Compact operators on spaces with a non-archimedean inner product

José Aguayo[1] and Miguel Nova[2]

We take a non-archimedean valued fields $\mathbb{K}$ for which its residue class field is formally real. We consider the natural inner product on c_0, the space of all null-sequences of elements of $\mathbb{K}$. Under these coditions, the sup-norm of c_0 is induced by the inner producto.

In a previous work, the authors characterized those closed subspaces of c_0 with an orthonormal complement respect to this inner product. Such subspaces, together with its orthonormal complement, defines some special projections, so called, normal projection.

In the present work, the authors present characterization of such normal projections and also characterization of another kind of operator called compact self-adjoint operator.

The main theorem is the following:

Theorem 1 *If $T : c_0 \to c_0$ is a compact and self-adjoint operator, then there exists an element $\lambda = (\lambda_n)$ in c_0 such that $T = \sum_{n=1}^{\infty} \lambda_n P_n$, where each P_n is a normal projection.*

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On the way to a spectral theorem on Form Hilbert Spaces

Carla Barrios*

Abstract

Form Hilbert spaces are generalizations of Hilbert spaces that maintain numerous properties, even though some striking differences have been found between such spaces and their classical counterpart, specially in the study of diverse classes of linear operators.
Here is presented part of an ongoing doctoral thesis work that aims to establish a spectral theorem for compact selfadjoint operators. Classical concepts and results of spectral theory are explored in the context of Form Hilbert spaces.

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Minimal non-Archimedean Algebraically Closed and Real-Closed Fields

Martin Berz

Abstract

Conditions are developed that are necessary for algeraic and real closure of non-Archimedean totally ordered fields and their complex extensions. It is shown that the Levi-Civita fields are the smallest extensions of the real and complex fields numbers that satisfy these conditions. We then embark on proving that these fields indeed are algebraically and real closed using only elementary methods from within the Levi-Civita fields. To this end, we first show that after after a variety of transformations and using the corresponding properties of the real and complex numbers, the corresponding roots can be found up to infinitely small error. Subsequently, using a second set of transformations, we determine an iterative sequence of corrections to any approximate solutions that improves its quality by an infinitely large amount. It is shown that the resulting sequence of corrected approximate solutions converges in the strong (order) topology of the fields, and that the resulting limit is indeed the advertised root.

Existence and Uniqueness of Solutions of Differential Equations on the Levi-Civita Field

Martin Berz and Khodr Shamseddine

Abstract

We consider differential equations on the Levi-Civita field with a right hand side that is infinitely often derivate differentiable (D infinity). We show that such an ODE admits solutions that are themselves D infinity. To this end, we develop a theory of multivariate D infinity functions and show that they can be locally represented as Taylor series. We then re-phrase the ODE problem as a fixed point problem of a Picard operator in the common way. After various transformations and utilizing well-known existence and uniqueness properties of ODEs in Reals, the problem is transformed to a fixed point problem with infinitely small contraction factor. It is shown that the sequence of functions obtained by iteration converges uniformely in the strong (order) topology, that the resulting limit is itself D infinity, and that this limit indeed solves the ODE. It is then shown that while there are other solutions with lesser smoothness requirements, the solution so obtained is unique among D infinity functions.

Identity theorem for bounded meromorphic P-adic functions

Kamal BOUSSAF

Abstract : In complex analysis, studies were made on the identity theorem for analytic functions of bounded characteristic in the unit disk. Here we place in a complete ultrametric algebraically closed field K and we consider the K-Banach algebra of bounded analytic functions $\mathcal{A}_b(d(0,R^-))$ in the disk $d(0,R^-)$. We define sets of uniqueness, identity sequences and analytic boundaries for $\mathcal{A}_b(d(0,R^-))$. We show the equivalence between the two last notions. We also show a connection with Blaschke sequences and T-polar sequences introduced by M.C. Sarmant.

Malmquist-Yosida Equation inside an open disk.

Abdelbaki BOUTABAA and Alain ESCASSUT

Abstract: Let K be a complete ultrametric algebraically closed field of residue characteristic q. Let m be an integer relatively prime with q. Let $\mathcal{A}(d(a,R^-))$ be the K-algebra of analytic functions inside the disk $\{x \in K \mid |x-a| < r\}$, let $\mathcal{M}(d(a,R^-))$ be its field of fractions and let $\mathcal{M}_u(d(a,R^-))$ be the subset of functions which are not of the form $\frac{\phi}{\psi}$ where $\phi,\ \psi$ lie in $\mathcal{A}(d(a,R^-))$ and are bounded. Let $(\mathcal{E})$ be the Malmquist-Yosida Equation with constant coefficients: $(y')^m = F(y)$, with $F(X) \in K(X)$. We show that $(\mathcal{E})$ has no solution in $\mathcal{M}_u(d(a,R^-))$.

Some subalgebras of the algebra of bounded linear operators of the one variable Tate algebra .

Bertin DIARRA

Let K be an ultrametric valued valued field. The one variable Tate algebra is the algebra of formal power series $K\{X\} = \{f = \sum_{n\geq 0} a_n X^n \in K[[X]] \ / \ \lim_{n\to+\infty} |a_n| = 0\}$. Endowed with the norm $\|f\| = \sup_{n\geq 0} |a_n|$ and the usual addition and product of formal power series it is a Banach algebra.

For K of characteristic zero, as in classical umbral calculus, one obtains a description of the algebra of continuous linear operators of $K\{X\}$ that commute with the translation $\tau_a(f)(X) = f(X+a)$, for a in the ring of integers of K. Also, we give a description of the algebra of continuous linear operators of $K\{X\}$ that commute with the translation $h_a(f)(X) = f(aX)$, for a in the ring of integers of K. One needs for this, to produce different kinds of orhonormal basis of the Banach algebra $K\{X\}$.

We are also interested to the study of commutation relations between some operators of $K\{X\}$ which give rise to Weyl algebra, quantum Weyl algebra as well as quantum plane algebra. In any case the completion with respect to the operator norm of these algebras is given, with explicit orthogonal basis of each other exhibited.

The Corona Problem on a spherically complete field

Alain Escassut

Abstract

Let K be an algebraically closed field complete with respect to an ultrametric absolute value $|\ .\ |$ and let A be the Banach K-algebra of bounded analytic functions in the "open" unit disk D of K provided with the Gauss norm. Let $Mult(A, |\ .\ |)$ be the set of continuous multiplicative semi-norms of A provided with the topology of simple convergence, let $Mult_m(A, |\ .\ |)$ be the subset of the $\phi \in Mult(A, |\ .\ |)$ whose kernel is a maximal ideal and let $Multa(A, |\ .\ |)$ be the subset of the $\phi \in Mult(A, |\ .\ |)$ whose kernel is a maximal ideal of the form $(x-a)A$ with $a \in D$. The Corona problem on such a field K lies in two questions: is $Mult_a(A, |\ .\ |)$ dense in $Mult_m(A, |\ .\ |)$? Is it dense in $Mult(A, |\ .\ |)$? In a previous work by N. Mainetti and the same author, it was shown that if each maximal ideal of A is the kernel of a unique $\phi \in Mult_m(A, |\ .\ |)$, then the answer to the first question is yes (which is the case when K is strongly valued). Here we show that this is also the case when K is spherically complete and that tends to back a positive answer to both questions. Conjectures are considered.

Hölder Differentiable Maps over Ultrametric Fields and the General Curve Lemma

by Helge Glöckner (Paderborn)

The General Curve Lemma is a tool of infinite-dimensional differential calculus, which allows pieces of a given sequence of smooth curves in a real locally convex space to be combined to a single smooth curve, which runs through each of the pieces in finite time (see [**?**, 12.2]). In the talk, I'll present an ultrametric version of the General Curve Lemma (taken from [**?**]), and use it to characterize k times Hölder differentiable maps on metrizable topological vector spaces over ultrametric fields (following [**?**]).

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Pseudo-stable manifolds for dynamical systems over ultrametric fields

by Helge Glöckner (Paderborn)

If E is a real Banach space, $\phi\colon E \to E$ a diffeomorphism fixing the origin and $a > 1$, then suitable conditions ensure that the set of all $x \in E$ such that $\phi^n(x) = o(a^n)$ is an immersed submanifold of E (the so-called *a-pseudo-stable manifold*). One proof is due to M. C. Irwin [**?**], who obtained the manifold structure using a special version of the Contraction Mapping Theorem for sequence spaces (cf. also [**?**]).

In my contribution to the proceedings volume, I'll construct pseudo-stable manifolds for diffeomorphisms of Banach spaces over ultrametric fields (and in more general situations), by an adaptation of Irwin's method. As a tool, the required specialized (non-archimedean) versions of the contraction mapping theorem and the C^k-dependence of fixed points on parameters will be discussed (in situations where more classical results, as in [**?**] or [**?**], would not apply).

The pseudo-stable manifolds, together with the a-stable manifolds ($a \in]0,1]$) constructed in [**?**], admit various applications in the theory of finite-dimensional Lie groups over local fields of positive characteristic (as announced in [**?**]).

In particular, these invariant manifolds can be used to transfer some classical results by J.S.P. Wang [**?**] concerning p-adic Lie groups to the case of Lie groups over local fields of positive characteristic, under suitable hypotheses [**?**]. For instance, they can be used to see that every Lie group admitting a contractive automorphism is nilpotent [**?**]. Under mild hypotheses, they also enable the "scale function" and "tidy subgroups" of a Lie group over a local field to be calculated [**?**], which are fundamental concepts in the structure theory of totally disconnected, locally compact topological groups initiated by G. Willis [**?**]. So far, this was only possible for p-adic Lie groups [**?**] and for semi-simple algebraic groups over local fields [**?**].

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VECTOR-VALUED P-ADIC MEASURES

A. K. KATSARAS

Abstract. For a separating algebra $\mathcal{R}$ of subsets of a set X and E a Hausdorff non-Archimedean locally convex space, we study the space $M(\mathcal{R}, E)$ of all E-valued bounded finitely-additive measures on $\mathcal{R}$ as well as its subspaces $M_\sigma(\mathcal{R}, E)$ and $M_\tau(\mathcal{R}, E)$ of all σ-additive and all τ-additive members, respectively. We also study integrals of scalar-valued functions on X with respect to members of $M(\mathcal{R}, E)$. We show that, if X is a Hausdorff zero-dimensional topological space, $C_b(X)$ the space of all bounded continuous scalar-valued functions on X and $K(X)$ the algebra of all clopen subsets of X, then, in case E is complete, $M_\tau(K(X), E)$ is algebraically isomporphic to the space of all linear maps from $C_b(X)$ to E which are continuous with respect to the strict topology β_o.

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On the Clifford algebra of orthomodular spaces over non-archimedean fields

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Abstract The orthogonal groups of *finite-dimensional* quadratic spaces (V, Ψ) are a classic, intensively studied mathematical topic and the Clifford algebras associated to te form Ψ play a vital role in the theory. In the case of *infinite dimensional* quadratic spaces, however, things look differently. The classic algebraic methods turn out to be weak and results on the group of isometries are scarce. The main reason is that in infinite dimension the subgroup generated by all reflexions covers only a small portion of the whole orthogonal group. It is natural to try to supplement the algebraic view by a topological approach. We will show that this can be done successfully in the case of infinite dimensional orthomodular spaces (E, Φ) constructed by standard procedures over certain non-archimedean fields.

Our talk begins by reviewing the base fields K together with their valuations, which are always of infinite rank. We then consider the construction of the spaces (E, Φ), which relies on a particular feature of the field K expressed by the notion of (algebraic) types. Let $C = C(E, \Phi)$ be the Clifford algebra associated to the form Φ in the usual sense. It is again the device of types which opens a way to introduce a non-archimedean norm on the algebra C. We pass to its completion $\tilde{C}$ and we show that $\tilde{C}$ is a remarkable counterpart of the classical Clifford algebras; in particular, we introduce infinite products of reflections, and we have a close look at the interrelation between $\tilde{C}$ and the group of isometries of the space.

AMS classificatión: 15A66, 12J25

On local linearization in utltrametric dynamics

Karl-Olof Lindahl, Växjö University

Abstract

We consider the problem of local linearization of power series defined over complete valued fields. The complex field case has been studied since the end of the nineteenth century, and renders a delicate number theoretical problem of small divisors related to diophantine approximation. Since a work of Herman and Yoccoz in 1981, there has been an increasing interest in generalizations to other valued fields like p-adic fields and various function fields. We present some new results in this domain of research. In particular, for fields of prime characteristic, the problem leads to a combinatorial problem of seemingly great complexity, albeit of another nature than in the complex field case. An introduction to the problem and its relation to the theory of dynamical systems can be found on http:w3.msi.vxu.seuserskli

About the Ultrametric Corona Problem

Alain Escassut and Nicolas Maïnetti

Abstract

Let K be a complete ultrametric algebraically closed field and let A be the K-Banach algebra of bounded analytic functions in the disk D : $|x| < 1$. Let $Mult(A, \| \, . \, \|)$ be the set of continuous multiplicative seminorms of A, let $Mult_m(A, \| \, . \, \|)$ be the subset of the $\phi \in Mult(A, \| \, . \, \|)$ whose kernel is a maximal ideal and let $Mult_a(A, \| \, . \, \|)$ be the subset of the $\phi \in Mult_m(A, \| \, . \, \|)$ whose kernel is of the form $(x-a)A, \ a \in D$ (if $\phi \in Mult_m(A, \| \, . \, \|) \setminus Mult_a(A, \| \, . \, \|)$, the kernel of ϕ is then of infinite codimension). The main problem we examine is whether $Mult_a(A, \| \, . \, \|)$ is dense inside $Mult_m(A, \| \, . \, \|)$ with respect to the topology of simple convergence:. This is a first step to the conjecture of density of $Mult_a(A, \| \, . \, \|)$ in the whole set $Mult(A, \| \, . \, \|)$, a question particularly asked by V. Berkovich: this is the corresponding problem to the well-known complex corona problem. We show that if a maximal ideal is the kernel of a unique $\phi \in Mult_m(A, \| \, . \, \|)$, then ϕ lies in adherence of $Mult_a(A, \| \, . \, \|)$. Particularly, if every maximal ideal is the kernel of a unique $\phi \in Mult_m(A, \| \, . \, \|)$, $Mult_a(A, \| \, . \, \|)$ is dense in $Mult_m(A, \| \, . \, \|)$. And particularly,this the case if K is strongly valued. In the general context, we find a subset of $Mult_m(A, \| \, . \, \|) \setminus Mult_a(A, \| \, . \, \|)$ which is included in the closure of $Mult_a(A, \| \, . \, \|)$. We also show, (more generally), if a $\psi \in Mult_a(A, \| \, . \, \|)$ does not lie in the closure of $Mult_a(A, \| \, . \, \|)$, then its restriction to the algebra of analytic elements on D, denoted by $H(D)$, is the norm of uniform convergence on D.

Rigorous Computation with the Levi-Civita Field

Kyoko Makino and Martin Berz

Abstract

The field of formal power series with left-finite fractional exponents first devised by Levi-Civita has the property of being the smallest non-Archimedean real-closed field. It further has the remarkable property that its elements can be represented on a computer up to arbitrarily small error within the framework of the weak topology, similar to the representability of the common real numbers to any desired precision. We present algorithms that implement the arithmetic on the field within the weak topology, which effectively unites the questions of accuracy within the underlying valuation and the accuracy of real number coefficients in the valuation classes. Using this arithmetic, we are able to employ common methods for functions on the Levi-Civita field that however have very useful applications for real functions. Among them are the exact differentiation of functions to any order using difference quotient formulas with infinitely small increments, the direct use of delta functions and their derivatives, and various related concepts.

A CRITERION FOR THE INVERTIBILITY OF LIPSCHITZ OPERATORS ON TYPE SEPARATING SPACES.

H. MORENO

Abstract. A Norm Hilbert space E with orthogonal base $e_1, e_2, \ldots$ is called a Type Separating space if the topological types of any two different basis vectors are also different. In this paper we study a criterion for the invertibility of a Lipschitz injective operator B in terms of a boundedness condition for the norms $\|Be_1\|, \|Be_2\| \ldots$. It generalizes a result in [1]. From this we derive, following the work done in [2], a handy criterion in terms of the (infinite) matrix of B. We show applications for the case of the undecomposable selfadjoint Lipschitz operators studied in [3].

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Summability Matrices of Type M in Non-Archimedian Analysis

P.N. Natarajan

Abstract

In this paper,summability matrices of type M in a complete,non-trivially valued,non-archimedean field K are defined and some of their properties are studied.

β_p topologies defined on the space $C_b(X, E)$

José Aguayo, Samuel Navarro and Miguel Nova

We define a new type of a strict topology which is denote by β_p. We compare this topology with other topologies and prove various properties like, for example, β_P has the same bounded sets than the uniform topology. We also study special properties for this locally convex space. For example, we prove that $\left(C_b\left(X, E\right), \beta_p\right)$ is a $gDF - space$. Finally, we study a new version of the Ascoli's Theorem in $\left(C_b\left(X\right), \beta_p\right)$.

Monomial Dynamics in Finite Field Extentions of the p-adic Numbers

Marcus Nilsson

April 21, 2008

Abstract

We study discrete dynamical systems over finite field extensions of the p-adic numbers. We especially study monomial and perturbed monomial systems. We find explicit formulas for the number of periodic points and describe the dynamics around these points. We also calculate the asymptotic mean value of the number of periodic points when $p \to \infty$. These results are generalizations of earlier results by the author, see [2], [3] and [1].

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On Monomial Dynamical Systems over the p-adic n-torus

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Abstract

We first study monomial systems on the n-torus modulo a prime number p. We investigate the dynamics by using group structures and directed graphs. Under certain conditions the systems we get by lifting to the p-adic n-torus inherits much of the structure of the system modulo p. We also discuss the characters of the periodic points.

Compact perturbations of Fredholm operators on Norm Hilbert spaces over Krull valued fields

H. Ochsenius W.H. Schikhof

Abstract

A continuous linear operator of a Banach space into itself is called **Fredholm** if its kernel and cokernel are finite-dimensional. The subject of compact perturbations of Fredholm operators on complex spaces is well-known, see e.g. [2]. For spaces over non-archimedean valued fields K of rank one (i.e. the range of the valuation is in $[0,\infty)$) the preservation of Fredholm operators under compact perturbations was proved in [1].

In this paper we allow K to have an arbitrary totally ordered abelian value group G rather than a subgroup of $(0,\infty)$, but we restrict our study to Norm Hilbert spaces (NHS) E over K, i.e. each closed subspace admits a projection of norm ≤ 1. We prove the striking fact that the index of a Fredholm operator is 0. Further, we consider a natural class $\Phi(E)$ of so-called Lipschitz-Fredholm operators and prove that the operators A for which $A+\Phi(E)\subset\Phi(E)$ form precisely the set of all nuclear operators. (An operator A is called **nuclear** if there exists a sequence $A_1, A_2, \ldots$ of continuous finite rank operators such that $\|Ax - A_n x\| < g_n\|x\|$ $(x \in E\backslash\{0\}$ for some sequence $g_1, g_2, \ldots$ in G, tending to 0. Here the strict inequality is essential!)

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On Hayman's conjecture over p-adics fields

Jacqueline OJEDA

ABSTRACT

This lecture is aimed at studying Hayman's conjecture for meromorphic functions in an ultrametric field.

In the complex plane, W. Hayman conjectured that :

If f is a meromorphic function, $a \in \mathbb{C} \setminus \{0\}$ and $m \in \mathbb{N}$, then $f' + af^m$ has infinitely many zeros which are not zeros of f.

The problem was studied for more than 25 years and it was proved that it is holds whenever f is transcendental and $m \geq 3$. For the cases $m = 1$ end $m = 2$ there exist counter-examples.

Here, we examine the problem in an algebraically closed complete ultrametric field $\mathbb{K}$ of characteristic zero. Considering the function $f' + Tf^m$ with $T \in \mathbb{K}(x)$, we show, using methods of ultrametric analysis and particularly the ultrametric Nevanlinna theory, that Hayman's statement holds for each $m \geq 5$ and $m = 1$. Further, if the residue characteristic of $\mathbb{K}$ is zero, then the statement holds for each positive integer m different from 2. Finally, we examine the case $m = 2$. We also study the problem inside an open disc.

Dedekind completions of a class of G-modules*

H. Ochsenius† and E. Olivos‡

Abstract

In Non-Archimedean Functional Analysis of infinite rank, the Dedekind completion of a linearly ordered group is the natural home for the norms of vectors as well as of linear operators. We will consider a class of Hahn products, called Γ_α (α an ordinal), whose rank is the order-type of α and give an operational representation of every element of the Dedekind completion of such a group in terms of the supremum and infimum of its convex subgroups. In the case of Form Hilbert Spaces we will study the interrelations between the Dedekinds completions of G and its overgroup $\sqrt{G}$.

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Tensor products of p-adic locally convex spaces having the strongest locally convex topology

C. Perez-Garcia and W.H. Schikhof

Abstract

In this note we study the question whether the tensor product of two p-adic locally convex spaces having the strongest locally convex topology, again has the strongest locally convex topology. We show that, surprisingly, the answer depends on the algebraic dimension of the two spaces involved.

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TENSOR PRODUCTS OF P-ADIC VECTOR MEASURES

C. G. PETALAS AND A. K. KATSARAS

Abstract. Tensor products of p-adic vector measures are introduced and some of their properties are investigated. It is shown that a Fubini's Theorem holds for tensor products of τ-additive measures.

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P-adic arithmetic coding

Anatoly Rodionov, Sergey Volkov, Spectrum Systems Inc.

Abstract

A new incremental algorithm for data compression is presented. For a sequence of input symbols algorithm incrementally constructs a p-adic integer number as an output. Decoding process starts with less significant part of a p-adic integer and incrementally reconstructs a sequence of input symbols. Algorithm is based on certain features of p-adic numbers and p-adic norm.

p-adic coding algorithm may be considered as of generalization a popular compression technique – arithmetic coding algorithms. It is shown that for p = 2 the algorithm works as integer variant of arithmetic coding; for a special class of models it gives exactly the same codes as Huffman's algorithm, for another special model and a specific alphabet it gives Golomb-Rice codes.

Analysis on the Levi-Civita Field, a Brief Overview

Khodr Shamseddine and Martin Berz

Abstract

In this paper, we review the algebraic properties of various non-Archimedean ordered structures, extending them in various steps which lead naturally to the smallest non-Archimedean ordered field that is Cauchy-complete and real closed. In fact, the Levi-Civita field is small enough to allow for the calculus on the field to be implemented on a computer and used in applications such as the fast and accurate computation of the derivatives of real functions as "differential quotients" up to very high orders.

We then give an overview of recent research on the Levi-Civita field. In particular, we summarize the convergence and analytical properties of power series, showing that they have the same smoothness behavior as real power series; and we present a Lebesgue-like measure and integration theory on the field. Moreover, based on continuity and differentiability concepts that are stronger than the topological ones, we discuss solutions to one-dimensional and multi-dimensional optimization problems as well as existence and uniqueness of solutions of ordinary differential equations.

On metrizability of compactoid sets in non-archimedean locally convex spaces

Jerzy Kąkol and Wiesław Śliwa
(Poznań, POLAND)

Abstract. In 2003 N. De Grande-De Kimpe, J. Kąkol and C. Perez-Garcia using t-frames and some machinery concerning tensor products proved that compactoid sets in non-archimedean (LM)-spaces (i.e. the inductive limits of a sequence of non-archimedean metrizable locally convex spaces) are metrizable.

In this paper we show a similar result for a large class of non-archimedean locally convex space with a $\mathfrak{L}$-base, i.e. a decreasing base $(U_\alpha)_{\alpha\in\mathbb{N}^\mathbb{N}}$ of neighbourhoods of zero. This extends the mentioned result since every non-archimedean (LM)-space has a $\mathfrak{L}$-base. We also prove that compactoid sets in non-archimedean (DF)-spaces are metrizable. Our proofs are short and elegant.

Criteria for non-repelling fixed points

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Let $g(x) \in \mathbb{Z}_p[x]$ be a monic irreducible polynomial. For each $a \in \mathbb{Q}_p$, let $h_a(x) = x + a\,g(x)$. We study the discrete dynamical system defined by the polynomial $h_a(x)$, on the extension $\mathbb{Q}_p(\alpha)$ of $\mathbb{Q}_p$, where α is a zero of $g(x)$ and thus a fixed point of h_a. For the special case when $a = 1$, these dynamical systems have previously been studied in [1], and our objective is to generalize those results. The investigation is divided into three parts, depending on whether $|a| = 1$, $|a| < 1$, or $|a| > 1$.

References

[1] P.-A. Svensson, *Dynamical Systems in Local Fields of Characteristic Zero*, Ph.D. thesis, Växjö University, 2004.

A p-ADIC q-DEFORMATION OF THE WEYL ALGEBRA, FOR q A p^N-TH ROOT OF UNITY

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Abstract. Let q be a principal unit of the ring of valuation of a complete valued field K, extension of the field of p-adic numbers $\mathbb{Q}_p$.
In a recent work, we have studied a model of q-deformation of the Weyl algebra generated by two variables, whenever q is not a root of unity. In this paper we study these algebras when q is a p^N-th primitive root of unity. Interesting orthonormal families are exhibit and providing p-adic completion of this algebra and his center.

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