Verified High-Order Optimal Control in Space Flight Dynamics

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Motivation

- Space trajectory design is always affected by uncertainties
  - Uncertainties due to navigation systems (errors on the knowledge of the vehicle position and velocity)
  - Uncertainties in modeling both the environment and the system performances (e.g. atmosphere density and vehicle aerodynamic parameters)
- The design of a space mission must take into account the expected uncertainties values: robust design

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Outline

- Robust guidance algorithm using Differential Algebra (DA)
  - Aerocapture maneuver
- DA solution of robust optimal control problem
  - Low-thrust transfer to Mars
- Verified optimal control via Taylor Model (TM)
  - Lunar landing

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Robust Guidance

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- The trajectory is computed in nominal conditions typically by solving an optimization problem
- The trajectory is characterized by
  - A nominal vector of parameters $\boldsymbol{p} = \{\boldsymbol{x}_i, p_1, p_2, \ldots p_n\}$ which includes also the initial state $\boldsymbol{x}_i$
  - A nominal control history $\boldsymbol{u} = \boldsymbol{g}(u_0, u_1, \ldots u_n, t)$ defined for example by a cubic spline interpolation
  - A nominal final state $\boldsymbol{x}_f$
- The goal of robust guidance algorithm is to find the corrections in the control law $\boldsymbol{u}$ to reach the final position $\boldsymbol{x}_f$ regardless of the uncertainties on $\boldsymbol{p}$

Robust Guidance: DA algorithm

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Simple *1D* problem:

- Initialize the uncertain parameter $[p] = p^0 + \Delta p$ and the control change $[u] = u^0 + \Delta u$ as DA variables
- By means of DA numerical integration obtain the n-*th* order map $\Delta x_f = \mathcal{M}(\Delta u, \Delta p)$
- Add the identity map $\Delta p = \mathcal{I}(\Delta p)$ and invert the complete map to gain $\Delta u = \mathcal{M}^{-1}(\Delta x_f, \Delta p)$
- By forcing $\Delta x_f = 0$ find $\Delta u = \Delta u(\Delta p)$

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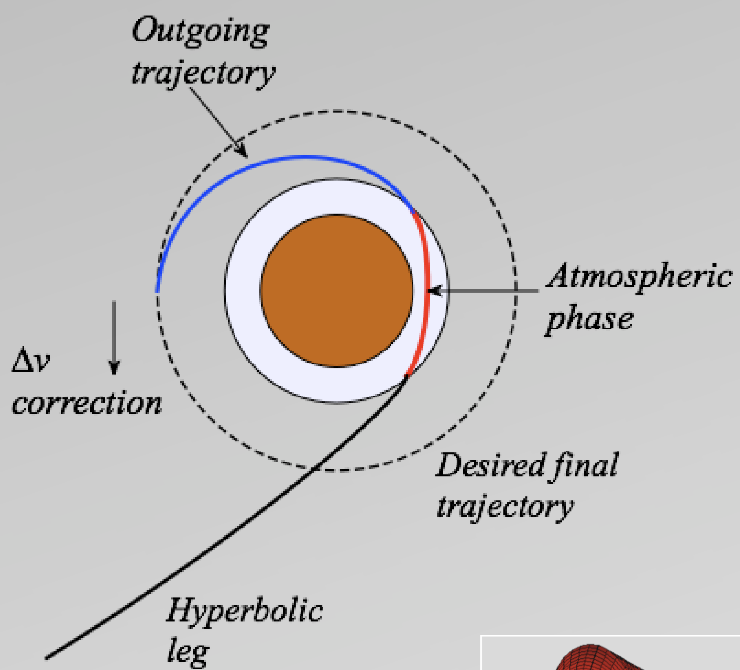
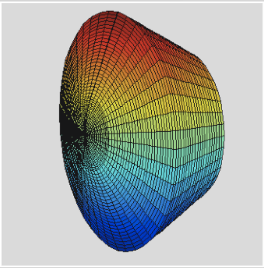
Aerocapture Maneuver

- Aerocapture is a way to reduce the propellant needed to gain the final planetary trajectory



$$\begin{cases} \dot{R} &= V \sin\gamma \\ \dot{\theta} &= \dfrac{V\cos\gamma\cos\psi}{R\cos\phi} \\ \dot{\phi} &= \dfrac{V\cos\gamma\sin\psi}{R} \\ \dot{V} &= \dfrac{D}{m} - G\sin\gamma \\ V\dot{\gamma} &= \dfrac{L\cos\sigma}{m} - G\cos\gamma + \dfrac{V^2\cos\gamma}{R} \\ V\dot{\psi} &= \dfrac{L\sin\sigma}{m\cos\gamma} - \dfrac{V^2\tan\phi\cos\gamma\cos\psi}{R} \end{cases}$$



with $D = \frac{1}{2}\rho V^2 S C_D, \;\; L = \frac{1}{2}\rho V^2 S C_L$

exponential density model $\rho = \rho_0 e^{-\beta h}$
and control parameter σ

Aerocapture Maneuver

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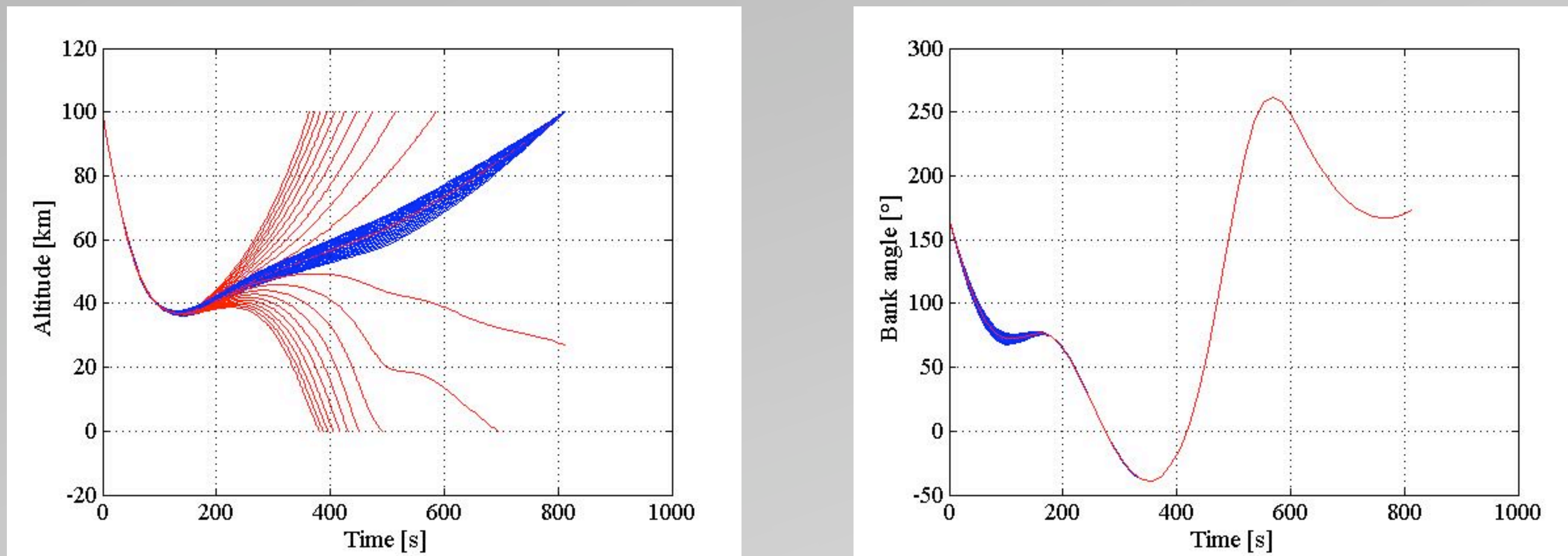
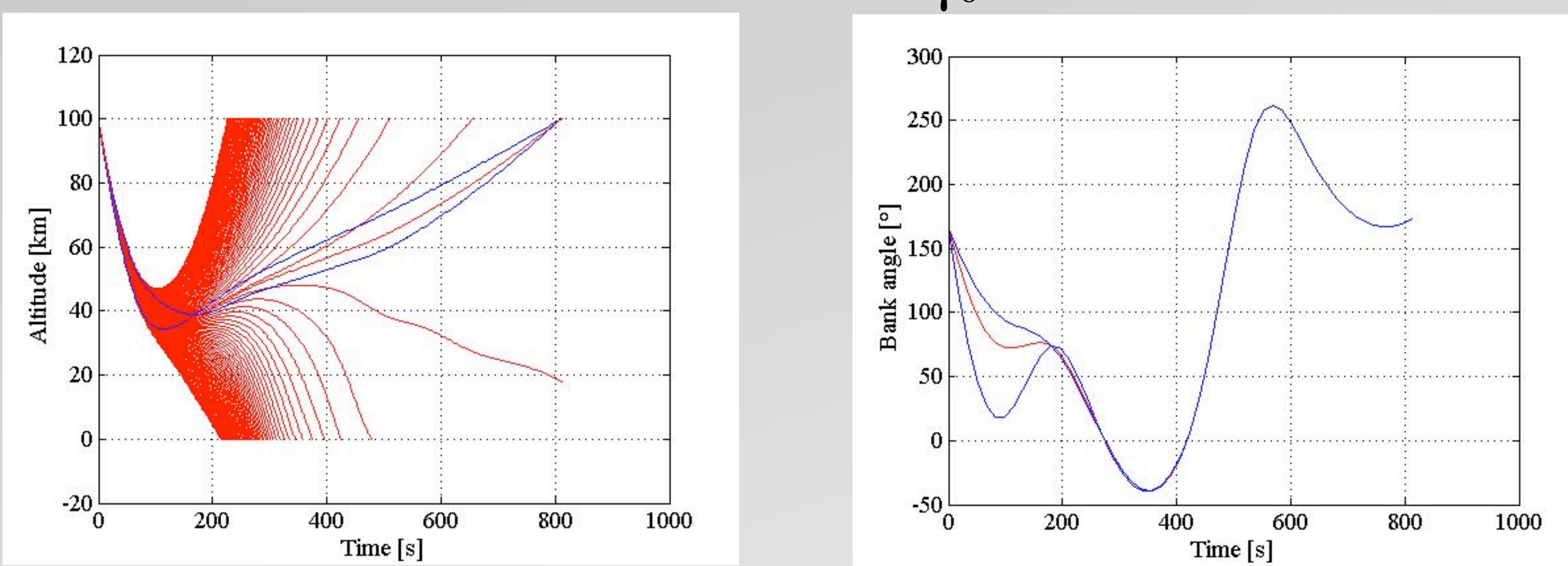
Uncertain C_D



Uncertain γ_0



Aerocapture Maneuver

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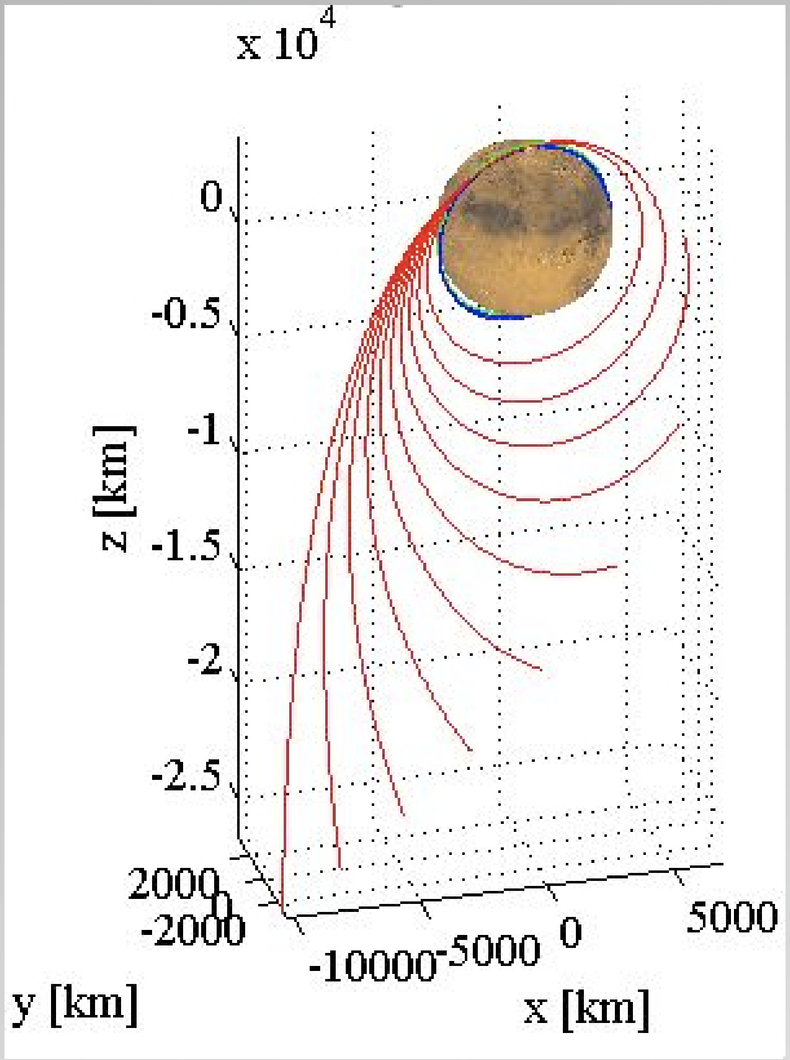

- Uncertainties considered: $C_D \pm 20$ %, $\varrho_0 \pm 20\%$, $\gamma_0 \pm 0.8$ deg
- Due to the high instability of the dynamics, a composition of several maps is required to manage γ_0 uncertainty
- 1 to 4 control points to match 1 to 4 final state components
- 7^{th} order Taylor expansions

I	COEFFICIENT	ORDER	EXPONENTS
1	7.640628215424944E-02	1	1 0
2	0.3760917116838308E-02	2	2 0
3	0.2420044808209212E-03	3	3 0
4	0.2012492098396635E-04	4	4 0
5	0.1944954823730704E-05	5	5 0
6	0.1750467514565079E-06	6	6 0
7	-.1841260173271442E-05	7	7 0

Robust Optimal Control of Trajectories

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Transfer between two fixed states

- Suppose we have a nominal solution of the optimal control problem:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t)$$
$$\boldsymbol{x}(t_i) = \boldsymbol{x}_i$$
$$\boldsymbol{x}(t_f) = \boldsymbol{x}_f$$

where $\boldsymbol{x} = \{x_1, x_2, ..., x_n\}$

Find a solution that minimizes:

$$J = \int_{t_i}^{t_f} \frac{1}{2} u^2 d\tau$$

Obtained by means of a control parameterization:

$$\boldsymbol{u} = \boldsymbol{g}(u_1, u_2, ..., u_m, t),\ m > n$$

- Suppose the presence of uncertainty on the initial state
- Find the new control function which solves the previous problem (i.e. reach $\boldsymbol{x}_f$ and minimize J)

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Algorithm:

- Initialize $\boldsymbol{x}_i$ and the control parameters as DA variables: $[\boldsymbol{x}_i] = \boldsymbol{x}_i^0 + \Delta \boldsymbol{x}_i$ and $[u_k] = u_k^0 + \Delta u_k$ for $k = 1, \ldots, m$

- A Runge-Kutta DA integration of the ODE leads to:

$$\Delta \boldsymbol{x}_f = \mathcal{M}_{\boldsymbol{x}_f}(\Delta \boldsymbol{x}_i, \Delta \boldsymbol{u}) \text{ and } \Delta \boldsymbol{u} = \{\Delta u_1, ..., \Delta u_m\}$$

→ Expansion of $\boldsymbol{x}_f$ w.r.t. $\boldsymbol{x}_i$ and the control parameters

- Select a subset of control parameters equal to the number of constraints on the final state (*n*), $\Delta \boldsymbol{u}_o$, and indicate the remaining ones (*m*-*n*) with $\Delta \boldsymbol{u}_a$

Robust Optimal Control of Trajectories

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- Expand the constraint manifold
  - Build the following map and invert it:

$$\begin{pmatrix}\Delta \boldsymbol{x}_f \\ \Delta \boldsymbol{u}_a \\ \Delta \boldsymbol{x}_i\end{pmatrix} = \begin{pmatrix}[\mathcal{M}_{\boldsymbol{x}_f}] \\ [\mathcal{I}_{\boldsymbol{u}_a}] \\ [\mathcal{I}_{\boldsymbol{x}_i}]\end{pmatrix} \begin{pmatrix}\Delta \boldsymbol{x}_i \\ \Delta \boldsymbol{u}_a \\ \Delta \boldsymbol{u}_o\end{pmatrix} \rightarrow \begin{pmatrix}\Delta \boldsymbol{x}_i \\ \Delta \boldsymbol{u}_a \\ \Delta \boldsymbol{u}_o\end{pmatrix} = \begin{pmatrix}[\mathcal{M}_{\boldsymbol{x}_f}] \\ [\mathcal{I}_{\boldsymbol{u}_a}] \\ [\mathcal{I}_{\boldsymbol{x}_i}]\end{pmatrix}^{-1} \begin{pmatrix}\Delta \boldsymbol{x}_f \\ \Delta \boldsymbol{u}_a \\ \Delta \boldsymbol{x}_i\end{pmatrix}$$

  - By imposing $\Delta \boldsymbol{x}_f = 0$ obtain the Taylor Series expansion of the constraint manifold:

$$\Delta \boldsymbol{u}_o = \mathcal{M}_{\boldsymbol{u}_o}(\Delta \boldsymbol{u}_a, \Delta \boldsymbol{x}_i)$$

- Substitute in the objective function and gain:

$$J = \mathcal{J}(\Delta \boldsymbol{u}_a, \Delta \boldsymbol{u}_o, \Delta \boldsymbol{x}_i) \rightarrow \bar{J} = \bar{\mathcal{J}}(\Delta \boldsymbol{u}_a, \Delta \boldsymbol{x}_i)$$

- Evaluate the gradient with respect to $\Delta \boldsymbol{u}_a$:

$$\nabla_{u_a} \bar{J} = \nabla_{u_a} \bar{\mathcal{J}}(\Delta \boldsymbol{u}_a, \Delta \boldsymbol{x}_i)$$

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- Build the following map and invert it:

$$\begin{pmatrix} \nabla_{\boldsymbol{u}_a} \bar{J} \\ \Delta \boldsymbol{x}_i \end{pmatrix} = \begin{pmatrix} [\nabla_{\boldsymbol{u}_a} \bar{\mathcal{J}}] \\ [\mathcal{I}_{\boldsymbol{x}_i}] \end{pmatrix} \begin{pmatrix} \Delta \boldsymbol{u}_a \\ \Delta \boldsymbol{x}_i \end{pmatrix} \Longrightarrow \begin{pmatrix} \Delta \boldsymbol{u}_a \\ \Delta \boldsymbol{x}_i \end{pmatrix} = \begin{pmatrix} [\nabla_{\boldsymbol{u}_a} \bar{\mathcal{J}}] \\ [\mathcal{I}_{\boldsymbol{x}_i}] \end{pmatrix}^{-1} \begin{pmatrix} \nabla_{\boldsymbol{u}_a} \bar{J} \\ \Delta \boldsymbol{x}_i \end{pmatrix}$$

- Given an uncertainty on the initial state $\Delta \boldsymbol{x}_i$, the previous map delivers, by imposing $\nabla_{\boldsymbol{u}_a} \bar{J} = 0$, the control correction $\Delta \boldsymbol{u}_a$, solution of the optimal control problem

- The corrections to be applied on the omitted variables, $\Delta \boldsymbol{u}_o$, are given by the previous explicit expression of the constraint manifold

Low-Thrust Transfer to Mars

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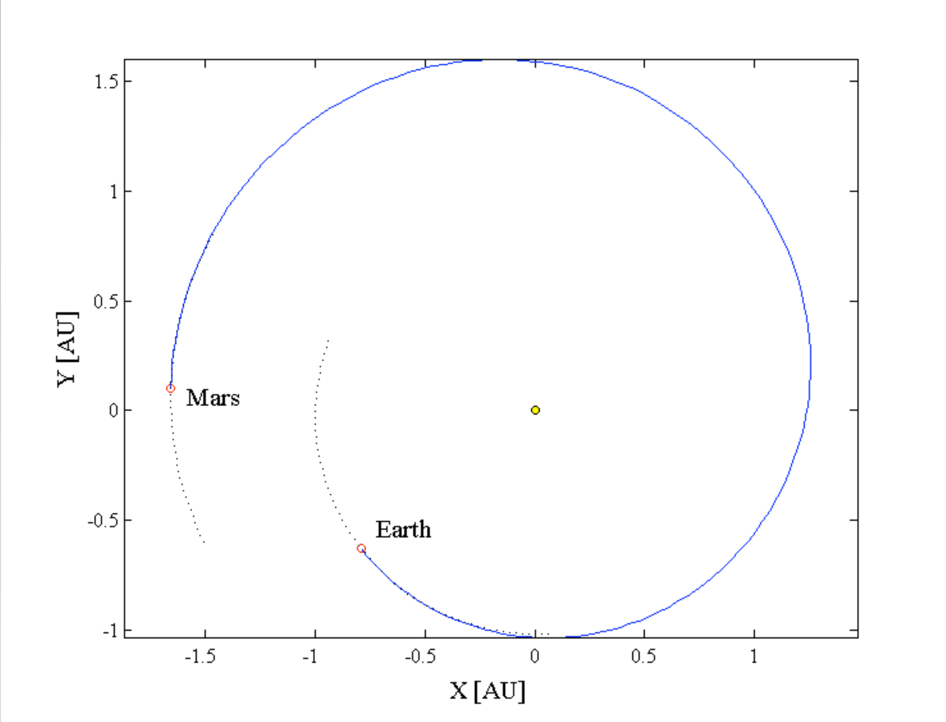
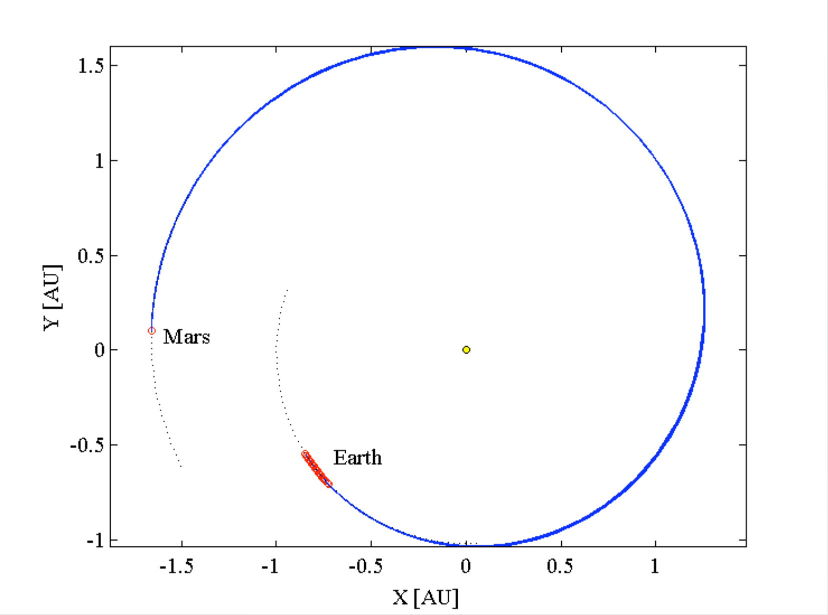
- The control profile is described by a cubic spline defined on 4 time-equally-spaced collocation points
- Nominal optimal solution
  - Departing time
    $t_0 = 1213.8$ MJD
  - Transfer time
    $t_{tof} = 513.210$ days

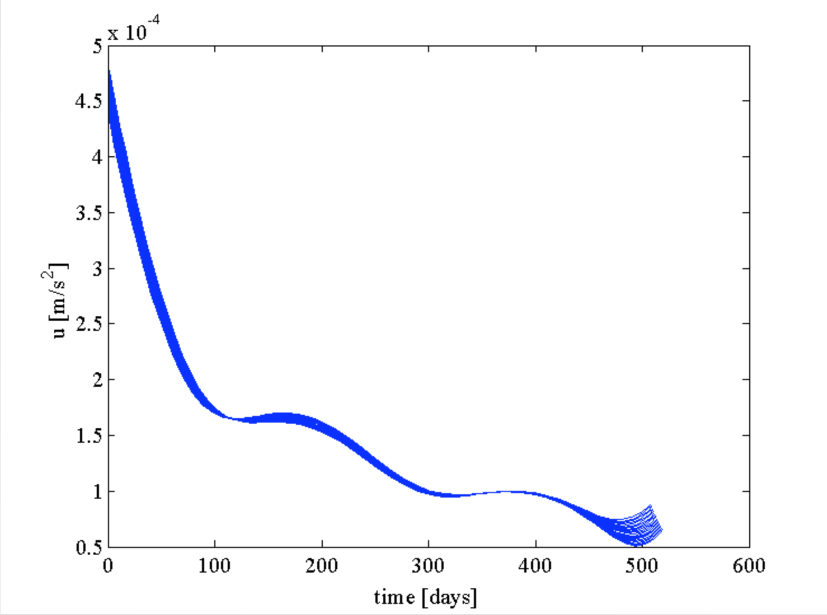


- Uncertainty considered
  - Launch window
    $t_0 = [1208, 1219]$ MJD

Low-Thrust Transfer to Mars

- DA techniques are used to evaluate the analytical ephemerides with departing date uncertainty
- As a result the ephemeris model delivers Taylor expansions of the Earth position and velocity (initial state)
- The time of flight is chosen to keep the arrival date fixed





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TM Validated Control

- The DA control law can be validated using TM
- Consider the generic ODE:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t)$$
$$\boldsymbol{x}(t_i) = \boldsymbol{x}_i$$

- The Taylor series are transformed into a Taylor model by composition with a Taylor model identity

e.g if $\Delta \boldsymbol{u} = \Delta \boldsymbol{u}(\Delta \boldsymbol{x}_i) \Rightarrow \Delta \boldsymbol{u}^{TM} = \Delta \boldsymbol{u} \circ \mathcal{I}_{\boldsymbol{x}_i}^{TM}$

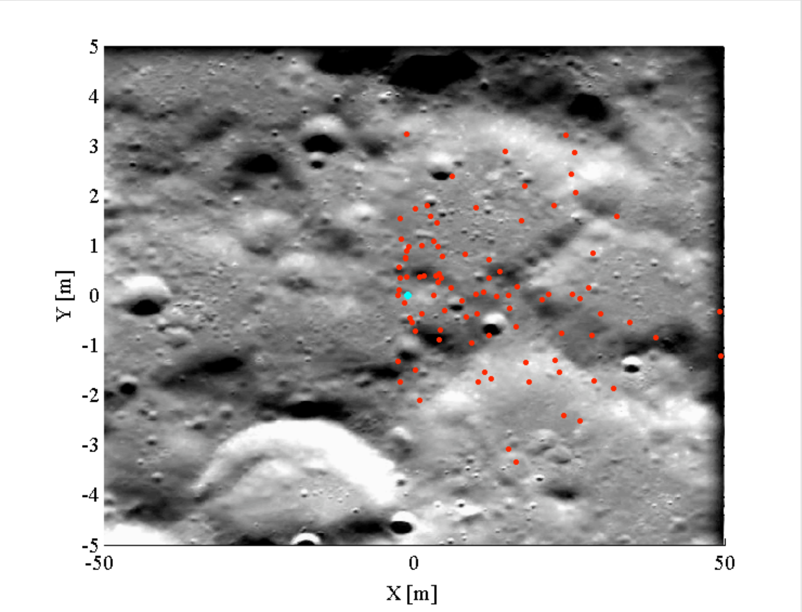
- The dynamics is then propagated using the TM control and validated Taylor integrator

Lunar Landing - 2BP

- Nominal initial conditions: pericenter of an elliptic orbit (20 km of altitude)
- The goal is to land at Moon South Pole
- The nominal optimal control is computed in a two-body dynamical model
- Firstly DA is used to find the control strategy that reacts to the uncertainties in two-body problem (2BP)

Example 1:

- Introduced uncertainty:
  30 m on initial position



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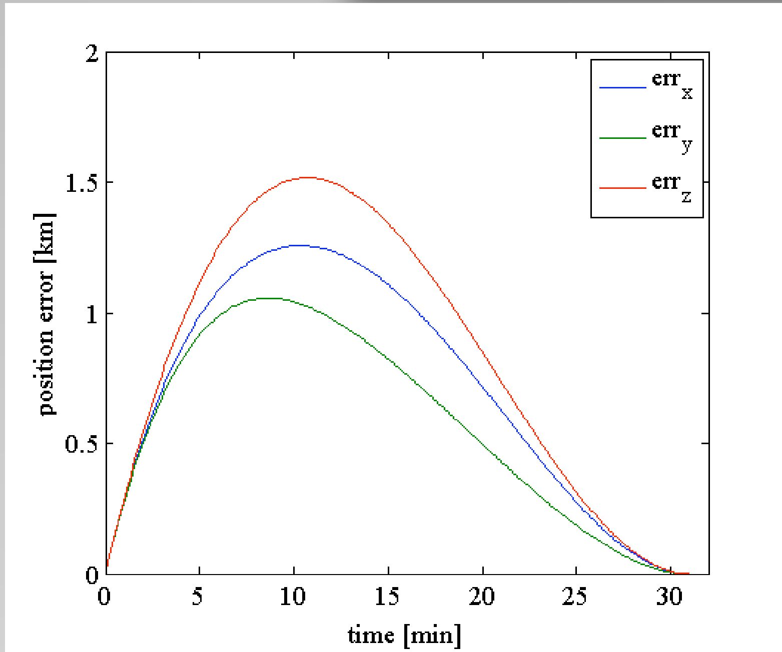
Lunar Landing - 2BP

Example 2:

- Introduced uncertainty:
  - 10 m on initial position
  - 10 m/s on initial velocity
- Control corrections of the order of 10^0 N



- Secondly Taylor models are introduced and the validated Taylor integration is used to address the validated control problem:

Variable	*Desired*	*Interval Enclosure*	*Width*
X [m]	-0.5512	[-0.5534 , -0.5491]	0.0043
Y [m]	1.6180	[1.6137 , 1.6220]	0.0083
Z [m]	0.0000	[-0.3648E-003, 0.3648E-003]	7.2960E-04
Vx [m/s]	-0.6553E-003	[-0.6575E-003, -0.6531E-003]	4.4000E-06
Vy [m/s]	-3.0006511	[-3.0006590 , -3.0006436]	1.5400E-05
Vz [m/s]	0.0000	[-0.3183E-006, 0.3183E-006]	6.3660E-07

Lunar Landing - Perturbed 2BP

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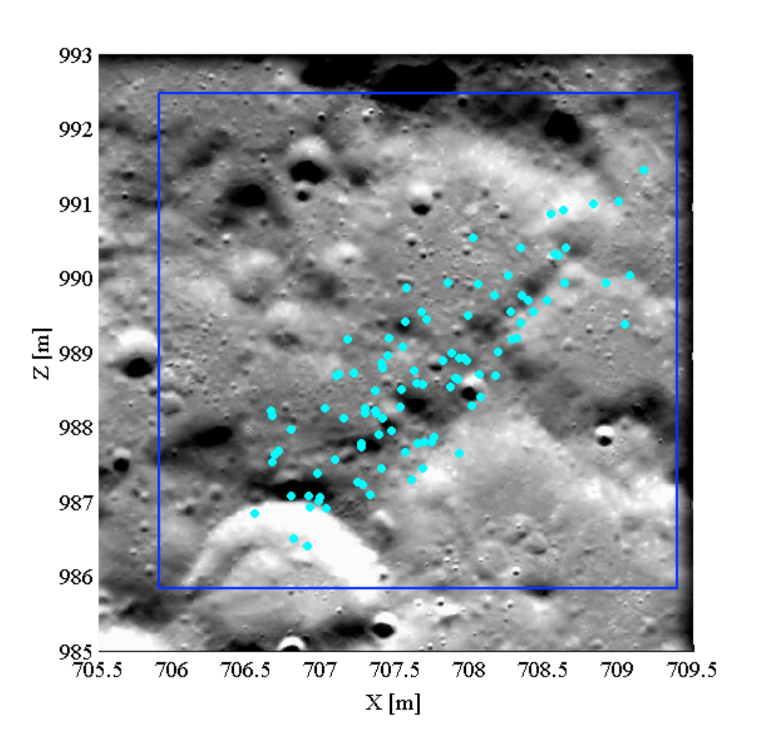
- Introduced perturbations:
  - Moon oblateness
  - Earth gravity field
  - Sun gravity field



- Validated Taylor Integration:

Variable	*Interval Enclosure*	*Width*
X [m]	[705.9079 , 709.3936]	3.4857
Y [m]	[2132.9812 , 2147.2011]	14.2199
Z [m]	[985.8375 , 992.4736]	6.6361
Vx [m/s]	[0.4504 , 0.4523]	0.0019
Vy [m/s]	[1.0267 , 1.0494]	0.0227
Vz [m/s]	[0.9344 , 0.9385]	0.0041

Conclusions and Future Work

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Conclusions:

- DA computation is a powerful tool not only to address uncertainties sensitivity but also to solve optimal robust control problems in space flight dynamics
- TM can be use to solve validated control problem thus avoiding any Monte Carlo simulation run

Future work:

- Extend TM validation to aerodynamic phases and to larger value of initial uncertainties
- Address the optimal feedback control problem

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