

Computer Assisted Proof of the existence of High Period Fixed Points in the Hénon Map

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Abstract

We will present several methods to numerically verify the existence of attracting and hyperbolic high period fixed points. The underlying mathematical methods used are the Miranda and Schauder fixed point theorems. For the numerical calculations we will use intervals and Taylor Models. Specifically we will prove the existence of fixed points of order 15 in a Hénon map near the standard parameters and several fixed points up to order 27 in an area preserving Hénon map to illustrate our methods. We will show how to use interval methods for the attracting fixed points and point out the numerical problems that automatically arise from that approach and how to mitigate them using higher precision intervals. Then we will use Taylor Models to perform the same verification. By doing so we can speed up the process significantly and reduce the precision needed to perform the calculations. In the last part we will present the limitations even Taylor Models suffer from due to limited floating point precision and how that problem will be solved by the advent of Higher Precision Taylor Models.