Rigorous Global Optimization of Impulsive Space Trajectories

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Motivation

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- Space activities are expensive:

  Ariane 5 launch cost: 200 M$ ÷
  Allowed Spacecraft Mass: 10000 kg =
  Cost per kilogram: 20000 $/kg

  → more expensive than gold (as expensive as saffron)

- Propellant represents the main contribution to s/c mass:
  - Propellant is on average 40% of spacecraft mass

  → we want to reduce the required propellant

- The goal of the trajectory design is to find the best solution in terms of propellant consumption while still achieving the mission goals

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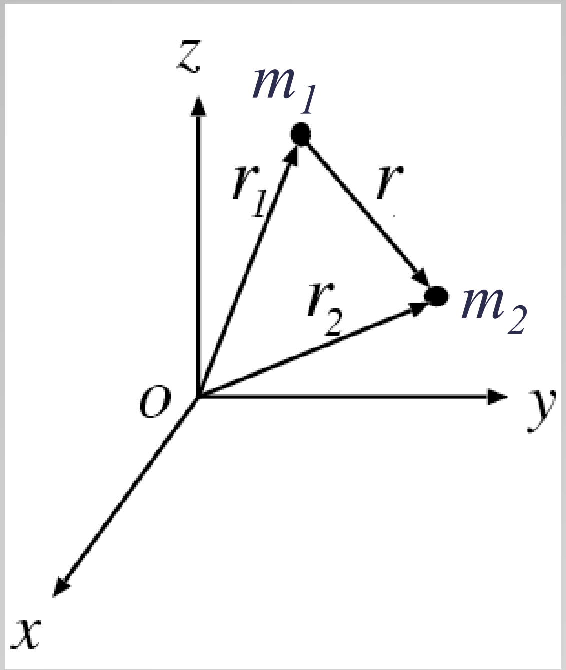
Outline

- Dynamical Model
- Patched-Conics Approximation
- Two-Impulse Transfers
  - Ephemerides Evaluation
  - Lambert´s Problem Solution
- Differential Algebra Based Global Optimization
- Rigorous Global Optimization with COSY-GO

Dynamical Model: 2-Body Problem

- The 2-Body Problem considers two point masses in mutual orbit about each other



The relative motion of the two masses is governed by:

$$\ddot{\vec{r}} = -\frac{k}{r^3}\vec{r}$$

E.g.

m_1 ⟶ Sun

m_2 ⟶ Spacecraft

Analytical solutions exist for the 2-Body Problem: *Conic Arcs*

- $\vec{r} = \vec{r}(\theta)$ ⟶ explicit
- $t = t(\theta)$ ⟶ implicit (Kepler´s equation)

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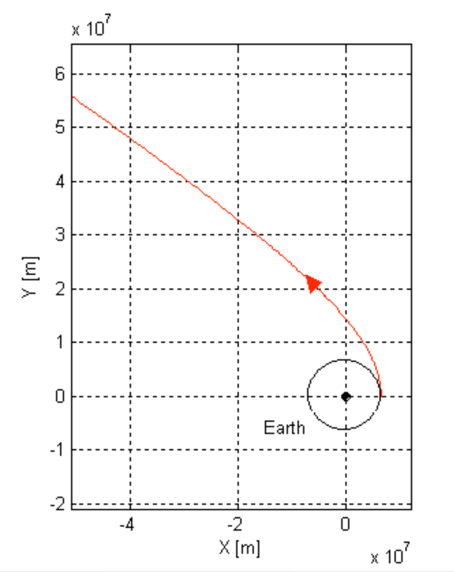
Patched-Conics Approximation

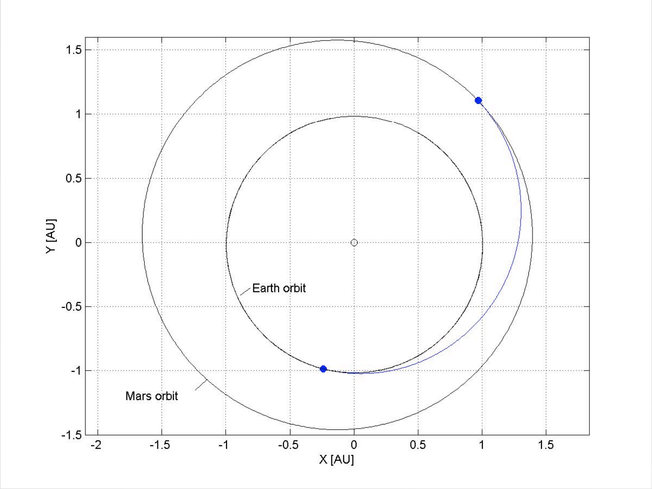
- The whole interplanetary transfer is divided in several arcs
- Each arc is the solution of a 2-Body Problem considering the spacecraft and only one other planet at a time

E.g.: 2-impulse Earth-Mars transfer ⟶ 3 conic arcs

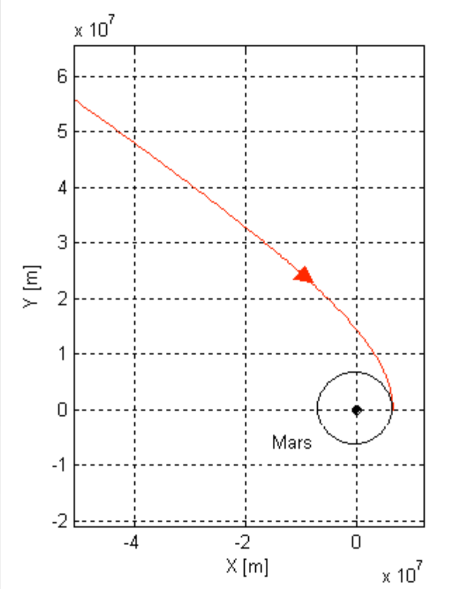
Earth escape



Heliocentric phase



Mars capture



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2-Impulse Planet-to-Planet Transfer

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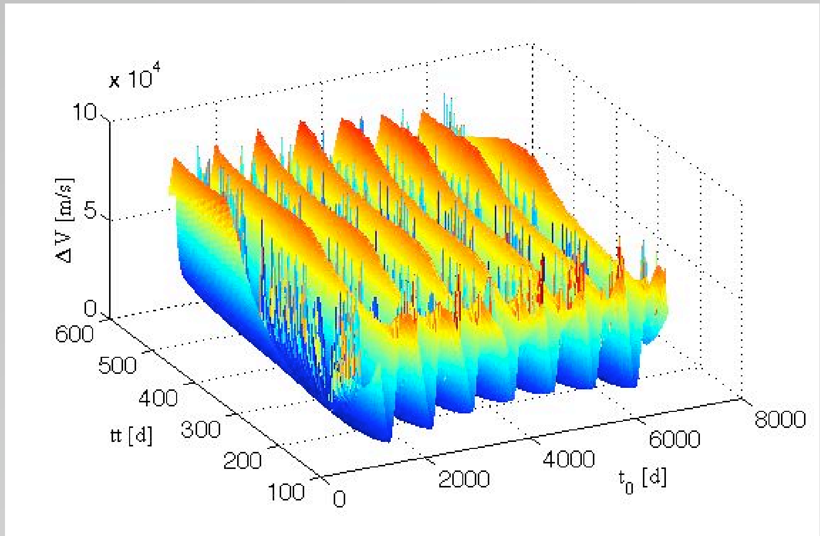
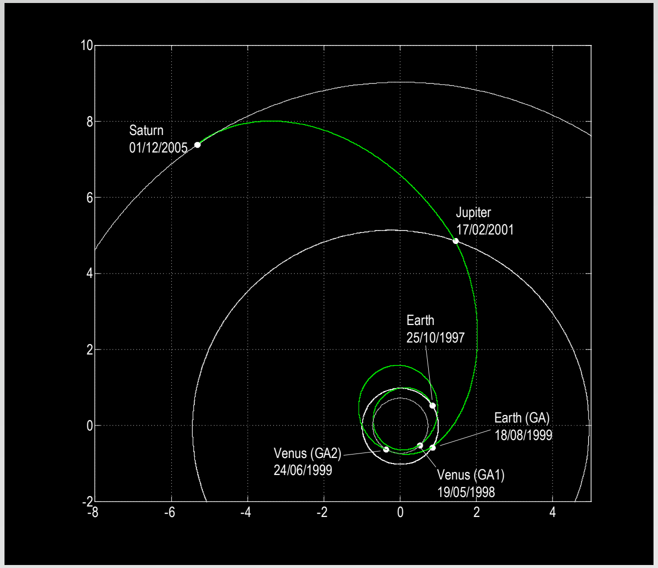
- 2-impulse Earth-Mars transfer has been selected as first benchmark problem
  - Applied for preliminary design of Earth-Mars interplanetary transfers
  - Objective function characterized by several comparable local minima



- Future benchmark problems
  - Multiple Gravity Assist interplanetary transfers

    E.g.: Cassini-Huygens (11 conic arcs)



Optimization Problem

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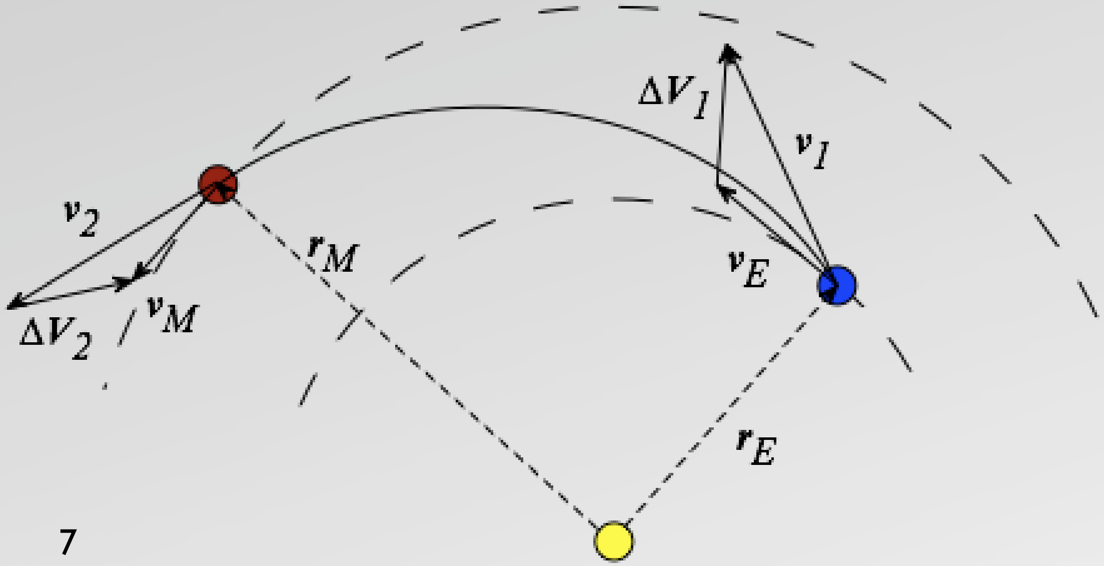
- The optimization variables are the time of departure t_0 and the time of flight t_{tof}
- The positions of the starting and arrival planets are computed through the ephemerides evaluation:

  $(\boldsymbol{r}_{E,}\ \boldsymbol{v}_E) = \text{eph}(t_0, \text{Earth})$ and $(\boldsymbol{r}_{M,}\ \boldsymbol{v}_M) = \text{eph}(t_0 + t_{tof}, \text{Mars})$
- The starting velocity v_1 and the final one v_2 are computed by solving the Lambert´s problem
- Objective function:

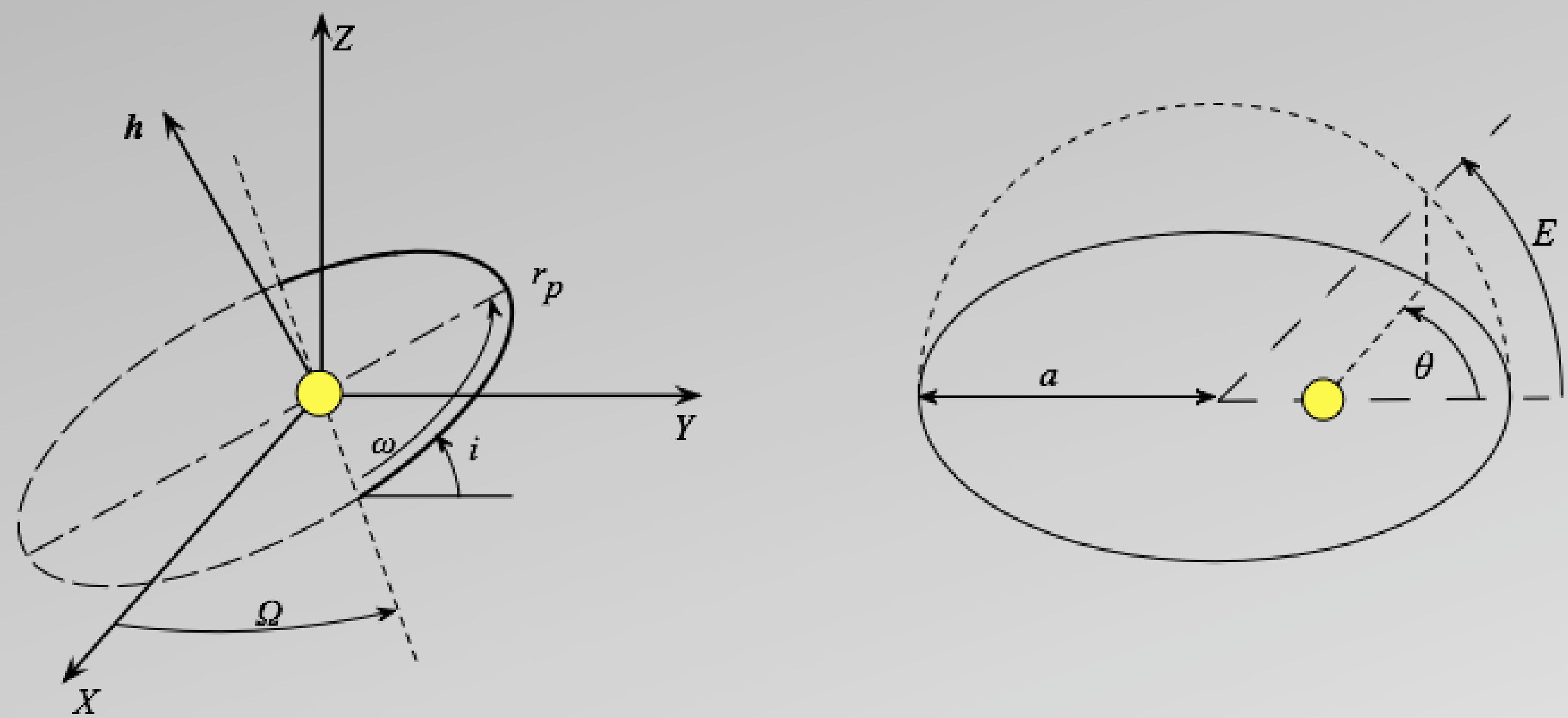
  $\Delta V = \Delta V_1 + \Delta V_2$
- Constraint:

  $\Delta V_1 < \Delta V_{1,max}$



Orbital parameters

- The orbital parameters are: $(a, e, i, \Omega, \omega, \theta)$



- The position and the velocity (***r***, ***v***) in cartesian coordinates are obtained from the orbital parameters by simple algebraic relations

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Ephemerides Evaluation

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- Polynomial interpolations of accurate planetary ephemerides (JPL-Horizon) are used for the preliminary phase of the space trajectory design
- Given an epoch and a celestial body, its orbital parameters $(a, e, i, \Omega, \omega, M)$ can be analytically evaluated
- The nonlinear equation $M = E - e\sin E$ (Kepler´s Eq) is solved for the eccentric anomaly E
- The relation $\tan\frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}$ delivers θ
- The position and the velocity (***r***, ***v***) of the celestial body in inertial frame reference frame are computed

→ We have to solve an implicit equation: Kepler´s equation

Lambert´s Problem

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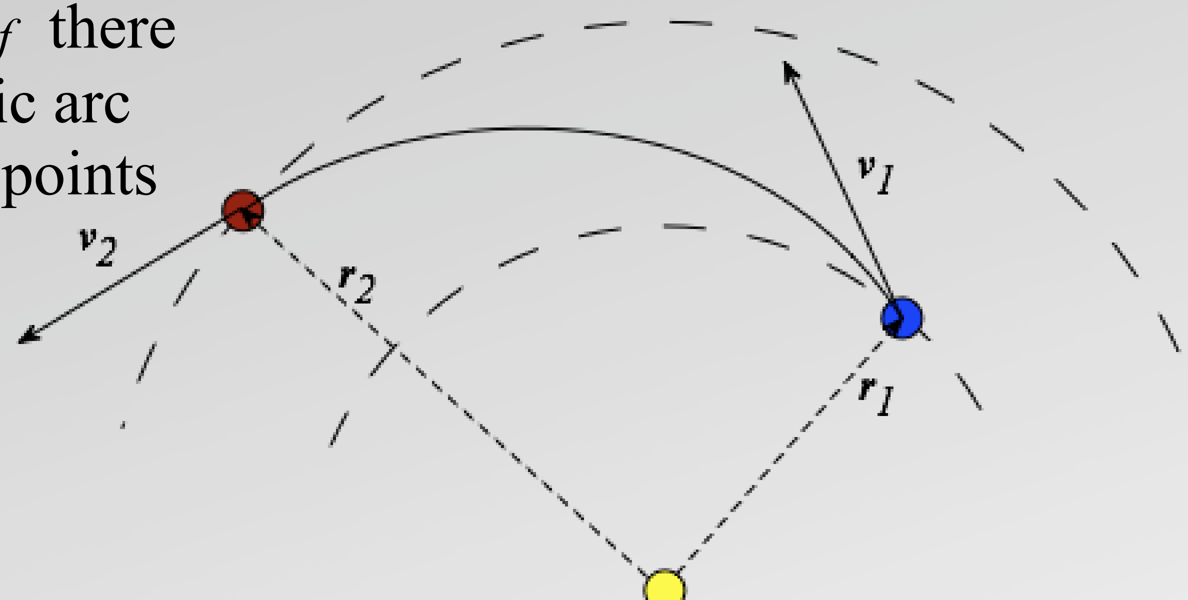
Given:

- initial position $\boldsymbol{r}_1$
- final position $\boldsymbol{r}_2$
- time of flight t_{tof}

Find the initial velocity, $\boldsymbol{v}_1$, the spacecraft must have to reach $\boldsymbol{r}_2$ in t_{tof}

- The solution of the BVP exploits the analytical solution of the 2-body problem

- Given $\boldsymbol{r}_1$, $\boldsymbol{r}_2$ and t_{tof} there exists only one conic arc connecting the two points in the given time



Lambert´s Problem

- Several algorithms have been developed for the identification and characterization of the resulting conic arc
- We used an algorithm developed by Battin (1960)
- A nonlinear equation must be solved (Lagrange´s equation for the time of flight):

$$f(x) = \log(A(x)) - \log(t_{tof}) = 0$$

in which $A(x) = a(x)^{3/2}((\alpha(x) - \sin(\alpha(x))) - (\beta(x)))$,

$\beta(x) = 2\arcsin\left(\frac{s-c}{2a(x)}\right)$, and $a(x) = \frac{s}{2(1-x^2)}$

- The value of s and c depend on $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$, so the nonlinear equation depends both on t_0 and t_{tof}

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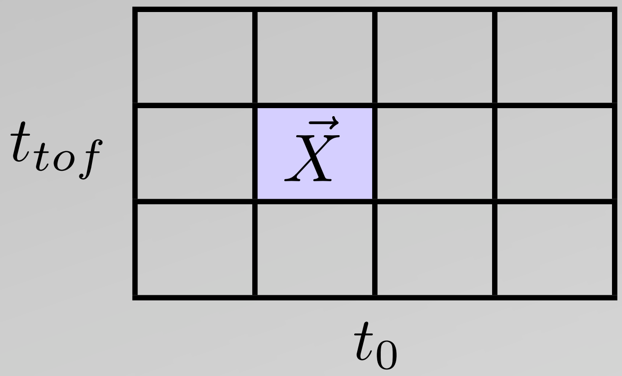
DA Based Global Optimizer (1/2)

DA based global optimization algorithm:

- Subdivide the search space in subintervals
- Suitably initialize the value of ΔV_{opt}



For each subinterval $\vec{X}$:

- Initialize t_0 and t_{tof} as DA variables and compute a Taylor expansion of the objective function ΔV and the constraint ΔV_1 on $\vec{X}$
- Bound the value of ΔV_1 on $\vec{X}$

  IF $\min \Delta V_1 > \Delta V_{1,max}$ ⟶ discard $\vec{X}$

- Bound the value of ΔV on $\vec{X}$

  IF $\min \Delta V > \Delta V_{opt}$ ⟶ discard $\vec{X}$

DA Based Global Optimizer (2/2)

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- Build and invert the map of the objective function gradient:

$$\begin{pmatrix} \nabla_{t_0} \Delta V \\ \nabla_{t_{tof}} \Delta V \end{pmatrix} = \mathcal{M} \begin{pmatrix} t_0 \\ t_{tof} \end{pmatrix} \rightarrow \begin{pmatrix} t_0 \\ t_{tof} \end{pmatrix} = \mathcal{M}^{-1} \begin{pmatrix} \nabla_{t_0} \Delta V \\ \nabla_{t_{tof}} \Delta V \end{pmatrix}$$

- Localize the zero-gradient point $\vec{x}^* = (t_0^*, t_{tof}^*)$

  IF $\vec{x}^* \notin \vec{X}$ $\longrightarrow$ discard $\vec{X}$

- Evaluate $\Delta V^* = \Delta V(\vec{x}^*)$

  IF $\Delta V^* < \Delta V_{opt}$ $\longrightarrow$ update ΔV_{opt} , and store $\vec{x}^*$ and $\vec{X}$

- If necessary, a more accurate identification of the actual optimum $\vec{x}^*$ can be finally achieved using a higher order DA computation on the last stored subinterval $\vec{X}$

DA Solution of Parametric Implicit eqs

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- Search the solution of $f(x,p)=0$ for *p* belonging to $p \in [p_l, p_u]$
- Use classical methods (e.g., Newton) to compute x^0 solution of $f(x,p^0)=0$
- Initialize $[x]=x^0+\Delta x$ and $[p]=p^0+\Delta p$ as DA variables and expand $\Delta f = \mathcal{M}(\Delta x, \Delta p)$
- Build the following map and invert it:

$$\begin{pmatrix}\Delta f\\ \Delta p\end{pmatrix} = \begin{pmatrix}[\mathcal{M}]\\ [\mathcal{I}_p]\end{pmatrix}\begin{pmatrix}\Delta x\\ \Delta p\end{pmatrix} \rightarrow \begin{pmatrix}\Delta x\\ \Delta p\end{pmatrix} = \begin{pmatrix}[\mathcal{M}]\\ [\mathcal{I}_p]\end{pmatrix}^{-1}\begin{pmatrix}\Delta f\\ \Delta p\end{pmatrix}$$

- Force $\Delta f = 0$ so obtaining the Taylor expansion of the solution w.r.t. the parameter: $\Delta x = \Delta x(\Delta p)$

Example: Mars Ephemerides

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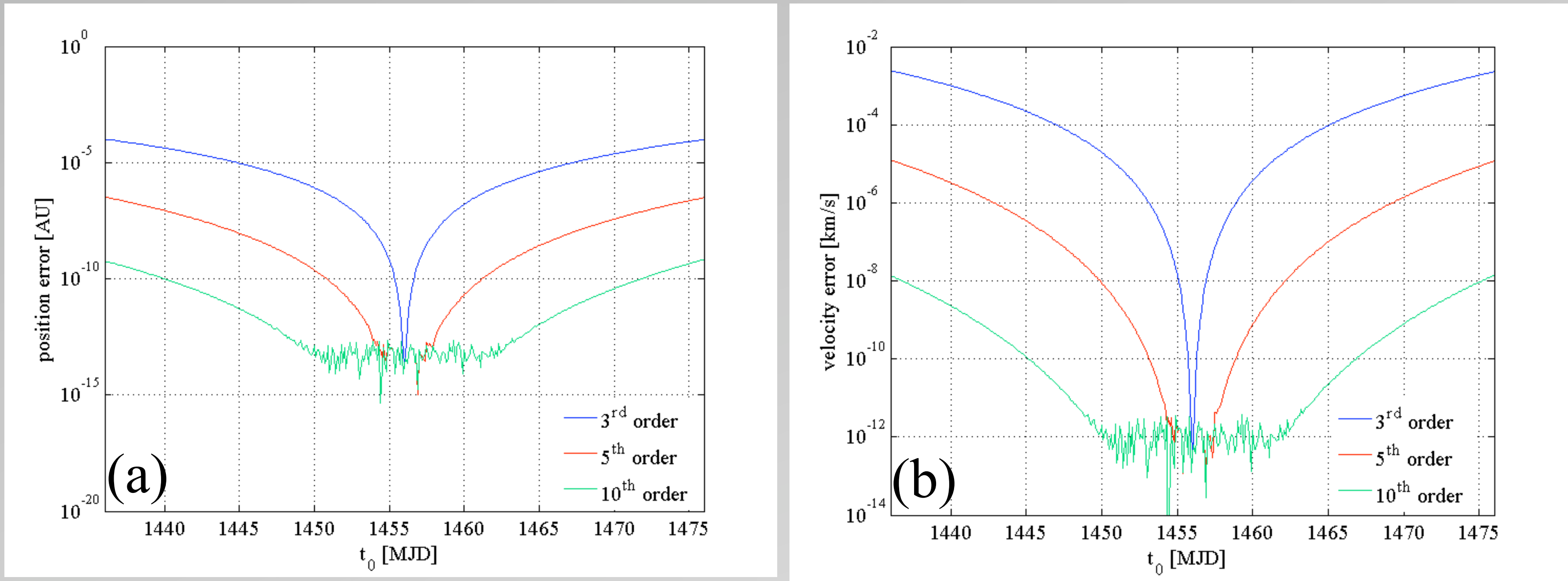
Epoch interval: 40 days



Errors on position, (a), and velocity, (b), between the DA and the point-wise evaluation of Mars ephemerides

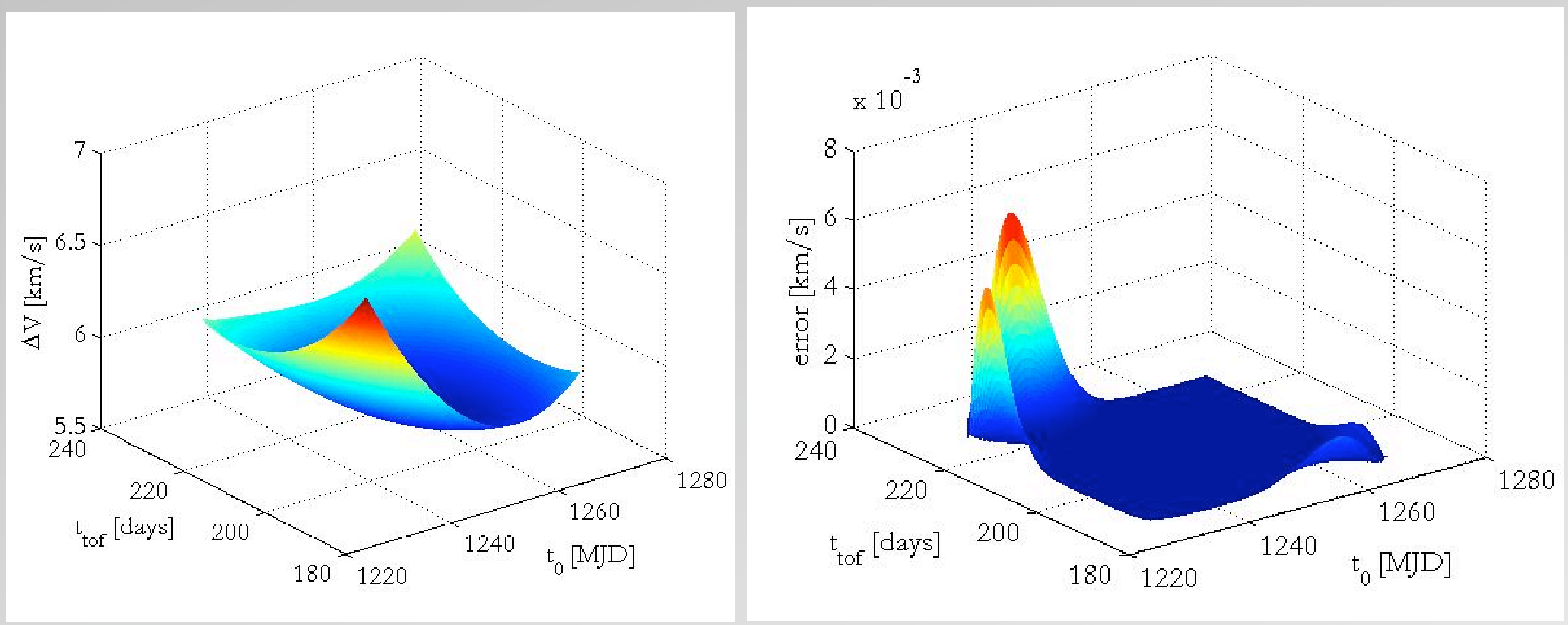
Errors drastically decrease when the order of the Taylor series increases

Example: Objective Function

- The DA evaluation of the planetary ephemerides and the Lambert´s problem solution enables the Taylor expansion of the objective function

Taylor representation of the objective function

Taylor representation error w.r.t. point-wise evaluation



Box width: 40 days

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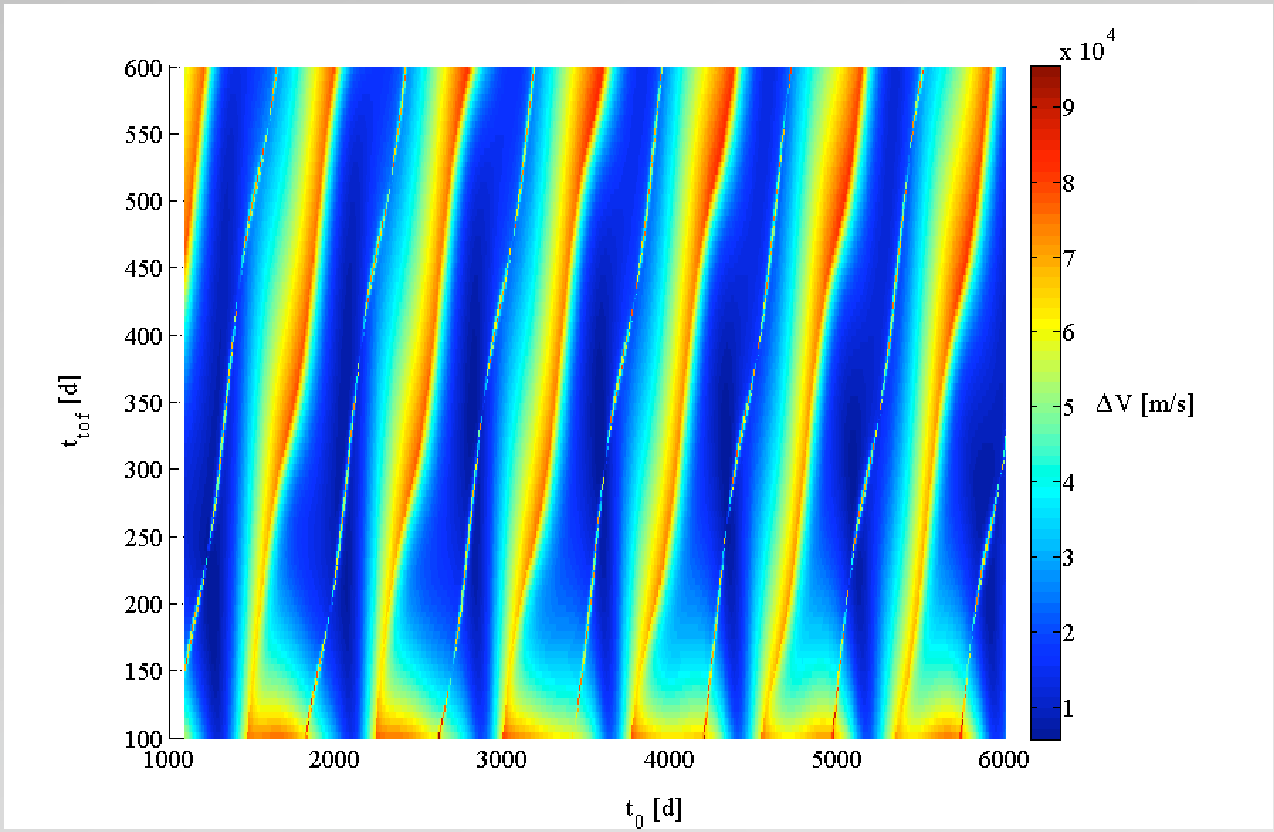
Earth-Mars Direct Transfer

Search space: $[1000, 6000] \times [100, 600]$

Maximum departure impulse: $\Delta V_1 < 5$ km/s

Platform: Pentium IV 3.06 GHz laptop

Objective function overview



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Search space: $[1000, 6000] \times [100, 600]$

Maximum departure impulse: $\Delta V_1 < 5$ km/s

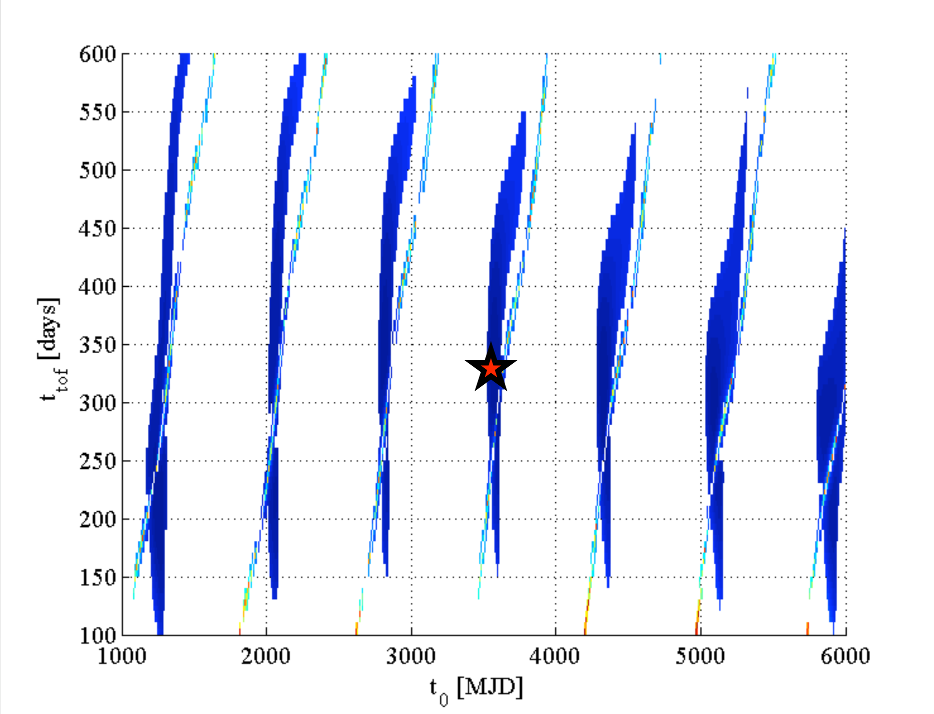
Platform: Pentium IV 3.06 GHz laptop

Solution 1:

- 10-day boxes + 5th order
- Pruning + Global Opt: 59.98 s
- $\Delta V^* = 5.6673$ km/s
- $x^* = [3573.188, 324.047]$

Solution 2:

- 100-day boxes + 5th order
- Pruning + Global Opt: 0.55 s
- $\Delta V^* = 5.6676$ km/s
- $x^* = [3573.530, 323.371]$



Verified Implicit Eq Solution - 1D

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- Suppose to have the $(n+1)$ differentiable function f over the domain $D = [-1, 1]$ and its n-th order Taylor model $P(x) + I$ so that

$$f(x) \in P(x) + I \quad \text{for all} \quad x \in D$$

- Consider the enclosure R of $P(x) + I$ over D and suppose $P'(x) > d > 0$ on D with $P(0) = 0$
- Find the Taylor Model $C(y) + J$ on R so that any solution of the problem $f(x) = y$ lies in $C(y) + J$

Algorithm:

- First compute $C(y)$, the n-th order polynomial inversion of $P(x)$, so that

$$P(C(y)) =_n y$$

- Using Taylor model computation, obtain $P(C(y)) \in y + \tilde{J}$ where $\tilde{J}$ includes the terms of order exceeding n in $P(C(y))$, and thus scales with at least order $n+1$

Verified Implicit Eq Solution - 1D

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- Use the consequences of small correction Δx to $C(y)$ to find the rigorous reminder J for $C(y)$ so that all the solutions of $f(x) = y$ lie in $C(y) + J$. According to the mean value theorem:

$$\begin{aligned} f(C(y)+\Delta x) - y &\in P(C(y)+\Delta x) - y + I \\ &= P(C(y)) + \Delta x \cdot P'(\xi) - y + I \\ &\subset y + \tilde{J} + \Delta x \cdot P'(\xi) - y + I \\ &= \Delta x \cdot P'(\xi) + I + \tilde{J} \end{aligned}$$

  for suitable $\xi \in [C(y), C(y) + \Delta x]$

- Since P' is bounded below by d, the set $\Delta x \cdot P'(\xi) + I + \tilde{J}$ will never contain the zero except for the interval

$$J = -\frac{I + \tilde{J}}{d}$$

  which is the desired interval

Verified GO of Earth-Mars Transfer

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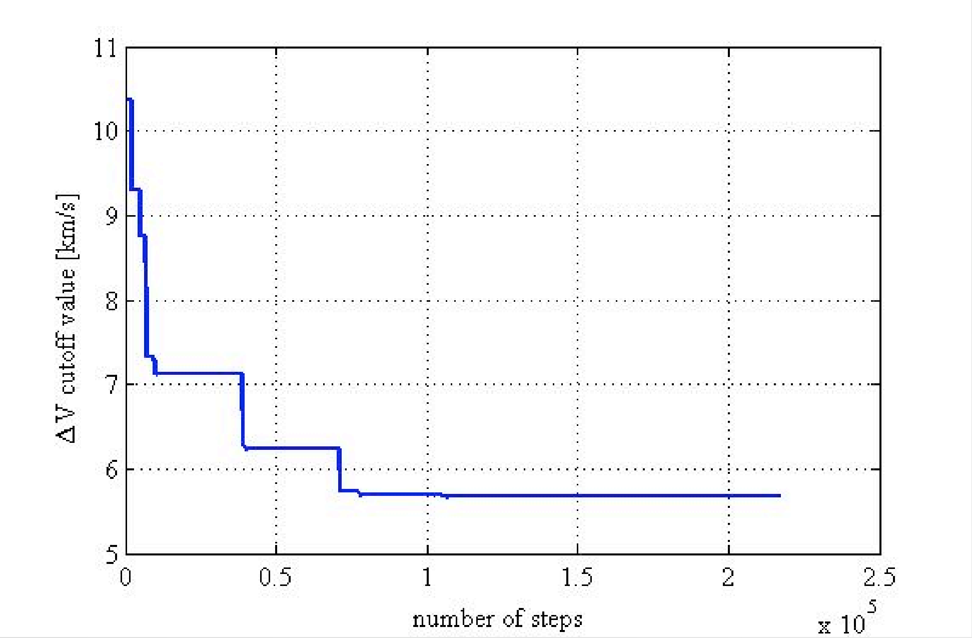
- The previous verified solver of implicit equations enabled the Taylor Model evaluation of the objective function
- COSY-GO has been applied for the global optimization of the impulsive Earth-Mars transfer

- Number of steps: 216911
- Computation time: 4954.39 s
- Enclosure of the minimum: [5.6673264, 5.6673272] km/s
- Enclosure of the solution:

$$t_0 \in [3573.176, 3573.212]$$
$$t_{tof} \in [324.034, 324.088]$$



Conclusions and Future Work

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Conclusions:

- DA and TM global optimizers are promising and efficient tools for the global optimization of a space mission
- Efficient management of discontinuities is needed for TM global optimization with COSY-GO

Future Work:

- Extend the models to Multiple Gravity Assist (MGA) interplanetary transfers and Deep Space Maneuvers
- Exploit the embedded domain box feature to prune the search space in MGA transfers
- Enable a dynamic selection of the box size based on a suitable definition of the trust region of a Taylor expansion in the DA based GO algorithm

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