Interval Methods and Taylor Model Methods for ODEs

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Traditional numerical methods for the integration of initial value problems for ODEs suffer from two kinds of errors. First, there is the approximation error, for which, in general, only some rough estimate describing the order of the method is available. Second, there is the accumulated roundoff error in a practical floating point calculation. The true amount of these errors is not known, and there are situations where they are badly underestimated.

Validated methods for ODEs aim at bounding all errors rigorously. Interval arithmetic has been used for almost 50 years for calculating accurate bounds for solutions of initial value problems. More than a decade ago, Taylor model methods for ODEs have been developed by Berz and Makino by combining interval arithmetic with computer algebra. Taylor models reduce both the dependency problem and the wrapping effect intrinsic to interval arithmetic. They also benefit from their capability of representing non-convex sets, which is especially advantageous for enclosing the flow of a nonlinear ODE.

In our introductory talk, we compare interval methods with Taylor model methods, outlining similarities and distinctions.