A New DA and TM Based Approach to Design Air-Core Magnets

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Taylor Model Methods VII,
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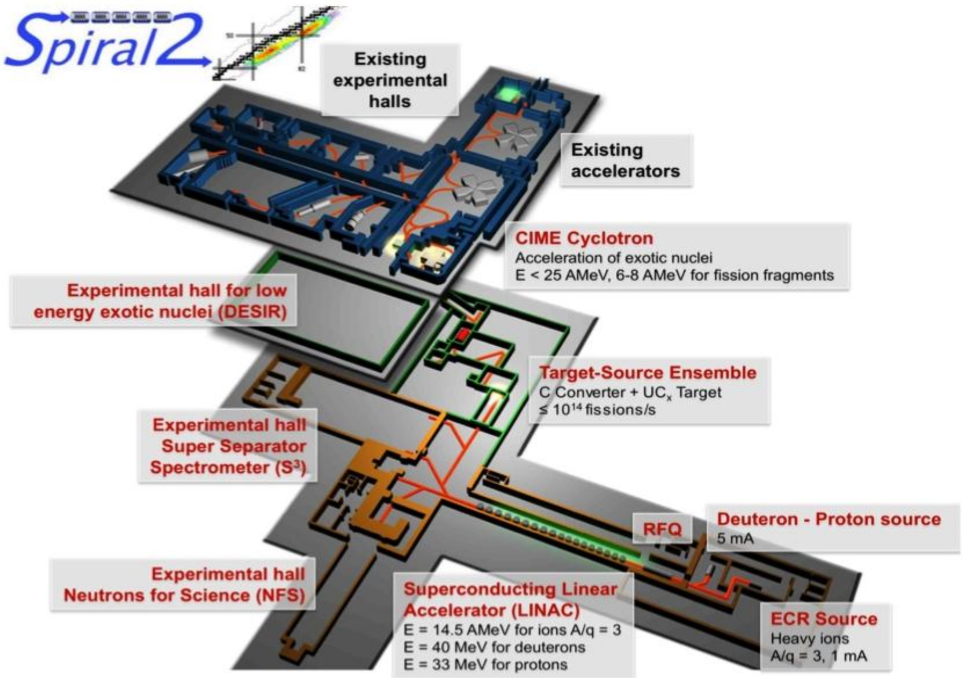
S3 device at SPIRAL2

- SPIRAL2 is a project to expand the capabilities of the GANIL, France facility in nuclear physics research with exotic beams
- One of the new instruments is the **Super Separator Spectrometer** (S^3) for high intensity stable heavy ion beams.

Technical challenges for S^3:

- Separation of very rare events from intense backgrounds
- Large beam acceptance and high selectivity for weak reaction channels are required



Interesting Experiments

The ^{100}Sn factory

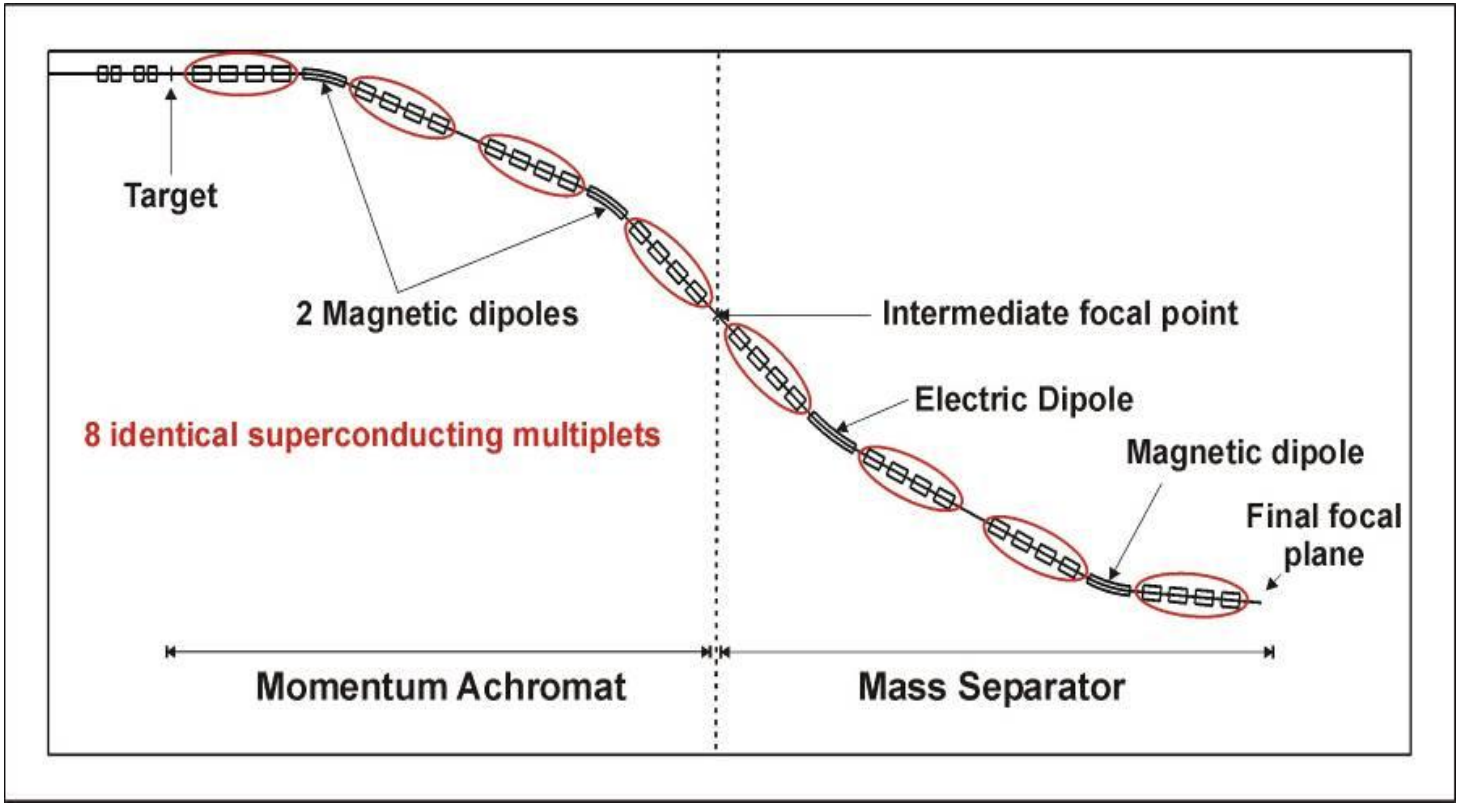
N = Z **^{58}Ni + ^{46}Ti → ^{100}Sn + 4n**	Ground state properties (half-lives, masses, spectroscopy)

SHE / VHE - Fusion-evaporation in direct kinematics

SHE / VHE **^{48}Ca+^{248}Cm → ^{292}116 + 4n**	Synthesis and delayed spectroscopy
	Chemistry
	Ground state properties (half-lives, masses, spectroscopy)

MAMS Layout for S^3

- 36m x 16m room layout
- Baseline MAMS configuration uses 8 quadruplets of mutipoles with quadrupole, sextupole, & octupole coils



Momentum **A**chromat followed by **M**ass **S**eparator design (MAMS)

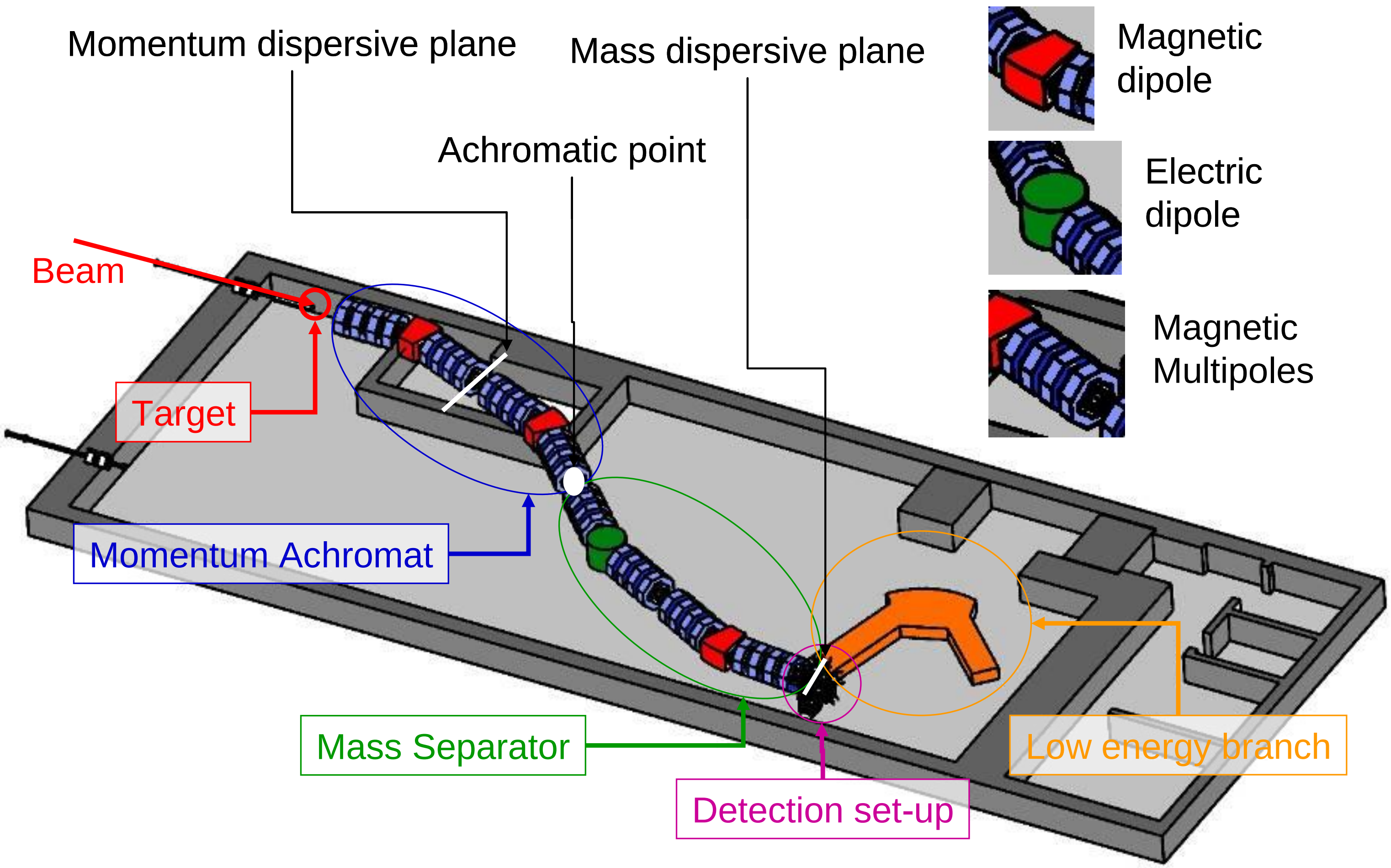
S3 Device Description

- Excellent primary beam suppression (10^{13}) at 0°
- Total transmission better than 50% for the two selected experiments
  - $^{48}Ca + ^{248}Cm \rightarrow ^{292}116 + 4n$
  - $^{58}Ni + ^{48}Ti \rightarrow ^{100}Sn + 4n$
  - *This corresponds to:*
    - *charge state acceptance of ± 10%, 5 charge states with <Q> = +20*
    - *momentum acceptance for each charge state of ± 10%*
    - *large angular acceptance in both planes of +/- 50 mrad*
- Maximum magnetic rigidity Br_{max} = 1.8 Tm *(momentum achromat)*
- Maximum electric rigidity Er_{max} = 12 MV
- Resolving power > 300 (FWHM) for physical separation in m/q
- Beam spot on the production target of S^3 of either:
  - $\sigma_x = 0.5$ *mm (Gaussian)* $\times \sigma_y = 2.5$ *mm (Gaussian) or*
  - $\sigma_x = 0.5$ *mm (Gaussian)* $\times \Delta y = 10$ *mm (uniform)*

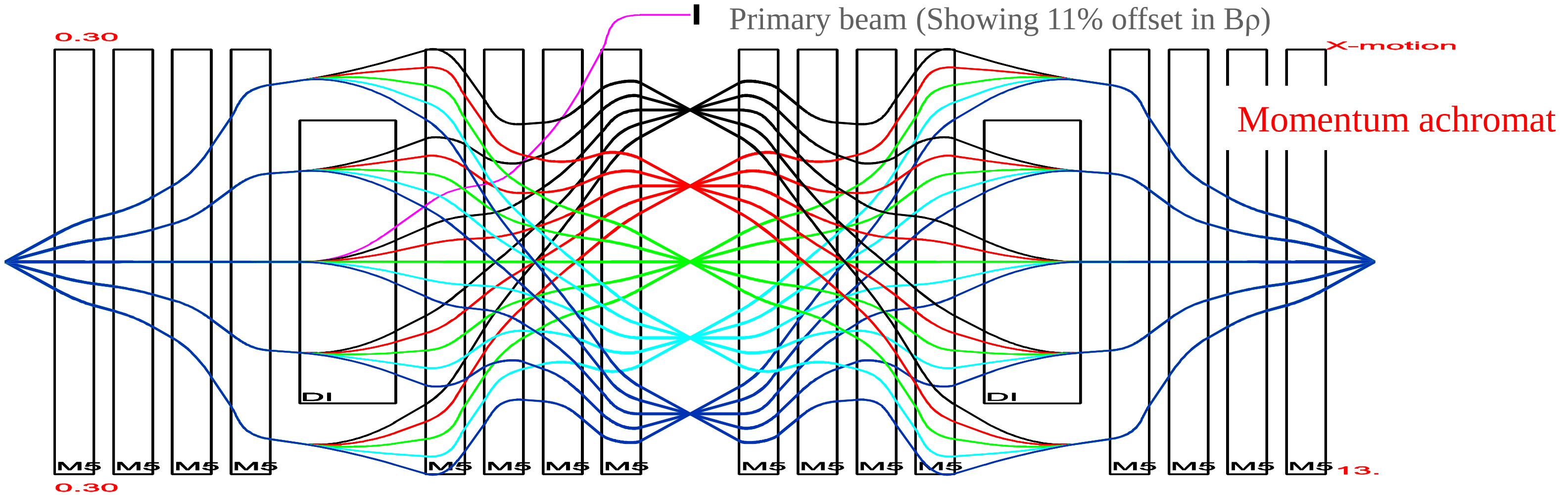
S3 Device Description (Continued)

- Final focal plane size depending on the experiment
  - *200 x 100 mm (maximum for high resolution mode, e.g. SHE synthesis)*
  - *100 x 100 mm (delayed gamma spectroscopy)*
  - *50 x 50 mm (low-energy branch gas catcher, GS properties)*
- Mass Achromat followed by Mass Separator (MAMS) layout choosen for S^3
  - Momentum achromat to suppress primary beam by at least 1:1000.
  - Further beam suppression and mass channel selection by a mass separator stage which is fully achromatic in momentum for each m/q value.
- Different operating modes are envisioned for performing experiments

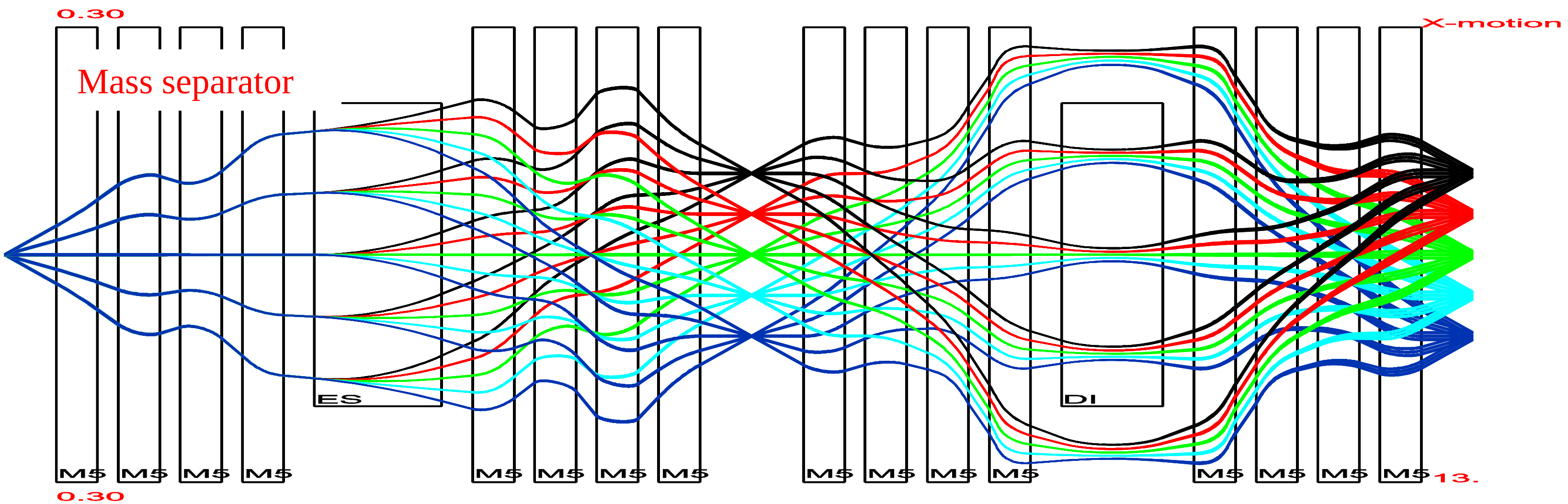
Layout in S3 room



First order optics



•Double mirror symmetric layout with 12mm per % of Bρ dispersion at the center

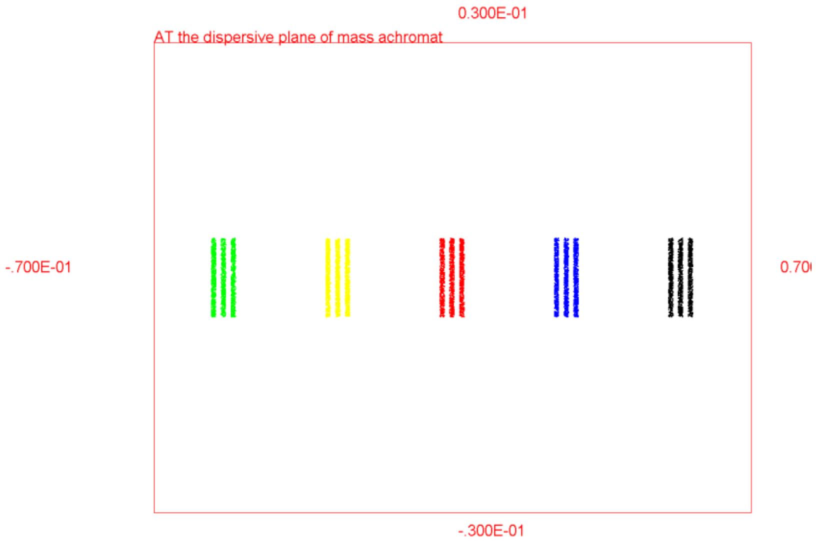


•Mirror symmetric layout with 6.7mm per % of m/q dispersion at the mass focal plane

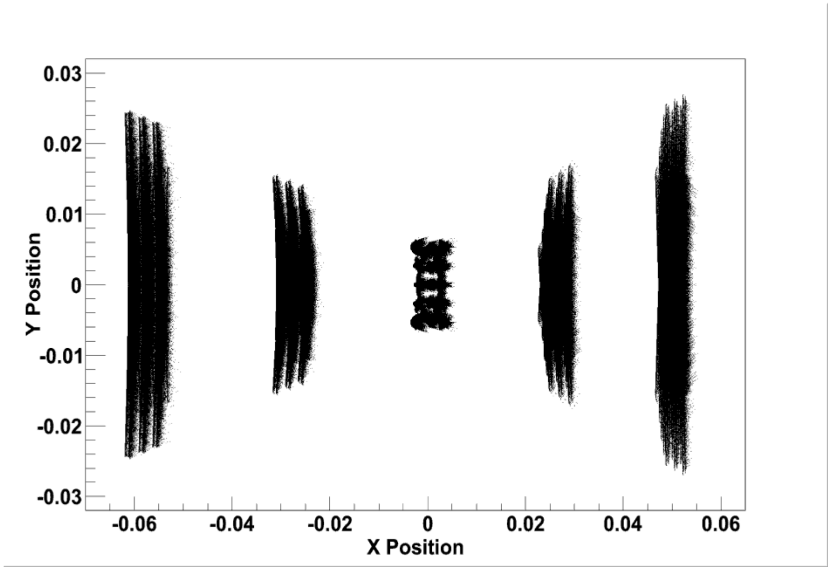
X-Y plot at the mass focal plane: SHE

$^{48}Ca + ^{248}Cm \rightarrow ^{292}116 + 4n$

5 charge states selected by slits into a 7-cm x 3-cm catcher or detector



First Order



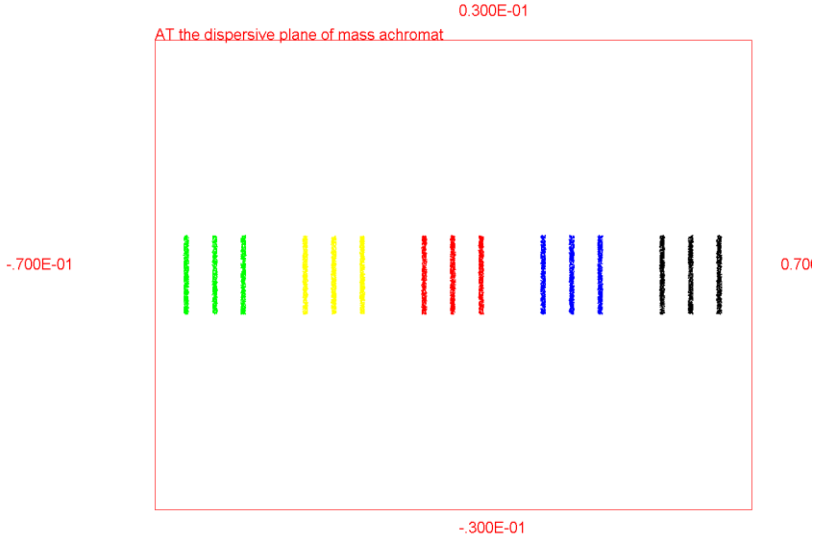
Second Order

Mass-energy aberrations corrected

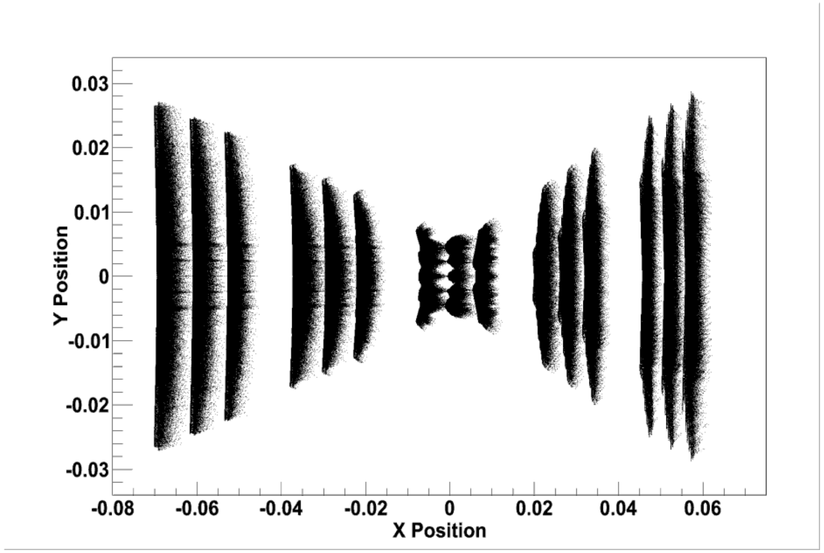
X-Y plot at the mass focal plane: NEZ

$^{58}\mathrm{Ni} + {}^{46}\mathrm{Ti} \rightarrow {}^{100}\mathrm{Sn} + 4n$

5 charge states selected by slits into a 7-cm x 3-cm catcher or detector



First Order



Second Order

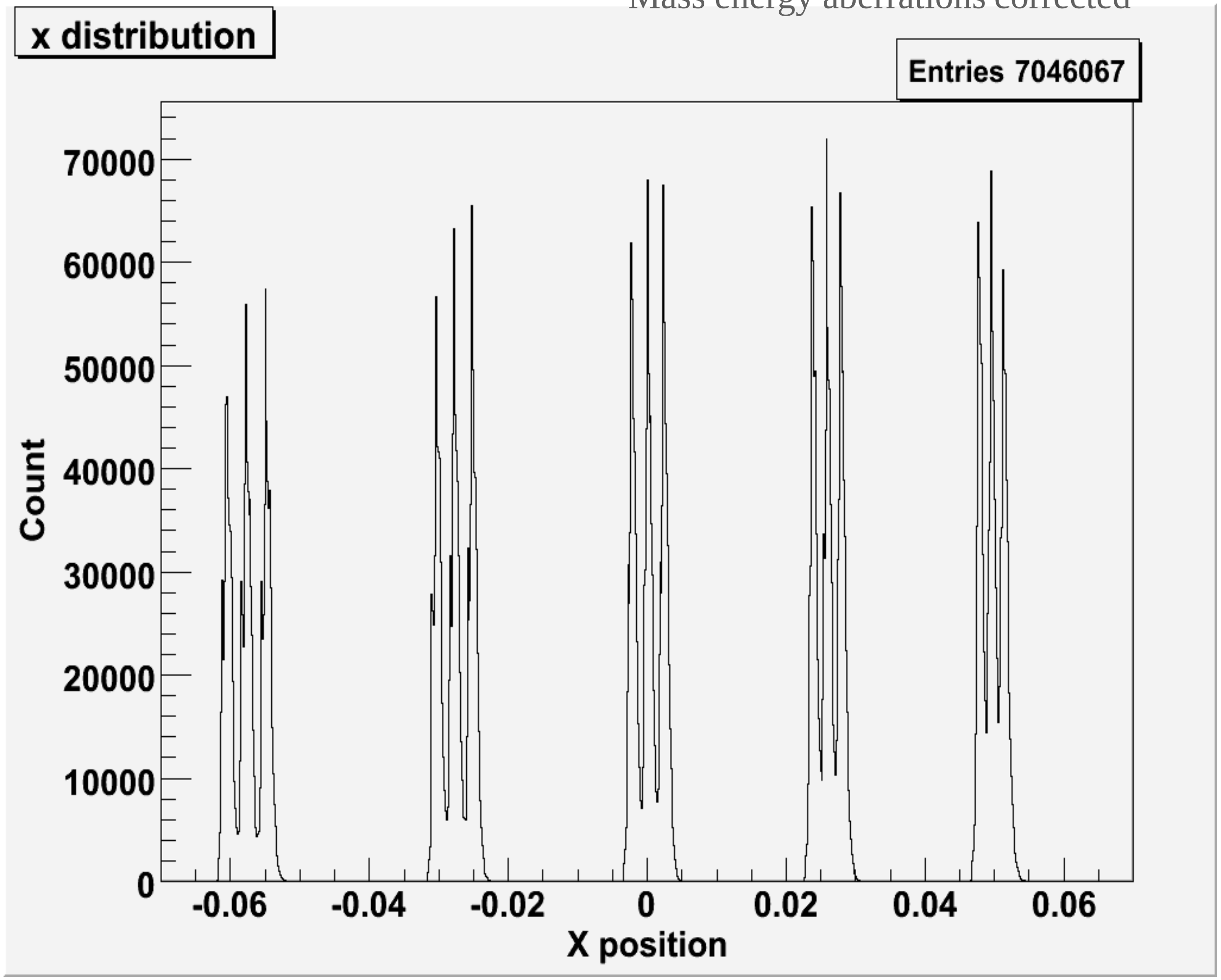
Mass-energy aberrations corrected

Histogram at mass focal Plane

$^{48}Ca + ^{248}Cm \rightarrow ^{292}116 + 4n$

$\delta Q = \pm 2$, $\delta m = \pm 1$, $\Delta B\rho = 4.6\%$

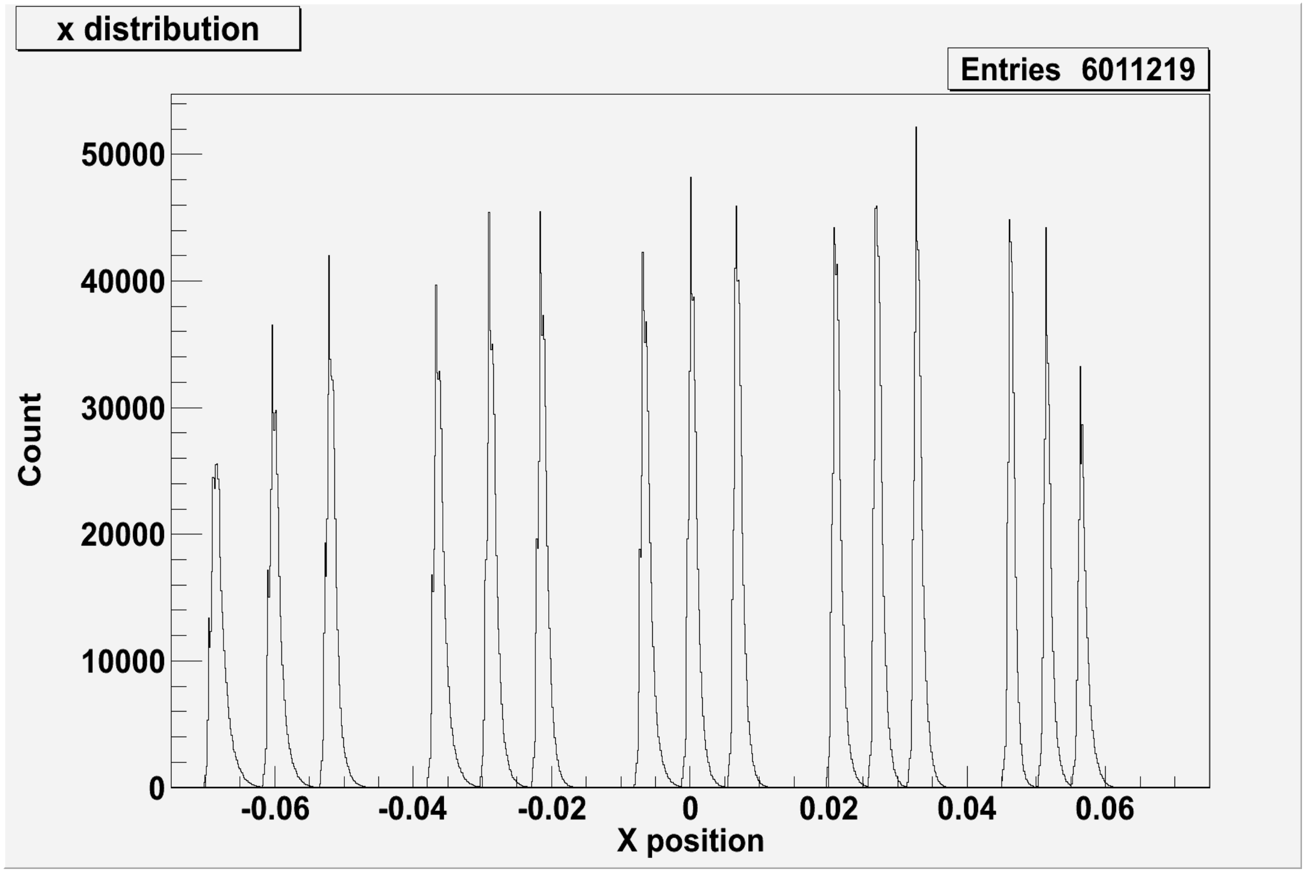
Mass energy aberrations corrected



Plot showing position of mass line

$^{58}Ni + ^{46}Ti \rightarrow ^{100}Sn + 4n$

$\delta Q=\pm 2$, $\delta m=\pm 1$, $\Delta B\rho=\pm 7.5\%$



Plot showing position of mass line

Magnet requirements for S3

- 8 SC quadruplets or triplets
- 3 dipoles and 1 electrostatic sector magnet
- Each singlet has quadrupole, sextupole, & octupole coils, with 30-cm warm bore diameter & 40-cm effective length (octupoles maay not be required)
- Fields required at 15-cm radius for 2 T-m rigidity (higher rigidity is easy):
  - Quadrupole: 1.0 T
  - Sextupole: 0.3 T
  - Octupole: 0.3 T
- Total power required for cryo-coolers of 8 quadruplets ~160 kW
  - Warm iron used to speed up cool down (~1 ton per multipole)
- Options for Multipole Magnet Design
  - Race track Coils
  - Double Helix Model by AML
  - 3D Cosine theta magnets

Type of Magnets

The electric and magnetic field will depend on the type of magnets we choose.
Some examples:

- Bending magnets (Dipole)
- Focusing magnets (Quadrupole)
- Steering magnets
- Kicker magnets (thin Quadrupole)
- Accelerating (Electric element)
- Corrector magnets (Hexapole, Octupole etc)

Accelerator lattice consists of array of magnets setup to attain certain goal. Complexity comes from the fact that there are **many** undetermined parameters. To arrive at a final (fully optimized) beam optic layout requires **several** iterations between beam optic design studies, magnet design studies and other practical constraints.

Magnets for Accelerator Physics Applications

Some factors influencing the choice of magnets and the design of magnets

- Bρ and Eρ of the beam/recoil
  - High Energy Physics: Only magnetic elements can be used
  - Low Energy Nuclear Physics (<10 Mev/nucleon): Both Electric and Magnetic elements can be used
- Field quality requirement
- Operating environment: Radiation Hardened magnets
- Super conducting or conventional: Depends on field strength requirements
- Tolerances to errors, misalignments, stress and strain in the support structures, heating
- Practical constrains like positioning of beam dumps, detectors, slits, monitors etc.
- Other factors: Reuse of existing magnets

- During the design phase:
  - Is the magnet practically feasible to build ?
    - Field quality requirement
    - Cost estimate
  - Beam optic properties (Transfer Maps)
  - Fringe Fields
  - Misalignment study
- After construction
  - Transfer maps with realistic fields

Some magnet Modelling codes/tools

- TOSCA module OPERA package
  - 3D code, uses Finite Element Method (FEM)
- ROXIE code from CERN
  - 3D code, uses many modes including hybrid Boundary Element Method & Finite Element Method
- POISSON
  - 2D Magnetostatic code, uses FEM
- SIMON
  - 3D code, uses Finite Difference method (FDM)
- RADIA (Free)
  - 3D Magnetostatic code, uses Boundary Element Method (BEM)

Pre-processor –> Field Solver –> Post-processor

Magnetic field due to arbitrary current distribution

- Magnetic field due to arbitrary current distribution is computed using the Biot-Savart law or Ampere's law

$$\vec{B}(\vec{r}) = \int \frac{1}{4\pi\mu_0} \frac{\vec{I} \times \overrightarrow{dl}}{|\vec{r} - \vec{r_s}|^2}$$

- Implementation:
  - Discretize the domain into current elements
  - DA framework is developed to describe a current element for the line, surface and volume case
  - Expand the kernel for the Biot-Savart law or Ampere's law
  - Integrate with respect to the variables describing the current elements
  - Sum over all the current elements
- The curl and the divergences for the field computed is always zero in the current free region.
- Number of current elements required is less due to use of High order
- Now the magnet design, beam optics and optimization can be done in the same code

Taylor model Integration

Let (P_n, I_n) be an n-th order Taylor model of f. From this we can obtain a Taylor model for the indefinite integral $\partial_i^{-1} f = \int f \, dx_i'$ with respect to variable x_i.

Taylor polynomial part: $\int_0^{x_i} P_{n-1} dx_i'$,

Remainder Bound: $(B(P_n - P_{n-1}) + I_n) \cdot B(x_i)$, where $B(P)$ is a polynomial bound.

So define the operator ∂_i^{-1} on space of Taylor models as

$$\partial_i^{-1}(P_n, I_n) = \left(\int_0^{x_i} P_{n-1} dx_i' \ , \quad (B(P_n - P_{n-1}) + I_n) \cdot B(x_i) \right)$$

$$\int_{x_{il}}^{x_{iu}} f(\vec{x})\, dx_i \in \left(P_{n,\partial^{-1} f}\left(\vec{x}|_{x_i = x_{iu} - x_{i0}} \right) - P_{n,\partial^{-1} f}\left(\vec{x}|_{x_i = x_{il} - x_{i0}} \right), I_{n,\partial^{-1} f} \right)$$

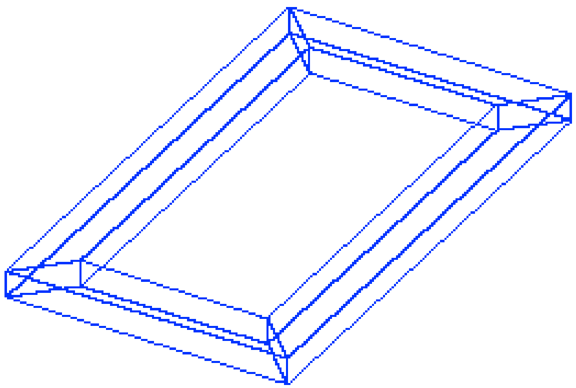
This method has following advantages:

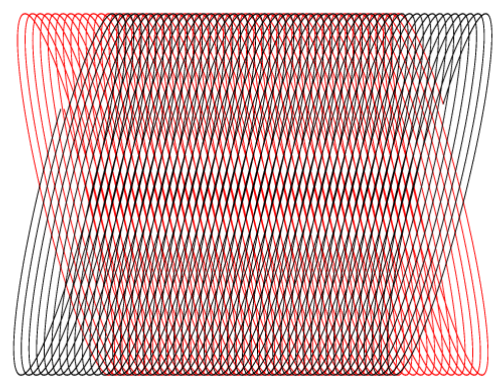
- No need to derive quadrature formulas with weights, support points x_i and an explicit error formula
- High order can be employed directly by just increasing the order of the Taylor model, limited only by the computational resources
- Rather large dimensions are amenable by just increasing the dimensionality of the Taylor models, limited only by computational resources

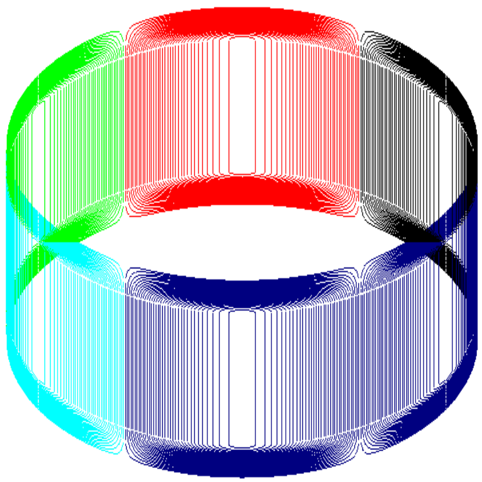
Tools

Due to their frequent use in the accelerator magnet applications, a dedicated set of tools has been written in the code COSY INFINITY for

- Infinitely long rectangular cross section current wire(2D design)
- Finite length rectangular cross section current wire
  - Current coil of rectangular cross section (3D design)
- Double Helix Model
- Cosine-theta type Magnet model

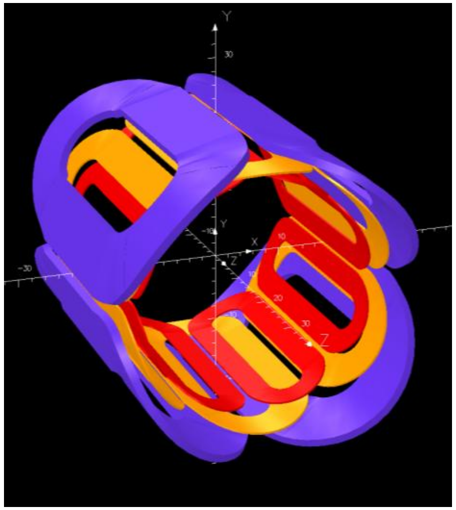




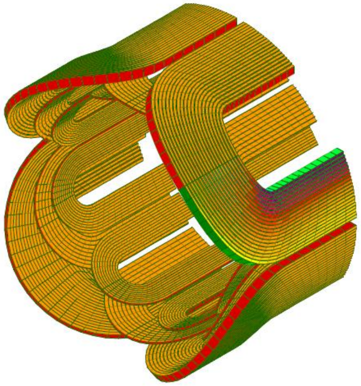


In addition to extracting the transfer maps these tools can be used to do conceptual design of magnets

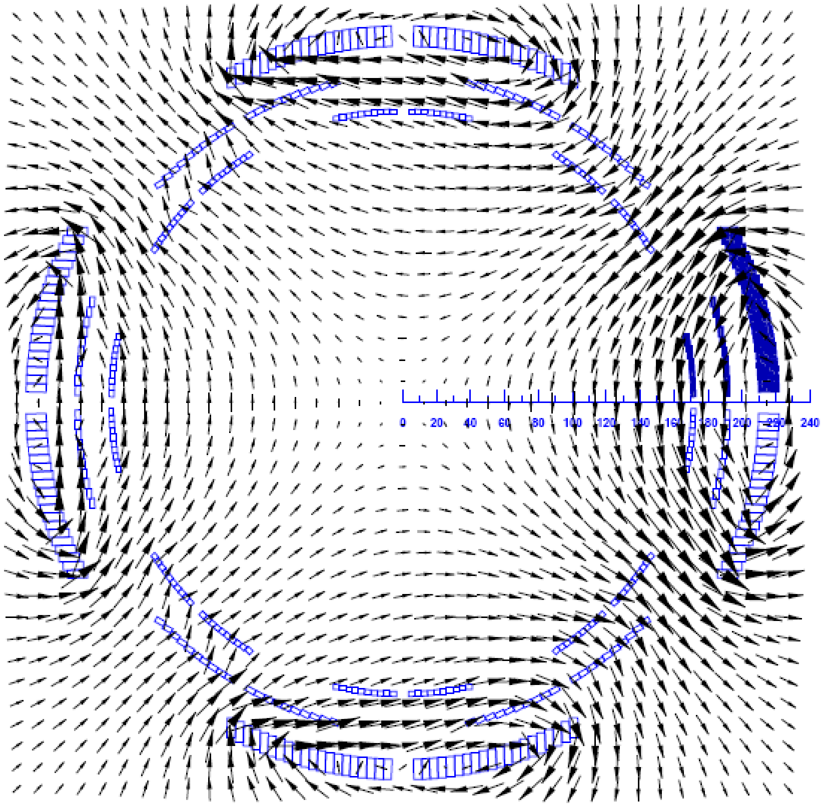
S3 Multipole magnet : using racetrack coils



3D coil configuration model using OPERA3D Code

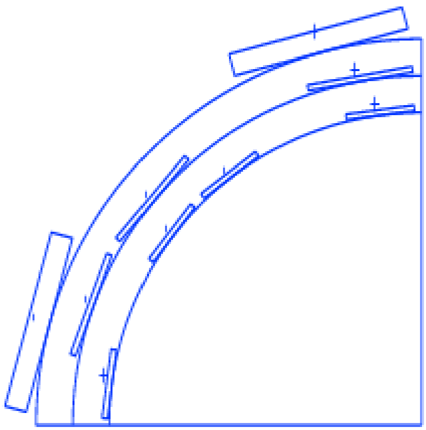


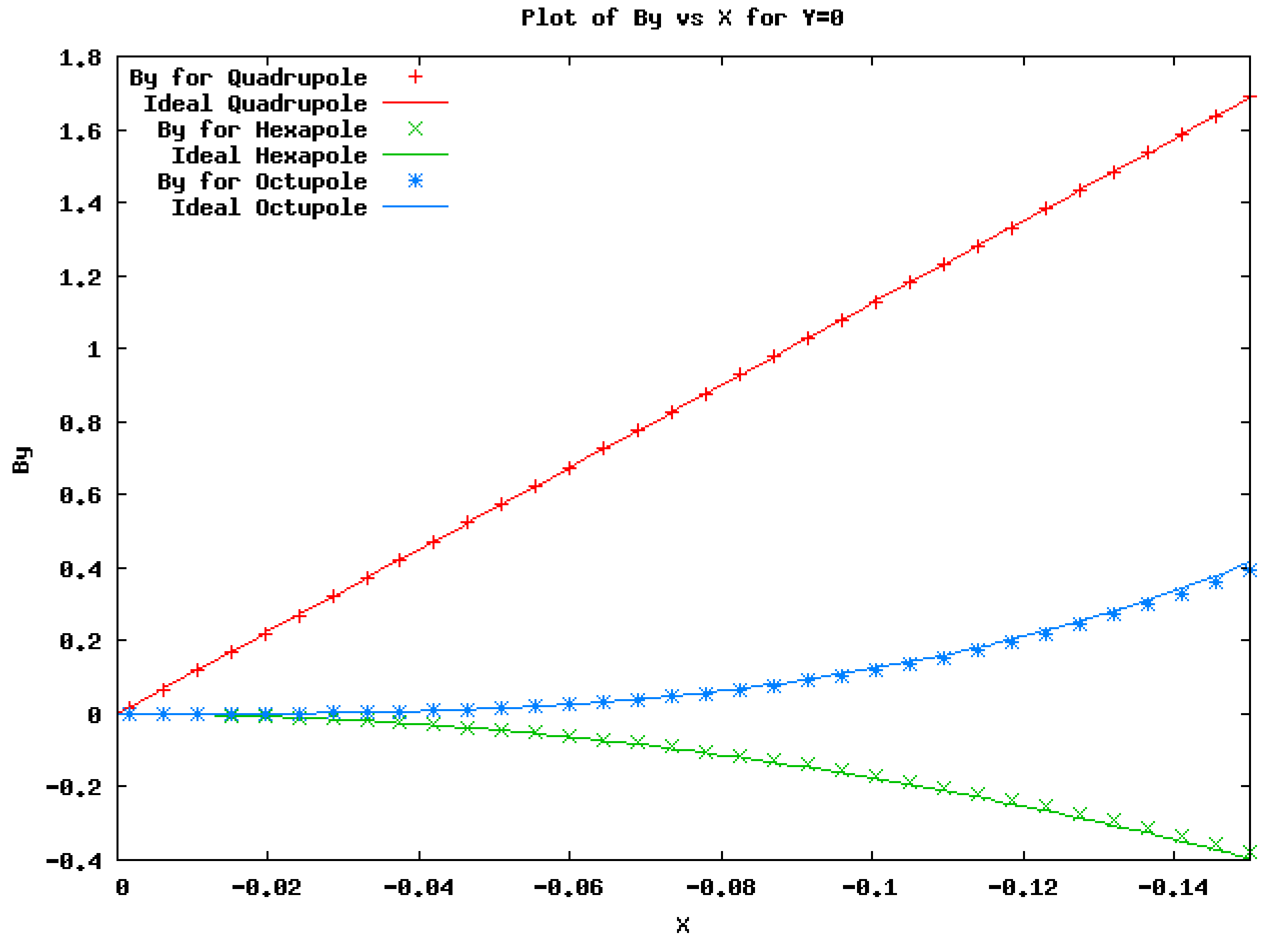
3D coil configuration model using ROXIE Code



Cross section layout and vector plot of the field

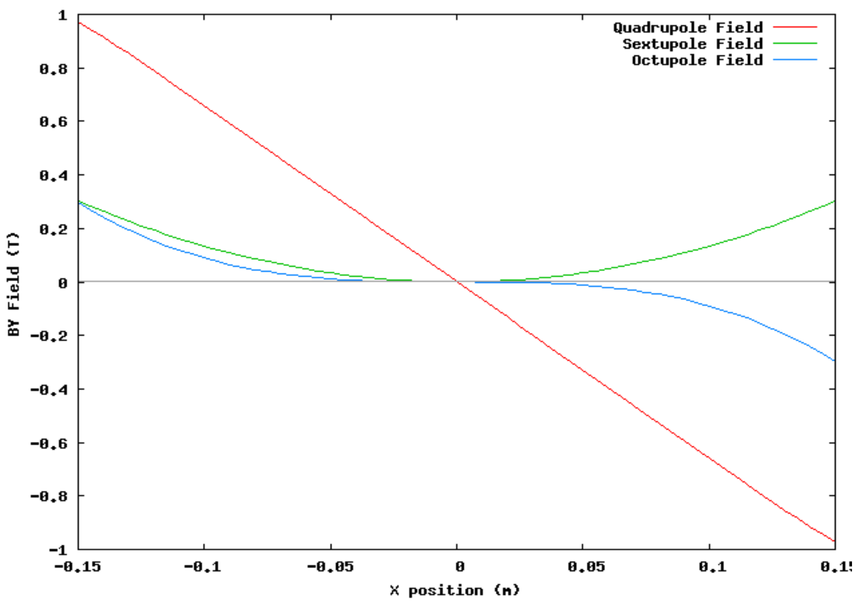
S3 Multipole magnet : using racetrack coils



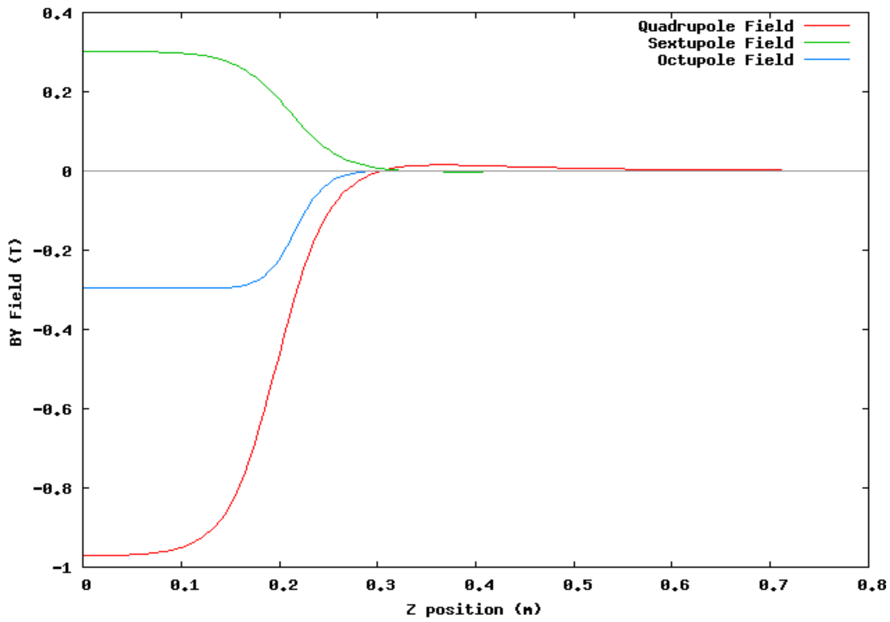


"Manikonda, S.; Nolen, J.; Berz, M. & Makino, K. (2009), 'Conceptual design of a superconducting quadrupole with elliptical acceptance and tunable higher order multipoles', Int. J. Mod. Phys. A24, 923-940."

Field Plots



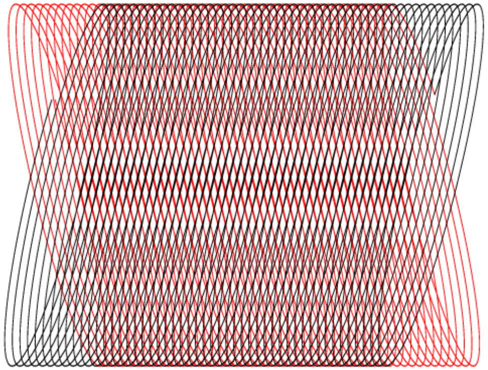
Field plot on the transverse "X" axis vs "By"



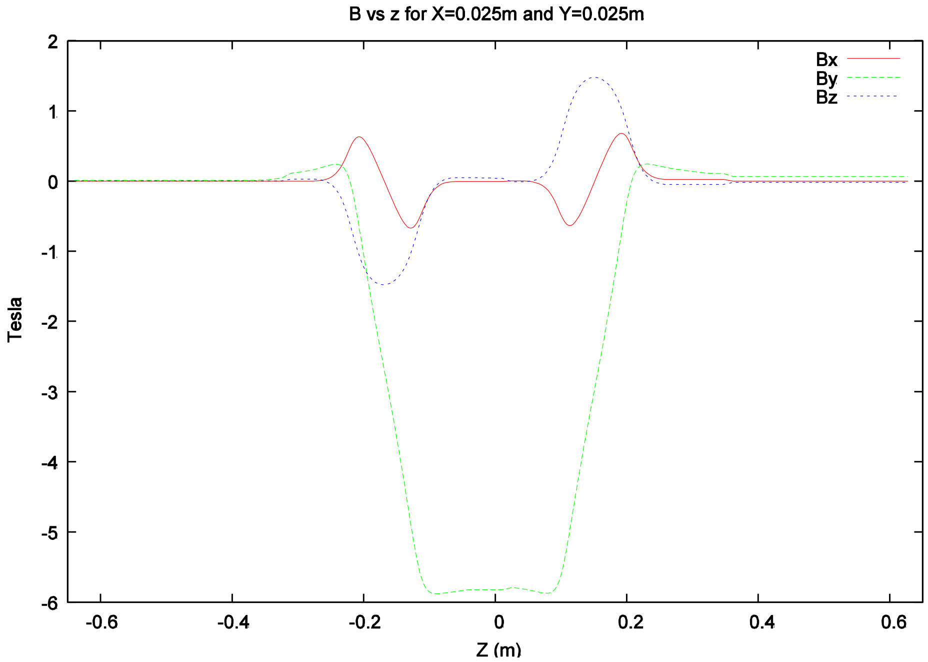
Field plot along "Z“ axis vs "By" at x=15cm and y=0cm

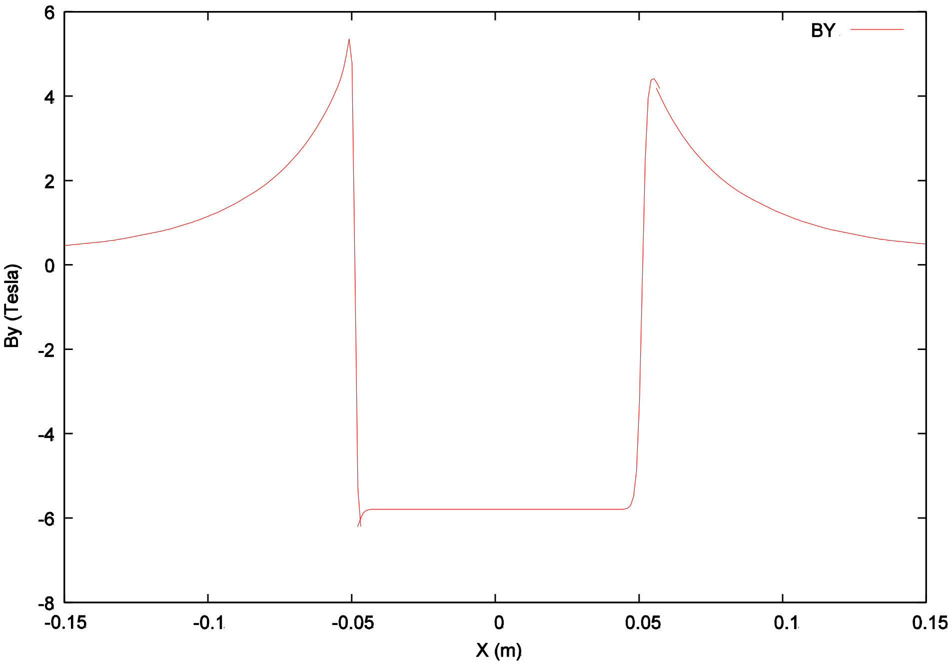
Double helix magnet design by Advanced Magnet Lab Inc.

$$z\left(\theta\right) = \frac{h\theta}{2\pi} + A_0\left(\sin\theta + \sum_{n=2}^{N}\epsilon_n \sin\left(n\theta + \phi_n\right)\right)$$



Dipole Example

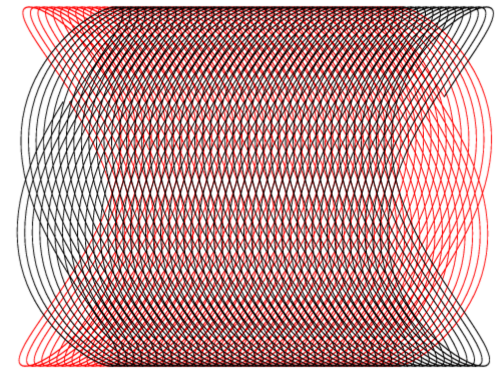


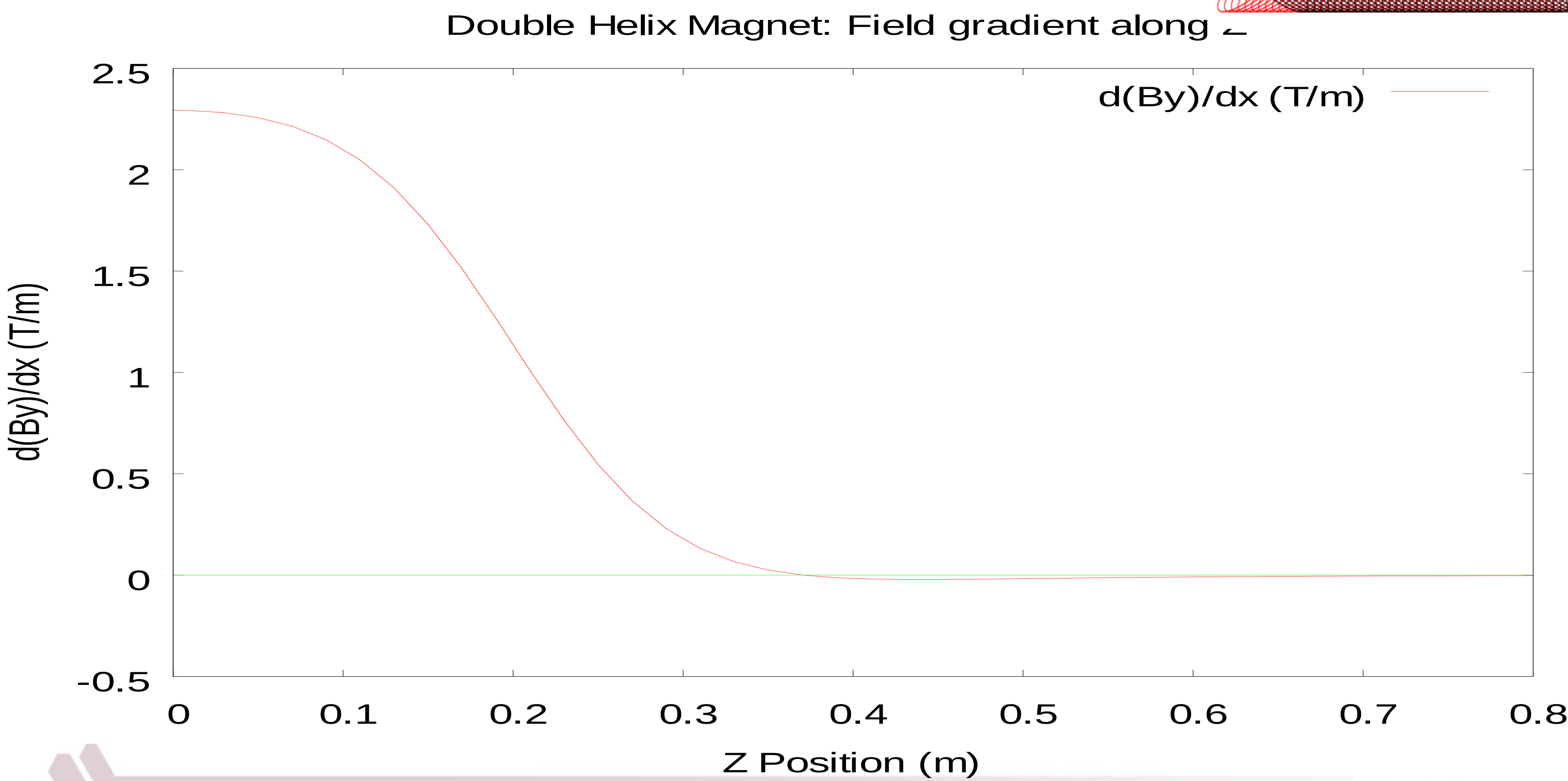


“Superconducting Double-Helix Accelerator Magnets, IEEE Proceedings of the 2003 Particle Accelerator Conference, 2003, Vol.3, pages 1996-1998. R.B. Meinke, M.J. Ball, C.L. Goodzeit”

S3 Quadrupole Magnet: Double Helix Model

- Effective Length = 0.197 m
- Field Gradient Used = 2.29 T/m
- Has negative field gradient outside magnet



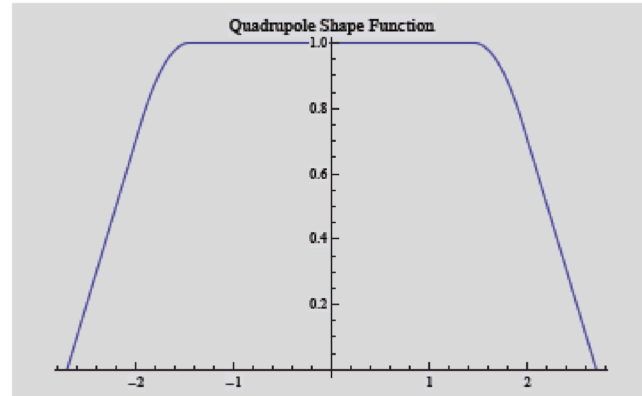


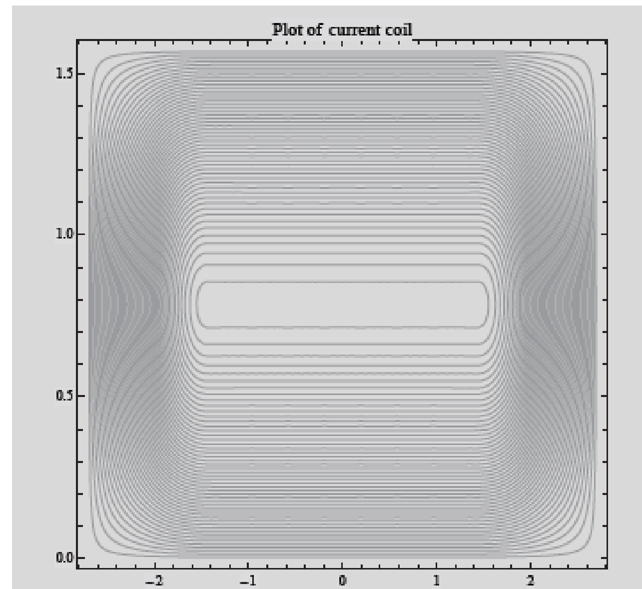
3-D cos mφ Magnets

- Proposed by P. L. Walstrom
- Based on using a shape function
- Produces pure cos mϕ magnetic field in 3D

$$F(x) * \sin(m\varphi) = (n - 1/2)/N$$

$$F[x] = \left(\begin{array}{cc} k\, x & k\,(0.75` + x) + k \\ 1 + (1.25` + k\, x)^2 & k\,(1.25` + x)\, k\, 0.75` \\ 1 & k\,(1.25` + x)\, k\, 1.25` \\ 1 + (1.25` - k\, x)^2 & k\,(0.75` + x)\, k\, 1.25` \\ k\, x & k\,(x)\, k\, 0.75` \end{array} \right)$$



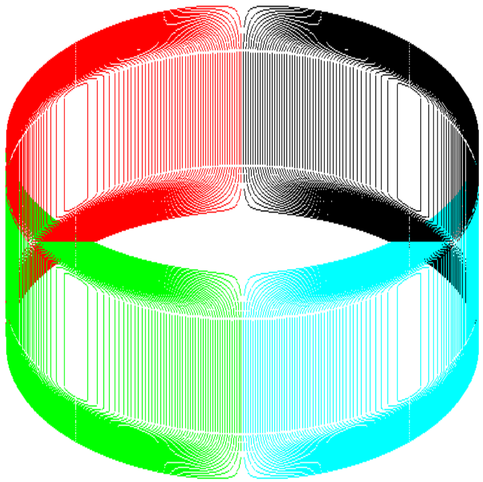


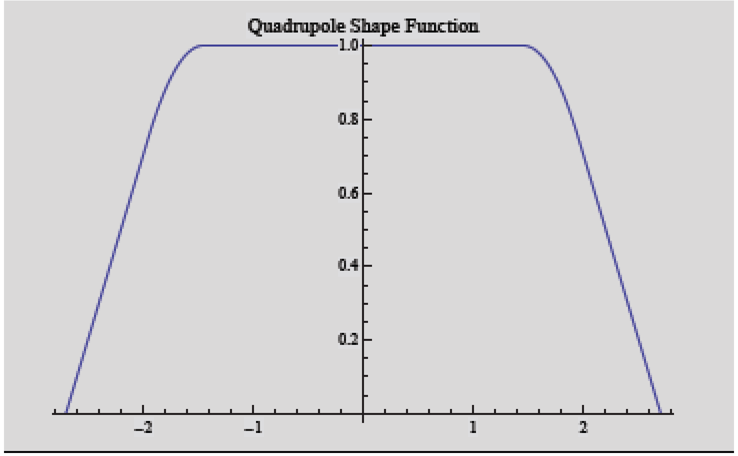
Winding that produces pure quadrupole field
(m=2 and K=2.7, N=50)

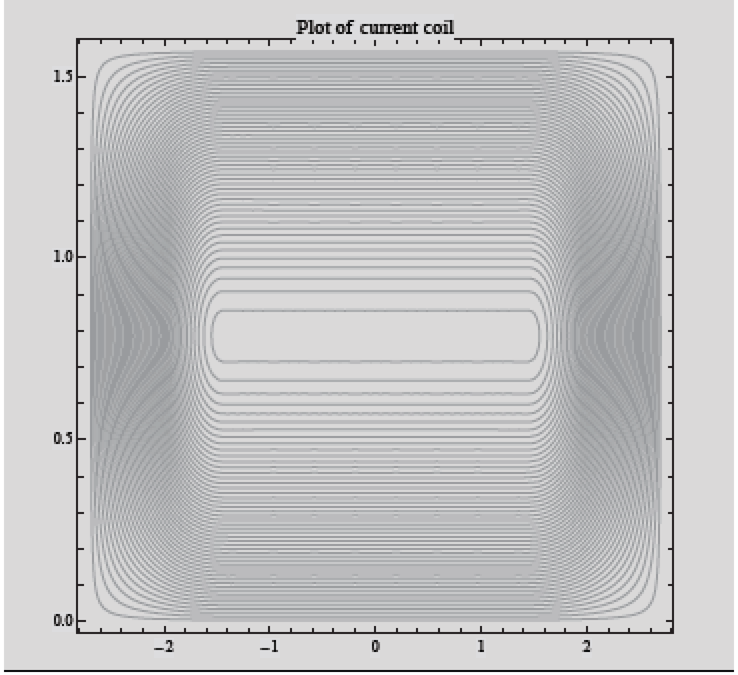
“P. L. Walstrom, Soft-edged magnet models for higher-order beam-optics map codes, Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment, Volume 519, Issues 1-2, Pages 216-221”

3D-Cos theta Quadrupole magnet

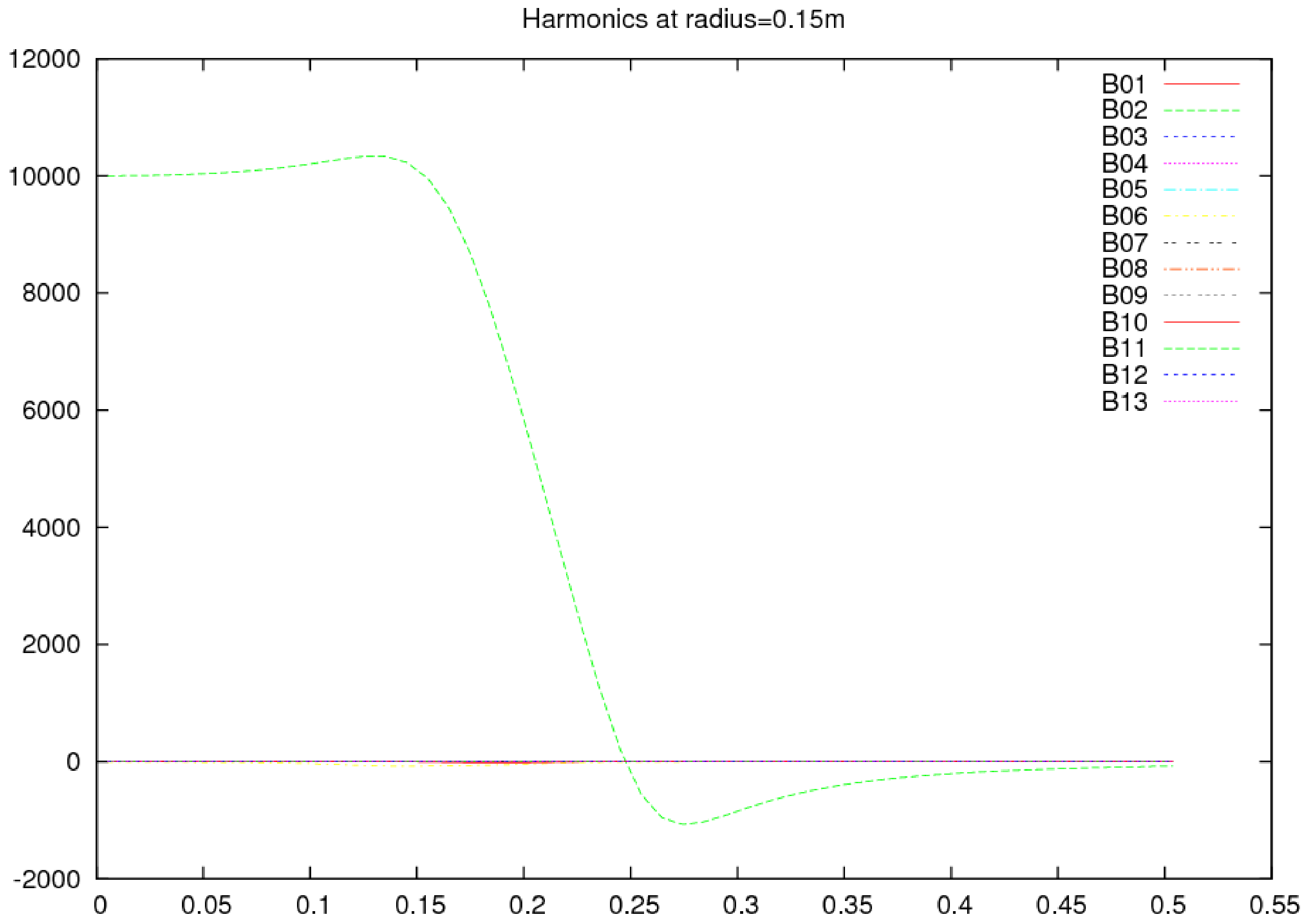
Winding Radius (m) = 0.17085
Total Number of turns = 50
Tip-to-tip total Coil Z Length (m) = 0.5



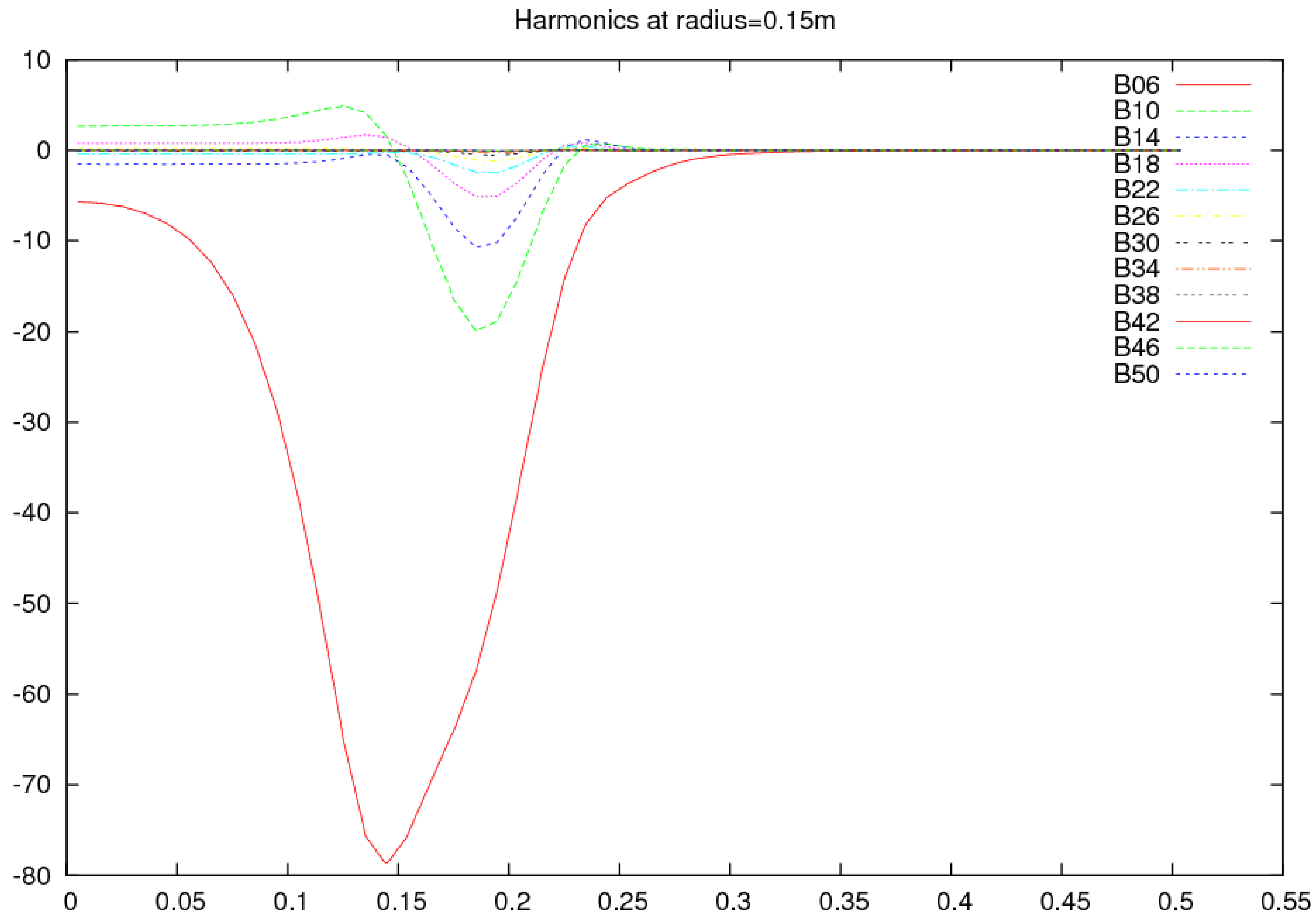




Harmonics for Quadrupole Magnet

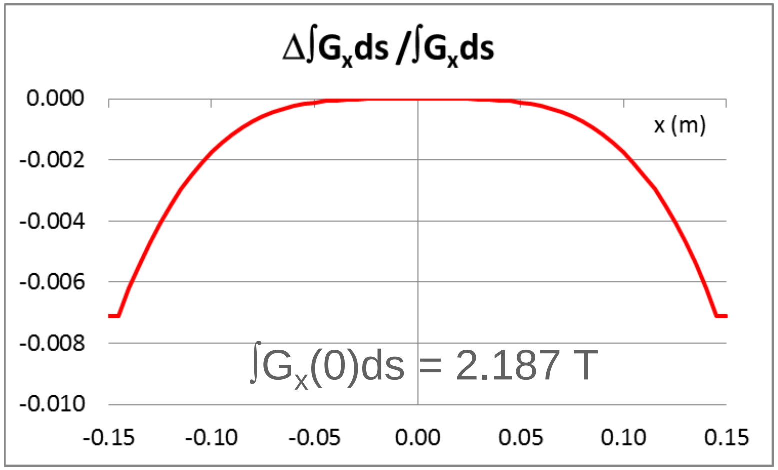


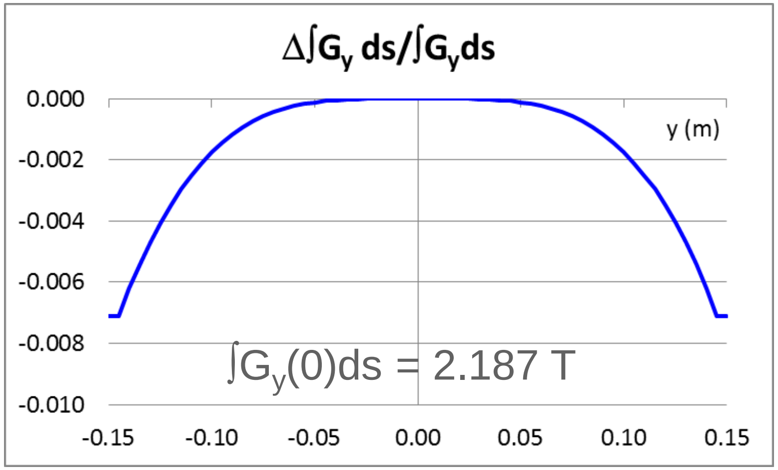
Allowed Higher order Harmonics for Quadrupole Magnet



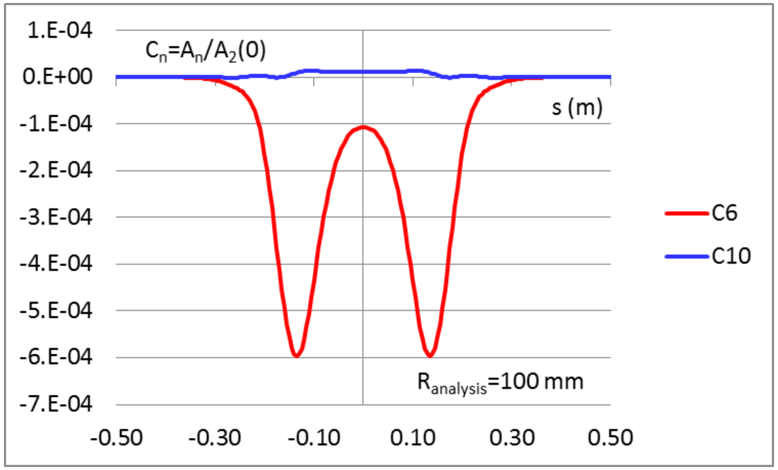
Quadrupole analysis

- J. Payet
- CEA/DSM/Irfu/SACM

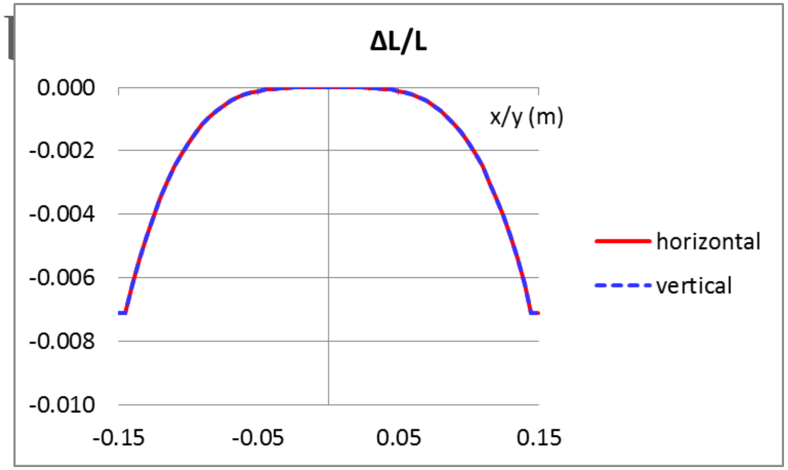




Gradient integral (from -1500 to +1500 mm) homogeneities



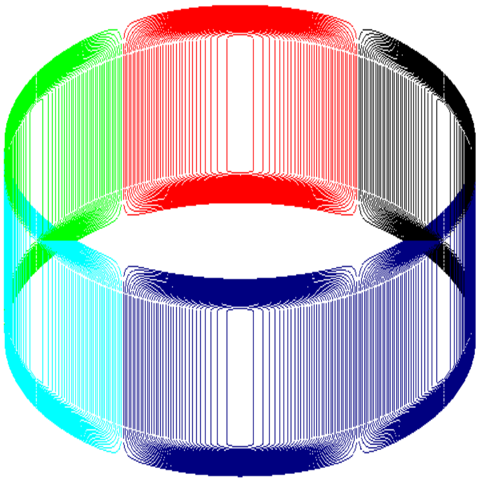
Harmonic analysis

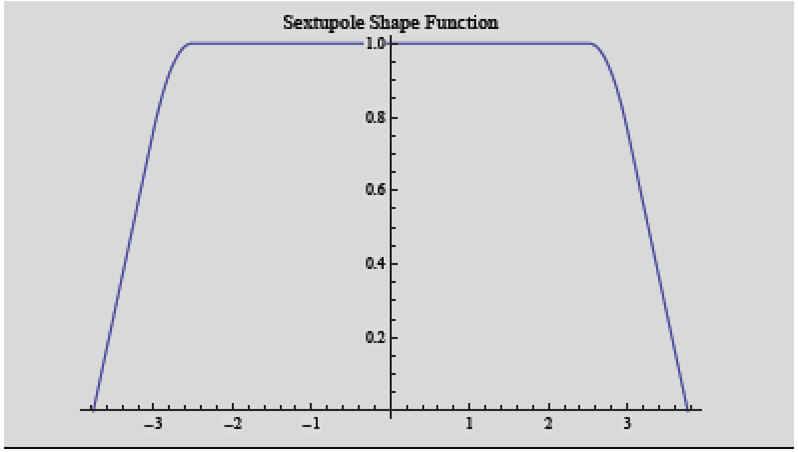


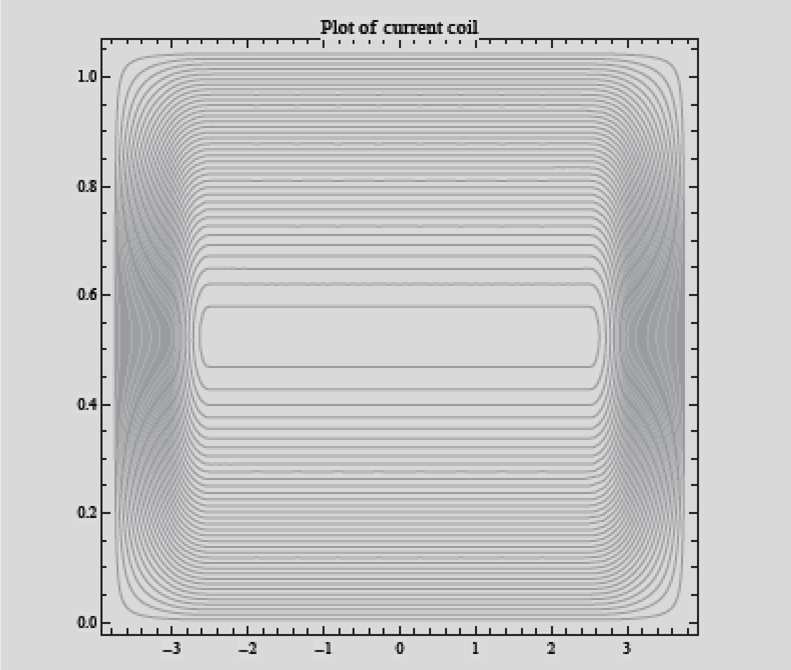
Magnetic length on the axis : 396 mm

3D cos-theta Sextupole magnet

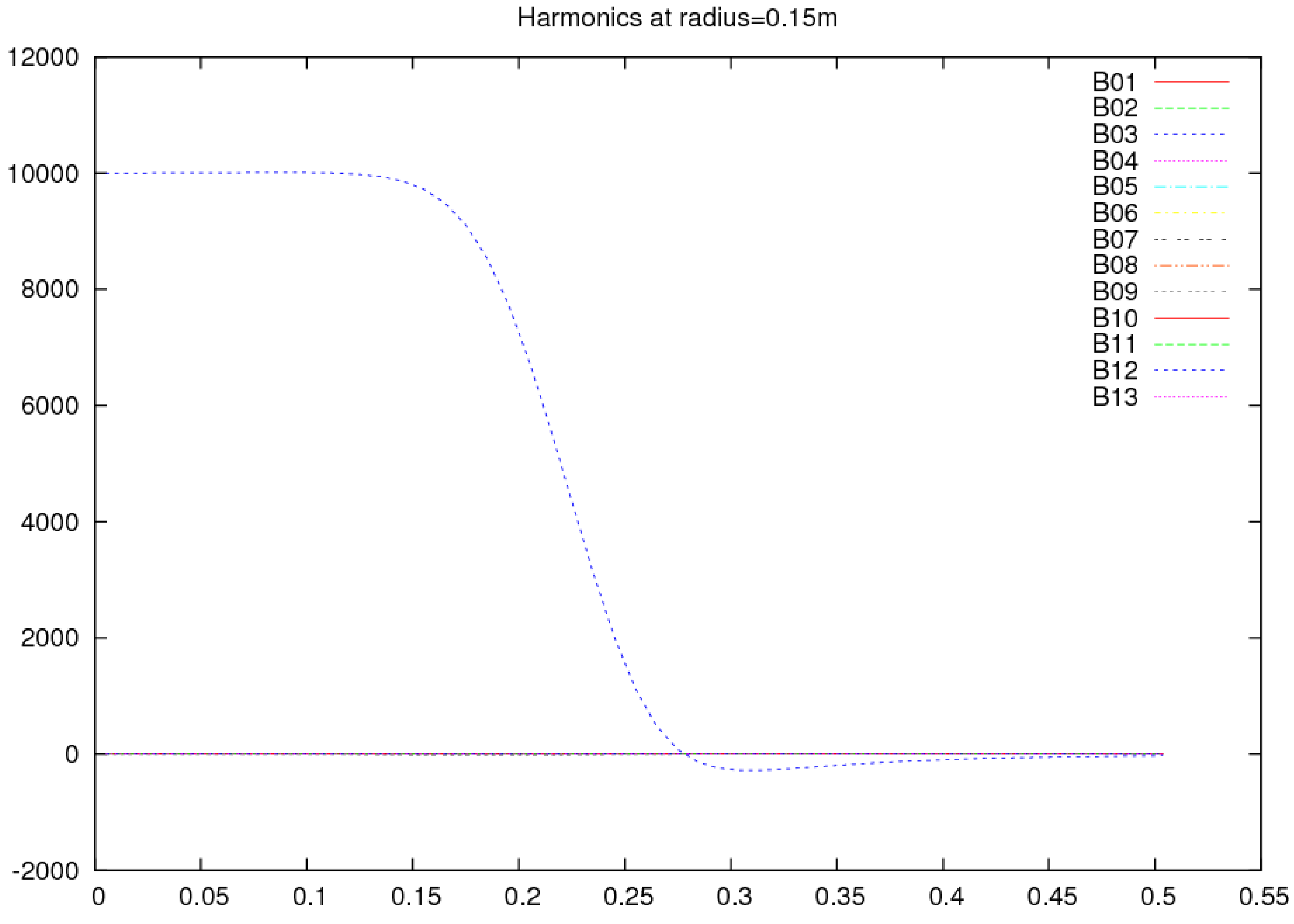
Winding Radius (m) = 0.20085
Total Number of turns = 36
Tip-to-tip total Coil Z Length (m) = 0.5



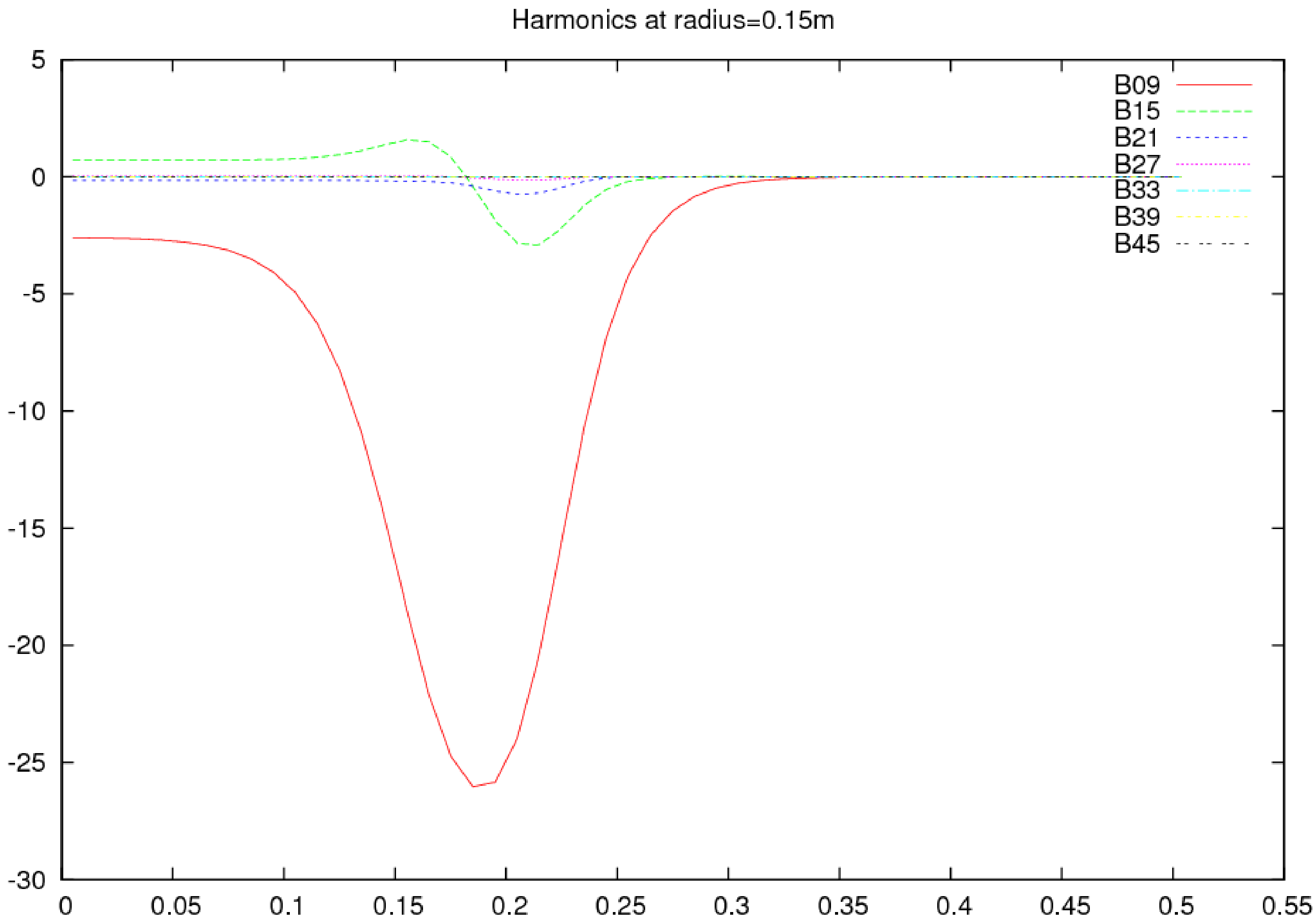




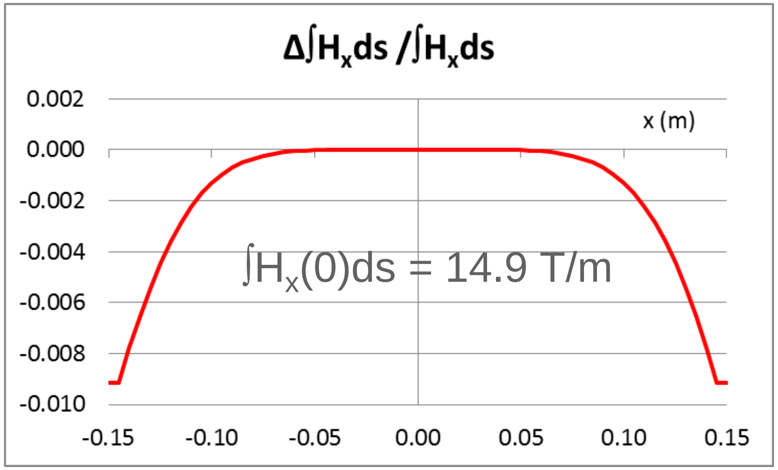
Harmonics for Sextupole Magnet

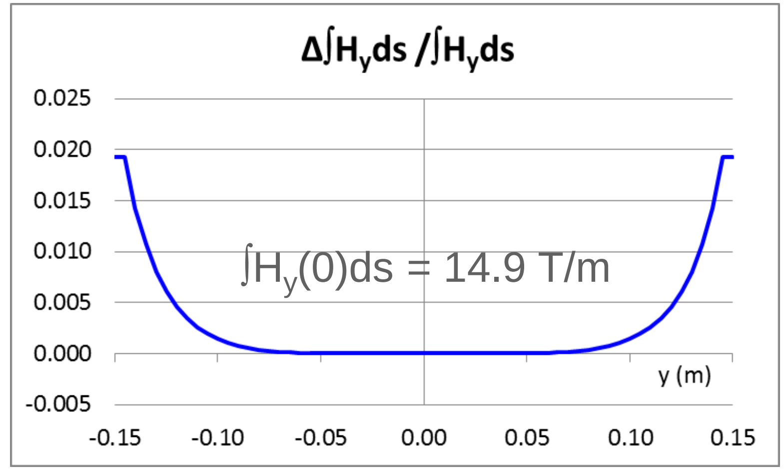


Allowed Higher order Harmonics for Sextupole Magnet

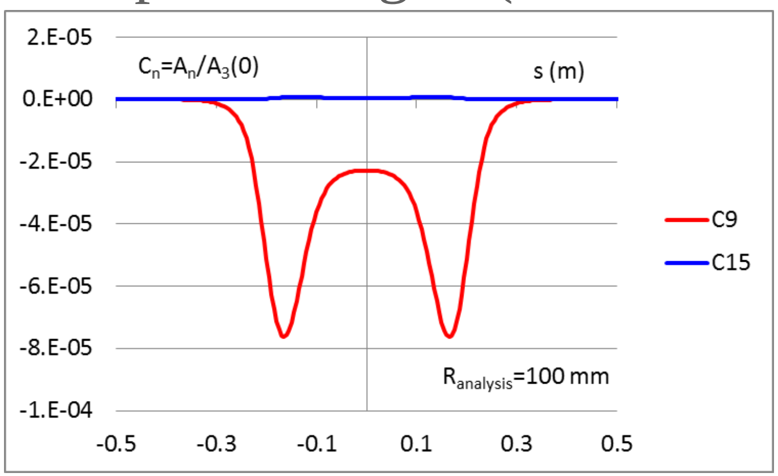


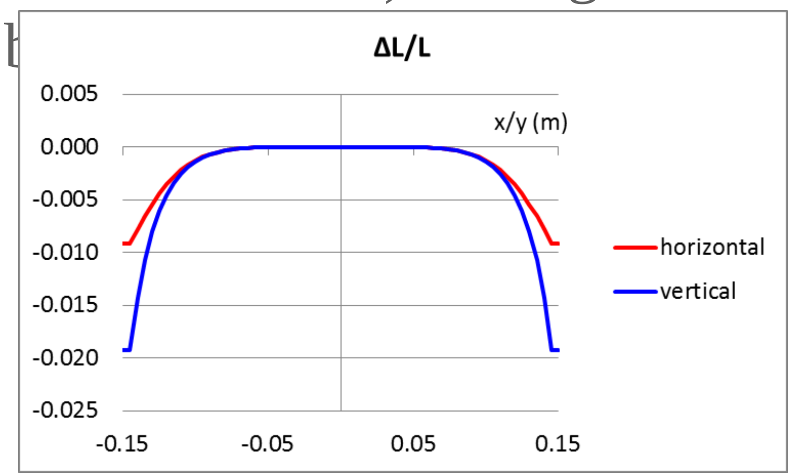
Sextupole analysis





Sextupolar integral (from -1500 to +1500 mm) homogeneities



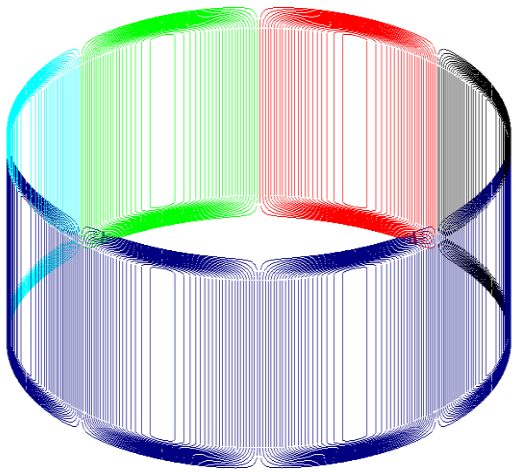


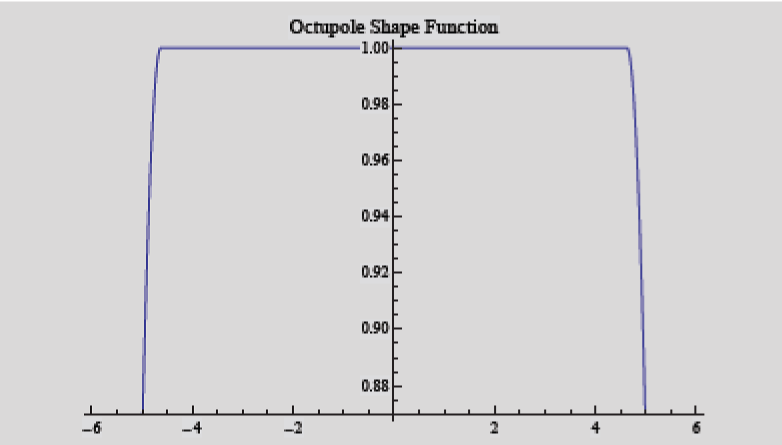
Harmonic analysis

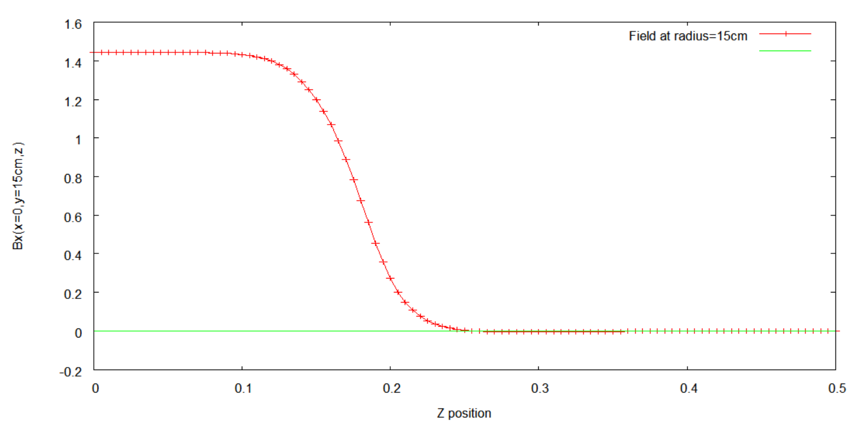
Magnetic length on the axis : 427 mm

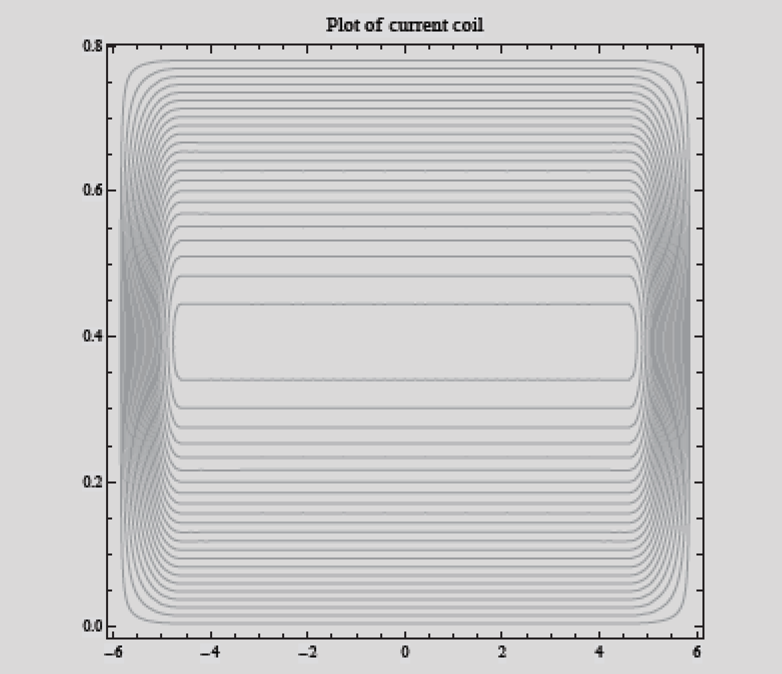
3D cost-theta octupole magnet

Winding Radius (m) = 0.1725
Total Number of turns = 23
Tip-to-tip total Coil Z Length (m) = 0.5

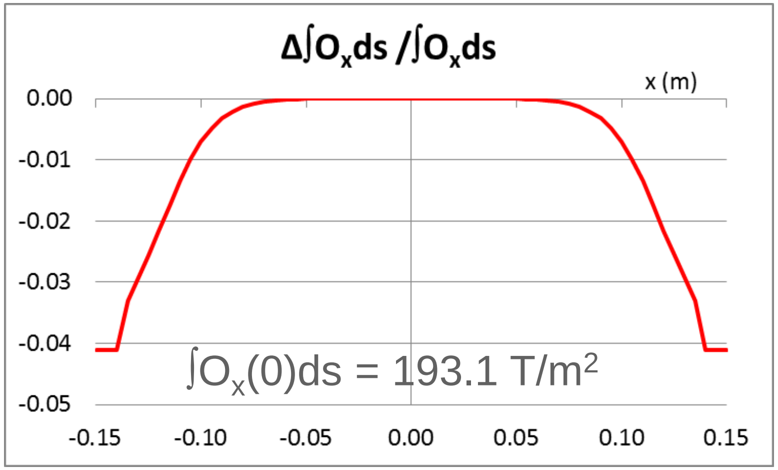


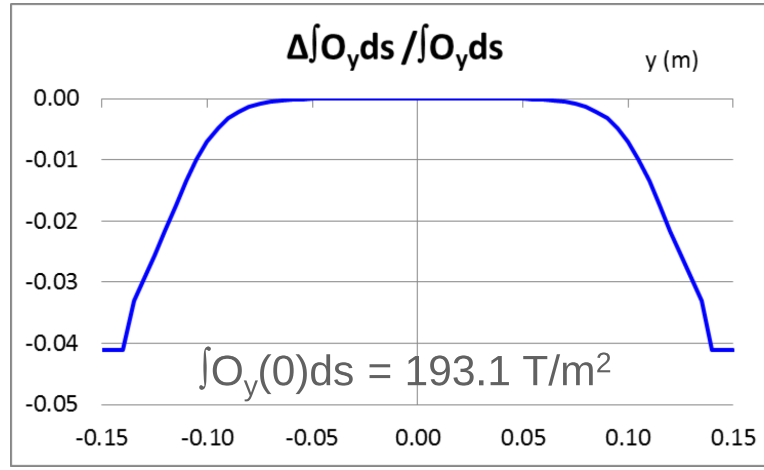




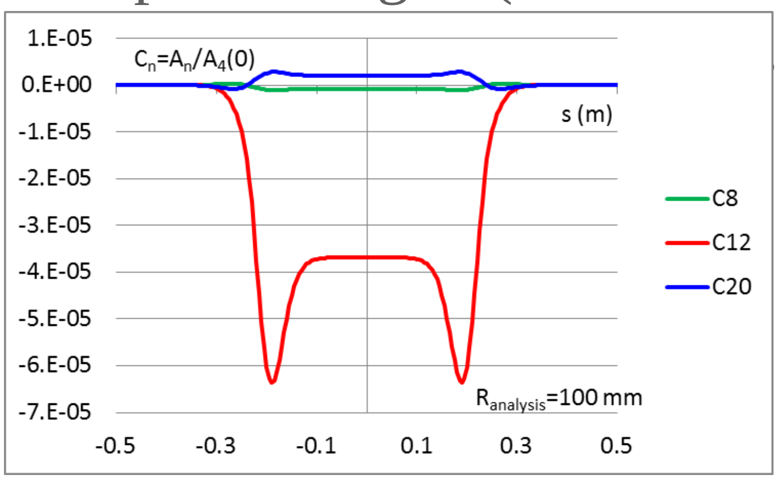


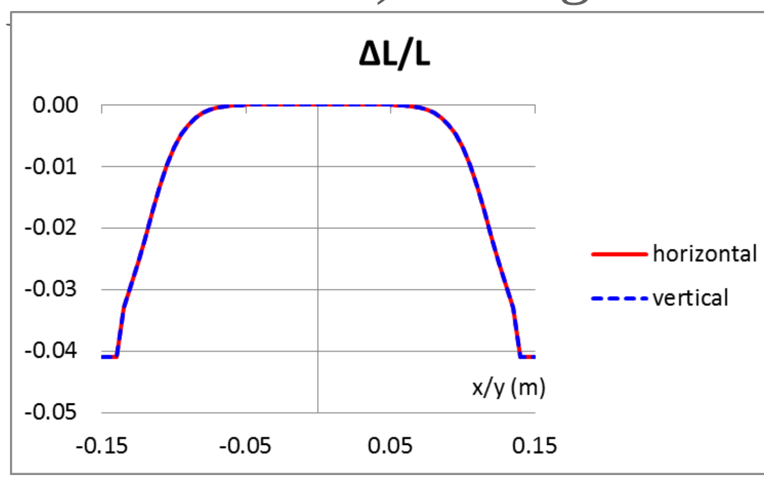
Octupole analysis





Octupolar integral (from -1500 to +1500 mm) homogeneities





Harmonic analysis

Magnetic length on the axis : 452 mm

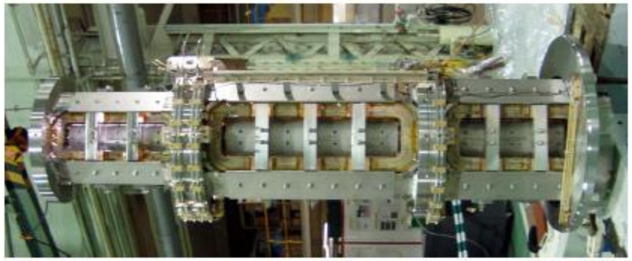
Conclusion

- Simulation studies were done to look at the feasibility of Superconducting option for S3 multipole magnets
- 3D cos-theta magnets were chosen as the basis for magnet bids
- New coil models have been implemented in COSY-Infinity code

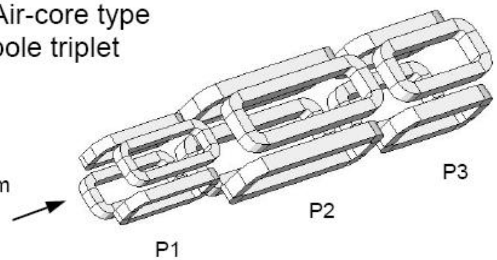
Thank You!

BigRIPS superconducting quad triplets

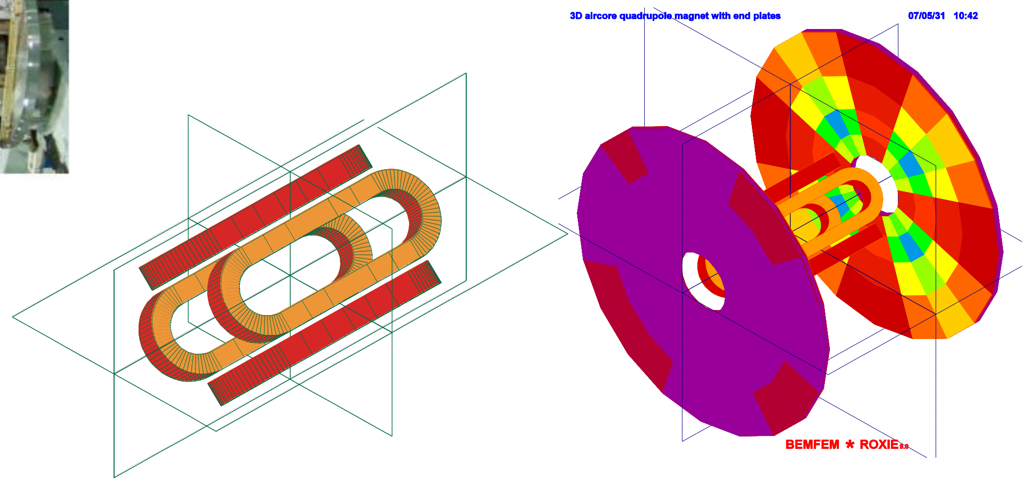
STQ1 cold mass

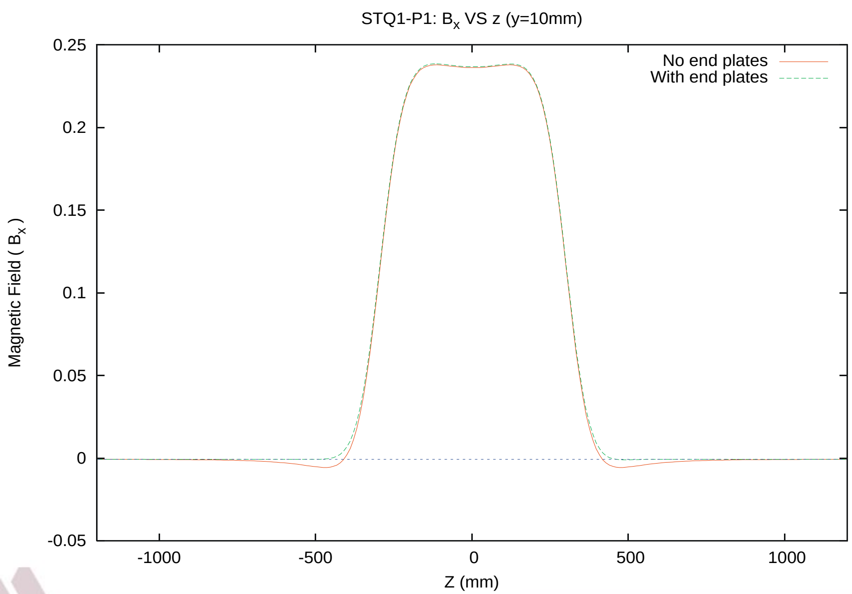


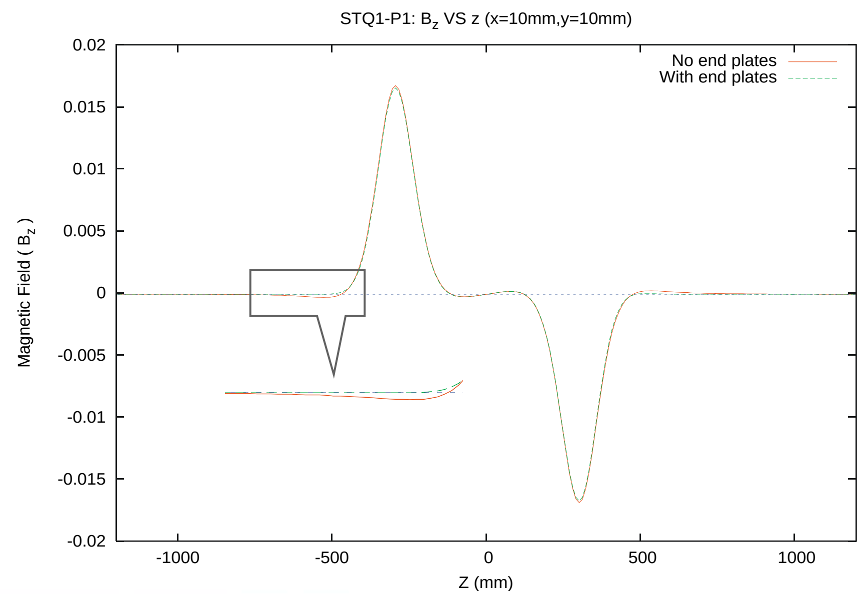
STQ1: Air-core type quadrupole triplet



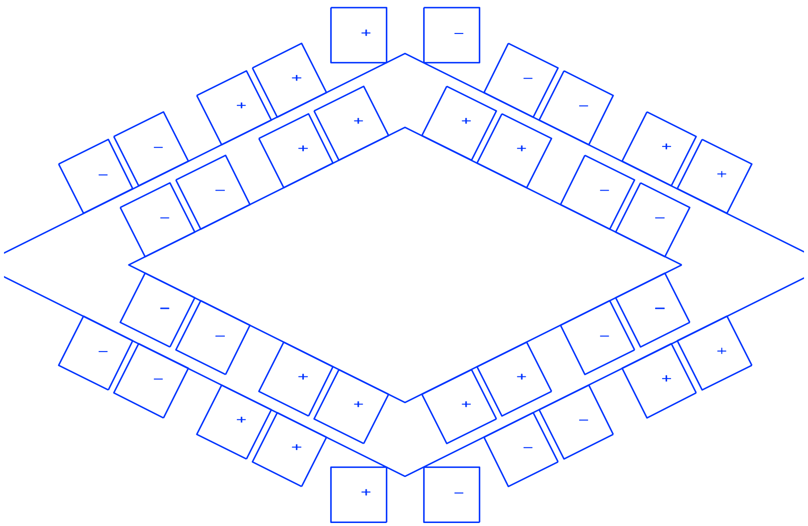
Roxie model to study the effect of end plates

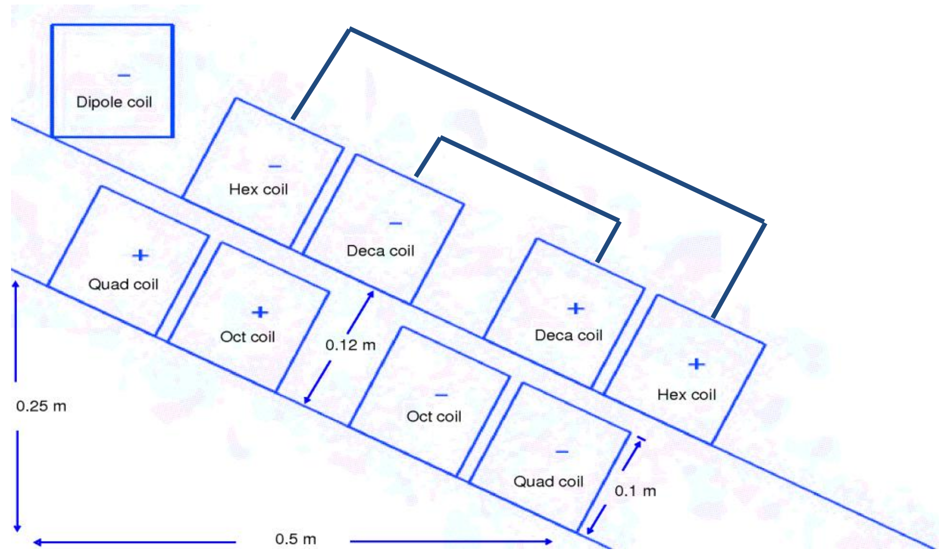






Design of quadrupole magnet with an elliptic cross section





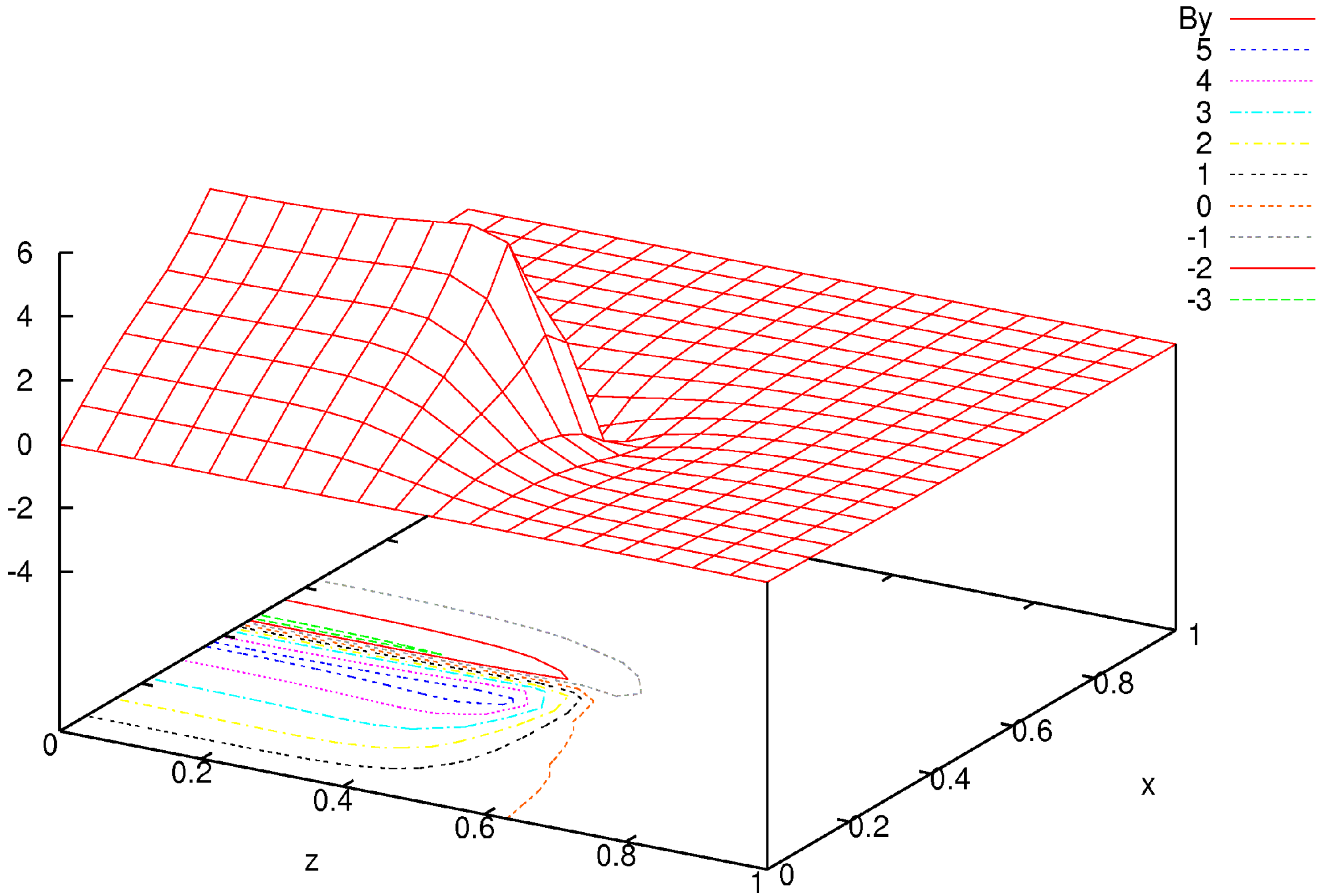
- 18 superconducting racetrack coils (±10^8A/m^2)
- Rhombic prism support structure (elliptic aperture 1:2)

- "+" produces a positive multipole term
- Inner wires produce quadrupole and octupole fields
- Outer wires produce hexapole and decapole fields
- 2D case: two Infinitely long current wires
- 3D case: Current Coil

Using DA we can make the currents as parameters and find the functional dependence
Of the multipole components on the coil currents.

3D Design: Fringe field

The plot of the magnetic field on the midplane, y = 0 m. Only the magnetic field in the first quadrant is shown.



The relationship between the currents and the principle multipole components can be given by a simple matrix

$$\begin{bmatrix} B_0^y \\ B_{(x)}^y \\ B_{(xx)}^y \\ B_{(xxx)}^y \\ B_{(xxxx)}^y \end{bmatrix} = \begin{bmatrix} 0 & 0 & -0.25 & -0.04 & +0.37 \\ +5.76 & +2.40 & 0 & 0 & 0 \\ 0 & 0 & -3.89 & -2.08 & -1.45 \\ -0.40 & +15.44 & 0 & 0 & 0 \\ 0 & 0 & +1.66 & -2.32 & +2.99 \end{bmatrix} \cdot \begin{bmatrix} QI \\ OI \\ HI \\ DI \\ I \end{bmatrix}$$

$$B_{(yy)}^y = -B_{(xx)}^y$$

$$B_{(xyy)}^y = -3B_{(xxx)}^y$$

$$-\frac{B_{(xxyy)}^y}{6} = B_{(yyyy)}^y = B_{(xxxx)}^y$$

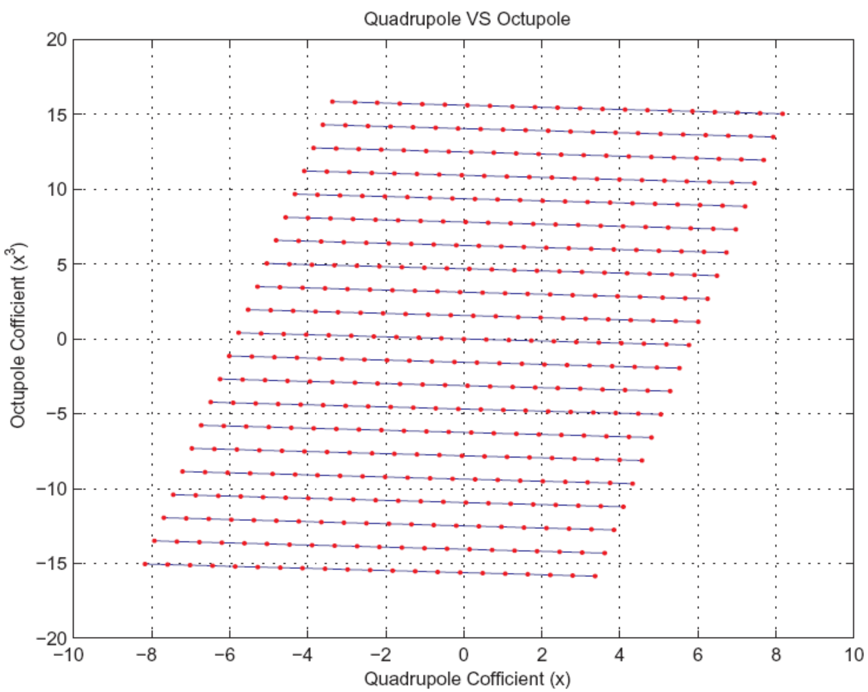
$$B_{(y)}^x = B_{(x)}^y$$

$$B_{(xy)}^x = 2B_{(xx)}^y$$

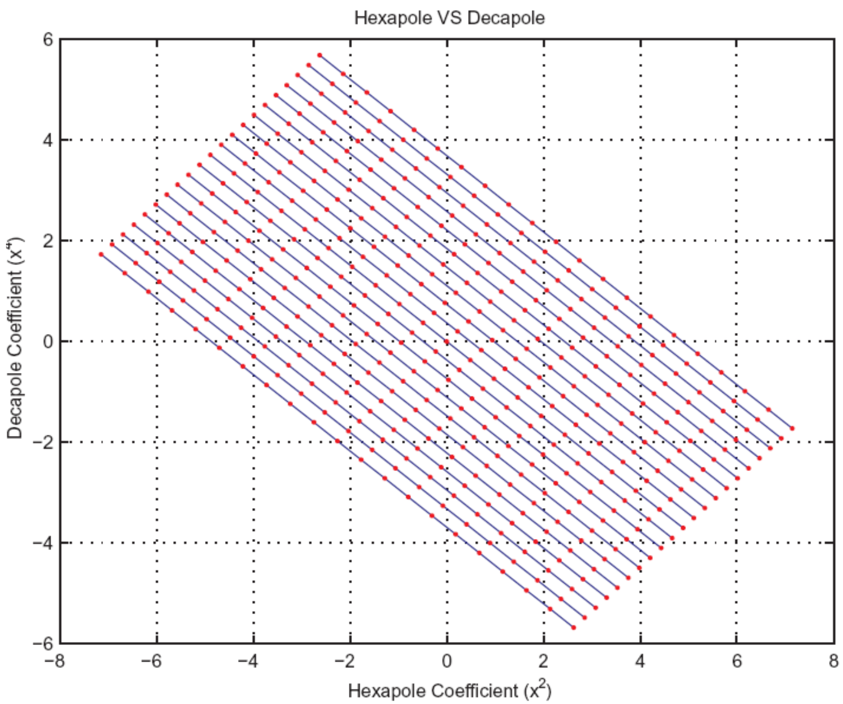
$$\frac{B_{(xxy)}^x}{3} = -B_{(yyy)}^x = B_{(xxx)}^y$$

$$B_{(xxxy)}^x = -B_{(xyyy)}^x = 4B_{(xxxx)}^y$$

Operational Plot



Quadrupole and the octupole terms

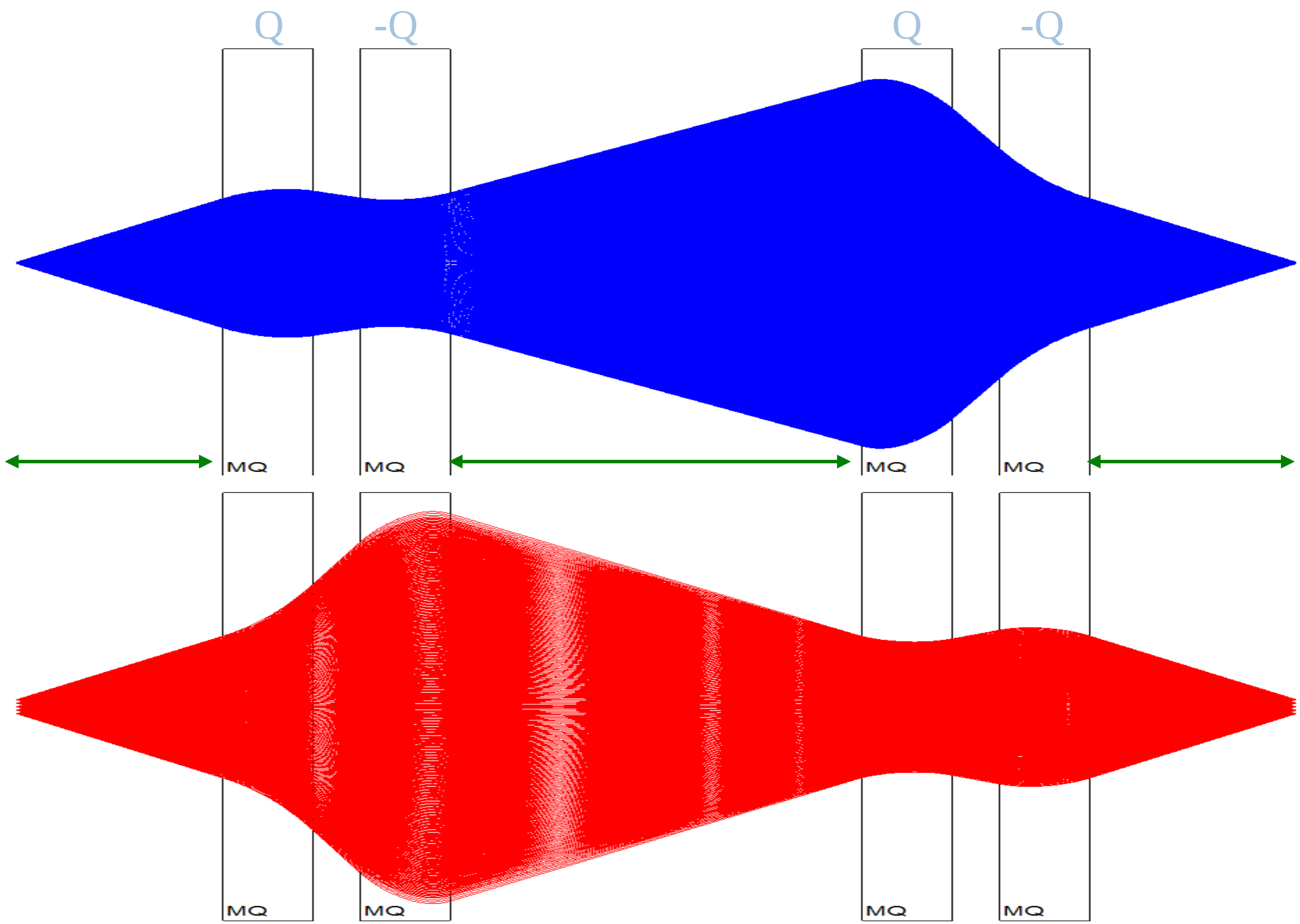


Hexapole and the Decapole terms

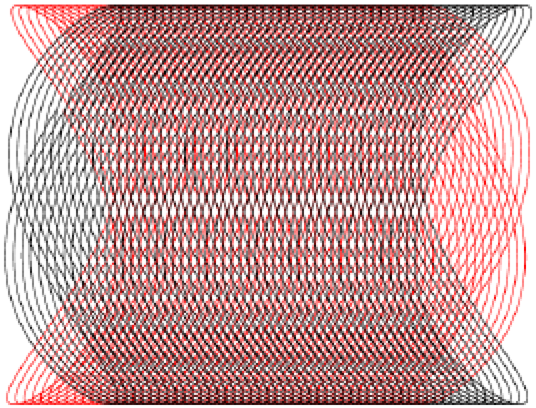
- The coefficients are computed at the horizontal half aperture
- The current density was varied between $\pm 10^8 A/m^2$

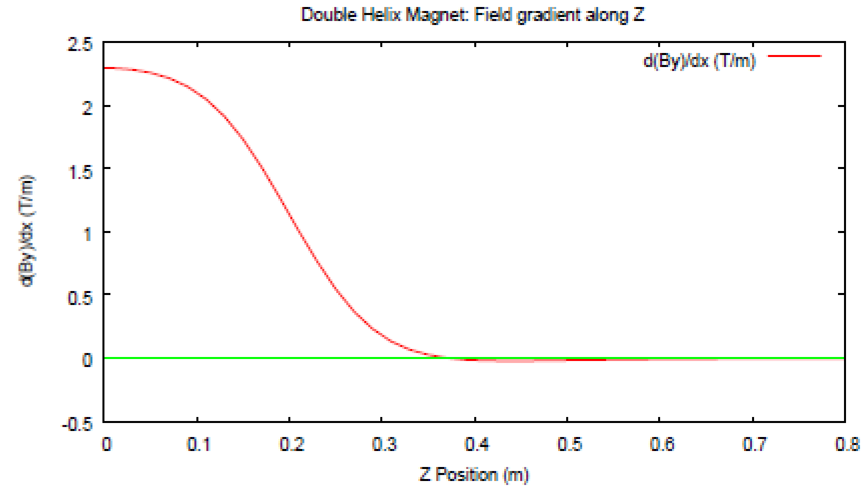
"Manikonda, S.; Nolen, J.; Berz, M. & Makino, K. (2009), 'Conceptual design of a superconducting quadrupole with elliptical acceptance and tunable higher order multipoles', Int. J. Mod. Phys. A24, 923-940."

Double Doublet System

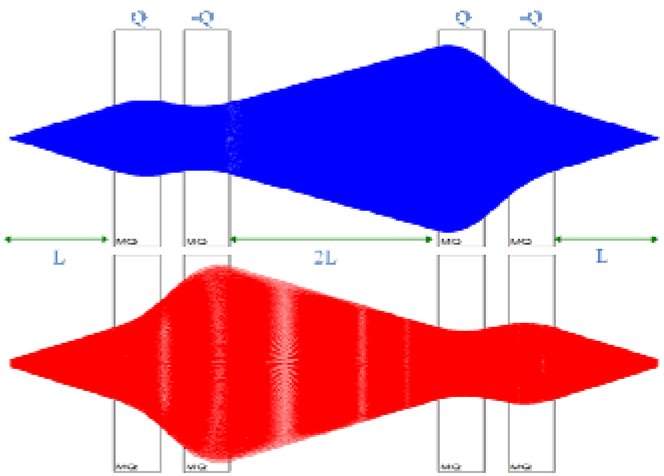


Negative Unit Transfer Map at First Order

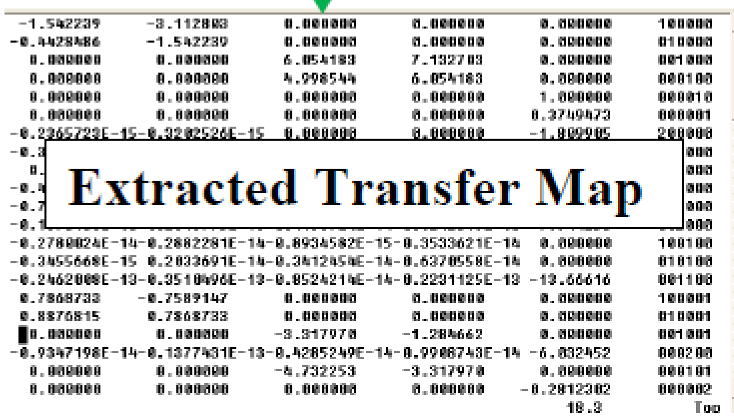




Double Helix Magnet: Field gradient along Z
d(By)/dx (T/m)
d(By)/dx (T/m)
Z Position (m)
2.5
2
1.5
1
0.5
0
-0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8



Q
-Q
Q
-Q
L
2L
L



-1.542239 -3.112803 0.000000 0.000000 0.000000 100000
-0.4428486 -1.542239 0.000000 0.000000 0.000000 010000
0.000000 0.000000 6.054183 7.132703 0.000000 001000
0.000000 0.000000 4.998544 6.054183 0.000000 000100
0.000000 0.000000 0.000000 0.000000 1.000000 000010
0.000000 0.000000 0.000000 0.000000 0.3749473 000001
-0.2365723E-15-0.3202526E-15 0.000000 0.000000 -1.809905 200000
Extracted Transfer Map
-0.2780024E-14-0.2882281E-14-0.8934582E-15-0.3533621E-14 0.000000 100100
-0.3455668E-15 0.2033691E-14-0.3412454E-14-0.6370558E-14 0.000000 010100
-0.2462008E-13-0.3510496E-13-0.8524214E-14-0.2231125E-13 -13.66616 001100
0.7868733 -0.7589147 0.000000 0.000000 0.000000 100001
0.8876815 0.7868733 0.000000 0.000000 0.000000 010001
0.000000 0.000000 -3.317970 -1.284662 0.000000 001001
-0.9347198E-14-0.1377431E-13-0.4285249E-14-0.9908743E-14 -6.832452 000200
0.000000 0.000000 -4.732253 -3.317970 0.000000 000101
0.000000 0.000000 0.000000 0.000000 -0.2812302 000002
18.3 Top

Comparison of double-helix with hardedge model

$^{48}Ca + ^{248}Cm \rightarrow ^{292}116 + 4n$

Aberration Term	Double-Helix (m)	Hard-edge Model (m)
(x|aδ)	0.8193619E-02	0.8142576E-02
(x|aaa)	-0.8679288E-02	-0.1505131E-02
(x|abb)	-0.8328710E-02	-0.2521999E-03
(x|abbδ)	-0.1361716E-02	small
(x|aδδ)	small	-0.3458687E-03
(x|aa)	0.1735195E-03	small
(x|bδδ)	-0.3518127E-03	small
(x|aaaδ)	0.2515068E-03	0.1440338E-03
(y|bbb)	-0.2327891E-01	-0.1505130E-02
(y|bbδ)	-0.8328712E-02	small
(y|ybb)	-0.4768876E-02	small
(y|bbbδ)	0.1896777E-02	small
(y|bδ)	0.8193619E-02	0.8142576E-02