34-Sabayon

Conclusions

- A method for the computation of minimum distance between two perturbed orbits is presented
- The method is based on Taylor differential algebra and COSY-GO global optimizer
- Test cases that account for zonal harmonics and atmospheric drag are presented
- Analysis of expansion order and minimum box size effects are considered

Future developments

- Analyze the impact of uncertainties on MOID
- From minimum distance between orbits to minimum distance between trajectories $\Rightarrow d^2 = f(t)$
- Implementation of alternative analytical solutions

Taylor Model Methods VII

Casa Marina Hotel

Key West, Florida, December 14-17, 2011

POLITECNICO DI MILANO

Collision risk assessment for perturbed orbits via rigorous global optimization

A. Morselli, R. Armellin, P. Di Lizia, F. Bernelli-Zazzera

Dipartimento di Ingegneria Aerospaziale, Politecnico di Milano, Milano, Italy

Differential Algebra

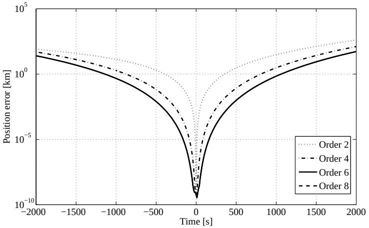
Differential Algebra (DA)

- Algebra of real numbers ⇒ Algebra of Taylor polynomials
- Automatic differentiation techniques
- Taylor expansions up to arbitrary order n for sufficiently regular functions

DA map of $\nu(t)$ of Kepler's equation solution

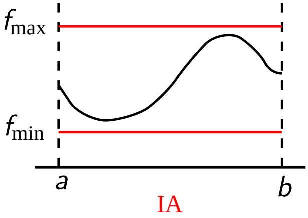
I	COEFFICIENT	ORDER	EXP t
1	1.197915993004572	0	0
2	-.3084298151229335E-03	1	1
3	-.9164915499374143E-06	2	2
4	0.7094672077674502E-09	3	3
5	0.5387967374899166E-12	4	4
6	-.2166159111274373E-14	5	5
7	0.1221617698455060E-17	6	6

Accuracy of expansion varying polynomial order



Interval arithmetics (IA)

- Extended domains of real numbers represented trough rigorous enclosures of intervals
- Evaluation of functions through interval arithmetics provides rigorous upper and lower bounds of a function inside an interval
  Disadvantage: overestimation of the result



Objective: combine advantages of DA and IA

Differential Algebra

Taylor polynomials contains functional dependance

Interval arithmetics

Intervals bound function deviation from polynomial approximation

→ Taylor model ←

$$f(\mathbf{x}) \in P_{n,\mathbf{x}_0,f}(\mathbf{x}-\mathbf{x}_0) + I_{n,\mathbf{x}_0,[a,b],f} \qquad \forall \mathbf{x}_0, \mathbf{x} \in [a,b]$$

$$T_{n,\mathbf{x}_0,[a,b],f} = \left(P_{n,\mathbf{x}_0,f}, I_{n,\mathbf{x}_0,[a,b],f}\right)$$