

$$M_k^n = \left\{ f \in C^k \text{ on } B_1^n, \|f\|_0 \leq 1 \right\}$$

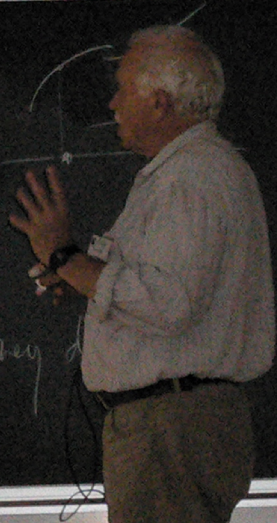
$$\|f\|_0 = \max |f|$$

$$\text{Th } H(M_k^n, \epsilon) \asymp \left(\frac{1}{\epsilon}\right)^{\frac{n}{k}}$$

$$M(\epsilon, n) \leq \left(\frac{C}{\epsilon}\right)^{\frac{n}{k}}$$

$$H \leq \left(\frac{1}{\epsilon}\right)^{\frac{n}{k}} \cdot \log\left(\frac{1}{\epsilon}\right)$$

⇒ Whitney



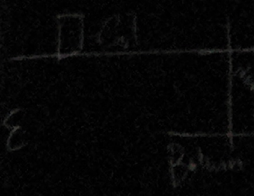
P^1 -Torus

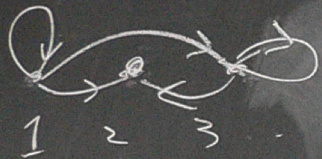
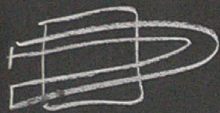


$$\left(\frac{1}{h}\right)^n = \left(\frac{1}{\epsilon}\right)^{\frac{n}{k}}$$

$$|T - P^1| \sim h^k = \epsilon$$

$$\Rightarrow h \sim \epsilon^{1/k}$$



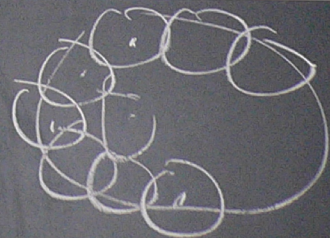


$$\begin{pmatrix} 1 & 1 & 7 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

- Singularities
- Noise
- Complexity

Metric entropy:

X ,

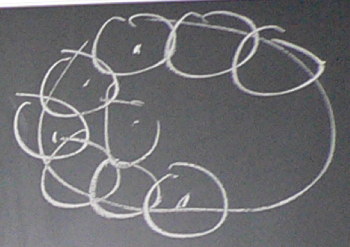


$$C_n \# M(\epsilon, X) = \dots$$

$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

Metric entropy:

X ,



Cov. # : $M(\epsilon, X) = \text{min \# of } \epsilon\text{-balls, covering } X$.

$$H(\epsilon, X) = \log_2 M(\epsilon, X)$$

$$M_k^n = \left\{ f \in C^k \text{ on } B_{\perp}^n, \|d^j f\|_0 \leq 1 \right\}$$

$$\|f\|_0 = \max |f|$$

Th. $H(M_k^n, \varepsilon) \asymp \left(\frac{1}{\varepsilon}\right)^{\frac{n}{k}}$

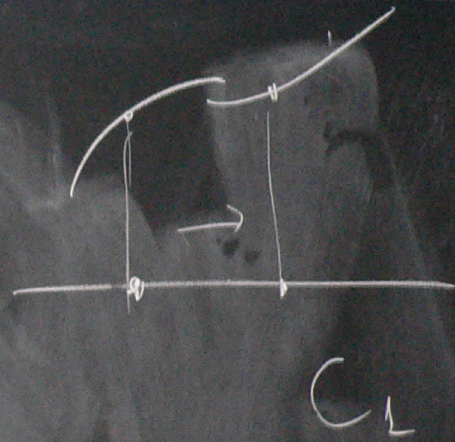
$$M(\varepsilon, M_k^n) \leq \left(\frac{c}{\varepsilon}\right) \left(\frac{1}{\varepsilon}\right)^{\frac{n}{k}}$$



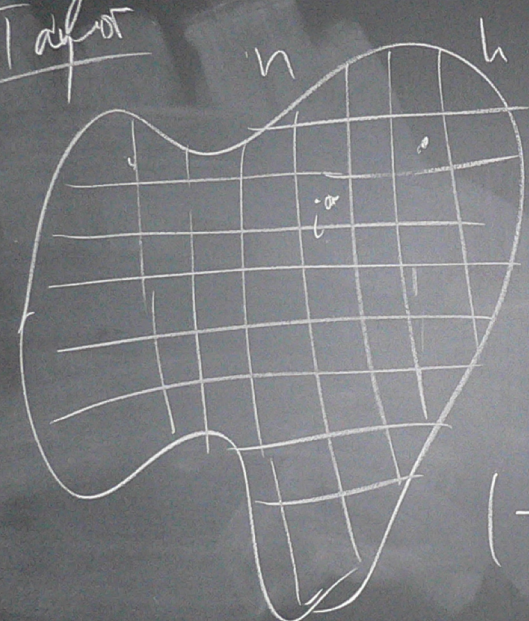
$$M(\varepsilon, M^*) \leq \left(\frac{C}{\varepsilon}\right) \left(\frac{1}{\varepsilon}\right)^{\frac{n}{k}}$$

$$\mathcal{H} \leq \left(\frac{1}{\varepsilon}\right)^{\frac{n}{k}} \cdot \log\left(\frac{1}{\varepsilon}\right)$$

\Rightarrow Whitney discrepancy



P^i - Taylor

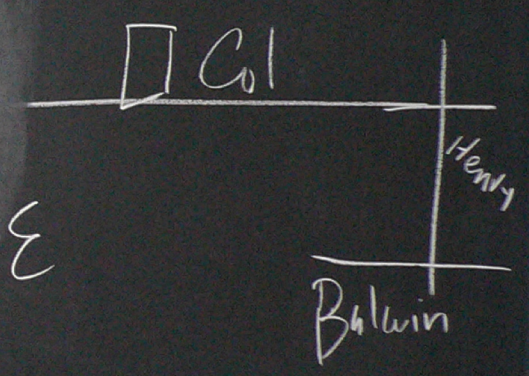


$$\left(\frac{L}{h}\right)^n = \left(\frac{L}{\varepsilon}\right)^{\frac{n}{k}}$$

$$\frac{C}{\varepsilon}$$

$$|f - P^i| \sim h^k = \varepsilon$$

$$\Rightarrow h \sim \varepsilon^{\frac{1}{k}}$$



Handwritten notes on the left chalkboard:

- Separability
- Noise
- Complexity

Handwritten notes on the middle chalkboard:

- Kolmogorov's ϵ -entropy C^k
- Whitney extension
- Generalized Kernel interpolation

Handwritten notes on the right chalkboard:

Horizontal	
	$\square \square \square$
Vertical	
	Diagonal

