

Rigorous Classification of Manifold Tangles and Bounds for Entropy

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Abstract

It is first shown how to obtain rigorous and tight enclosures of extended pieces of unstable and stable manifolds using Taylor model methods. This is achieved by enclosing manifold pieces in the set sum of the range of a high order polynomial and an small remainder bound box. The manifolds are trapped in a "thin" strip or higher-dimensional sheet, and it is assured that the manifold enters and exits them through the thin ends. The collection of these Taylor model representations is rigorously iterated further and further, resulting in an enclosure of extended pieces of manifold. Using these manifold enclosures in combination with a rigorous global optimizer, it is possible to determine rigorous and sharp enclosures of all homoclinic points generated by the manifolds. As a result, we obtain a collection of tiny boxes, each of which is assured to contain at least one homoclinic point. Furthermore, using the collections of enclosures of all homoclinic points of the n -th and $(n+1)$ st iterate of the manifolds, it is possible to establish rigorous mapping properties of all homoclinic points of the n -th iterate with corresponding homoclinic points of the $(n+1)$ st iterate. As a result, we obtain a homeomorphic representation of the tangle called a "brain", from which topological rectangles can be constructed automatically from neighboring homoclinic points and two stable and two unstable manifold pieces connecting them that have the same orientation. Using the mapping properties of homoclinic points, Markov crossings can be identified, and the symbolic dynamics of the rectangles can be determined automatically. Results utilizing the approach are shown, resulting in lower bounds for entropy for the standard Henon map that exceed all other currently known values.