Chaos in the Lorenz System for Many Parameters

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We consider the famous Lorenz system of differential equations given by the three dimensional vector field $X(x, y, z) = (s(y-x), (r-z)x-y, xy-bz)$. This is a vector field which depends on the three real parameters r, s, b. We consider the line segment in the parameter space given by $s = 10, b = 8/3$ and $25 \leq r \leq 80$. We prove that for all of these parameters the system exhibits chaotic motion. The precise definition is that the topological entropy is bounded below by $log(2)/8$. This implies, in particular, that there exist closed invariant sets with dense orbits and exponentially many periodic orbits. The methods of proof combine techniques of topological dynamics and verified computation using the COSY INFINITY integrator. This lecture will focus on the topological dynamics parts of the proof. Other lectures will give details on the verified computation.