High-Order Taylor Model Enclosures of Invariant Manifolds of ODEs

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A method to construct very accurate polynomial approximations of invariant manifolds of hyperbolic critical points in sufficiently smooth autonomous ordinary differential equations is presented. This construction is performed using differential algebra methods and also works in resonant cases as well as in arbitrary dimensions.

The constructed polynomial manifold approximation is then outfit with a tight remainder bound (typically of size 10^{-12} or less), the correctness of which is verified using Taylor Model techniques. While the methods presented work in arbitrary dimension, special attention is given to the three dimensional case. The result is a very sharp, verified high-order enclosure of the local invariant manifolds. Using verified integration, these local manifold pieces can be integrated to obtain rigorous enclosures of large parts of the invariant manifolds.

We show various examples of the use of the methods, including manifolds of the three dimensional Lorenz ODE.