

An Interval Method for Enclosing All Solutions of Two-Point Boundary Value Problems for ODEs

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Ordinary differential equations (ODEs) arise in mathematical models throughout science and engineering. When an explicit condition (or conditions) that a solution must satisfy is specified at one initial point, we have an initial value problem (IVP). For a boundary value problem (BVP), however, information about a solution may, in general, be specified at more than one point. For the problem with two such points, we have a two-point boundary value problem (TPBVP). A TPBVP may have no solution, or may have finite number of solutions, or may have infinitely many solutions. In some cases, it may be of interest to determine parameters in the ODE so that the TPBVP has a solution. The standard techniques for the numerical solution of a TPBVP for an ODE can be divided into two classes. The first class is based on repeated solution of a related IVP in order to obtain the TPBVP solution. Typical of this class are various shooting and multi-shooting approaches. The other class of methods is based on various versions of finite difference or collocation.

A common limitation of all of these approaches is that they can, at best, achieve convergence to a local solution to the TPBVP, which may result in missing other solutions of interest.

In this study, we will present a new approach that is guaranteed to enclose all solutions to the TPBVP. This method is based on interval analysis and employs a type of shooting approach. A key feature of the method is the use of a new validated solver for parametric ODEs, based in part on the use of Taylor models, which is used to produce guaranteed bounds on the solutions of IVPs with interval-valued initial conditions and parameters.

Some numerical experiments will be used to demonstrate the proposed approach.