Parametric Resonance Cooling – Simulations

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Overview

- Final transverse ionization cooling - Parametric resonance enhancement
- Resonant transport channels for final cooling - lattice prototypes
  - quadrupole based
  - solenoid based
- Transverse beam dynamics in the cooling channel – tracking studies
  - 'soft-edge' solenoid
    - liner transfer matrix
    - nonlinear corrections (in tracking)
  - thin 'ideal' absorber model
- G4BL simulation of a solenoid channel with absorbers – basic tools
  - Hydrogen absorber
  - Solenoid transfer matrix from G4BL simulation - symplectification

Transverse parametric resonance cooling

- Transport channel (between consecutive absorbers) designed to replenish large angular component, x', sector of the phase-space, 'mined' by ionization cooling process.
- Parametric resonance in an oscillating system - perturbing frequency is equal to the harmonic of the characteristic (resonant) frequency of the system.
- Normal elliptical motion of a particle's transverse coordinate in phase space becomes hyperbolic – resulting beam emittance has a wide spread in x' and narrow spread in x – sector of the phase-space where ionization cooling is most effective

Transfer matrix of a periodic resonant lattice

- Symplectic transfer matrix, M(s), for a beamline (in x or y)

$$M(s) = \begin{bmatrix} e^{-\lambda s}\cos\psi & g\sin\psi \\ -\frac{1}{g}\sin\psi & e^{\lambda s}\cos\psi \end{bmatrix}$$

- Lattice period can be designed in such a way that $\sin\psi = 0 \Rightarrow \psi = n\pi$, n = 1, 2.

$$M(s) = \begin{bmatrix} e^{-\lambda s}\cos\psi & g\sin\psi \\ -\frac{1}{g}\sin\psi & e^{\lambda s}\cos\psi \end{bmatrix} \rightarrow \begin{bmatrix} e^{-\lambda s} & 0 \\ 0 & e^{\lambda s} \end{bmatrix}$$

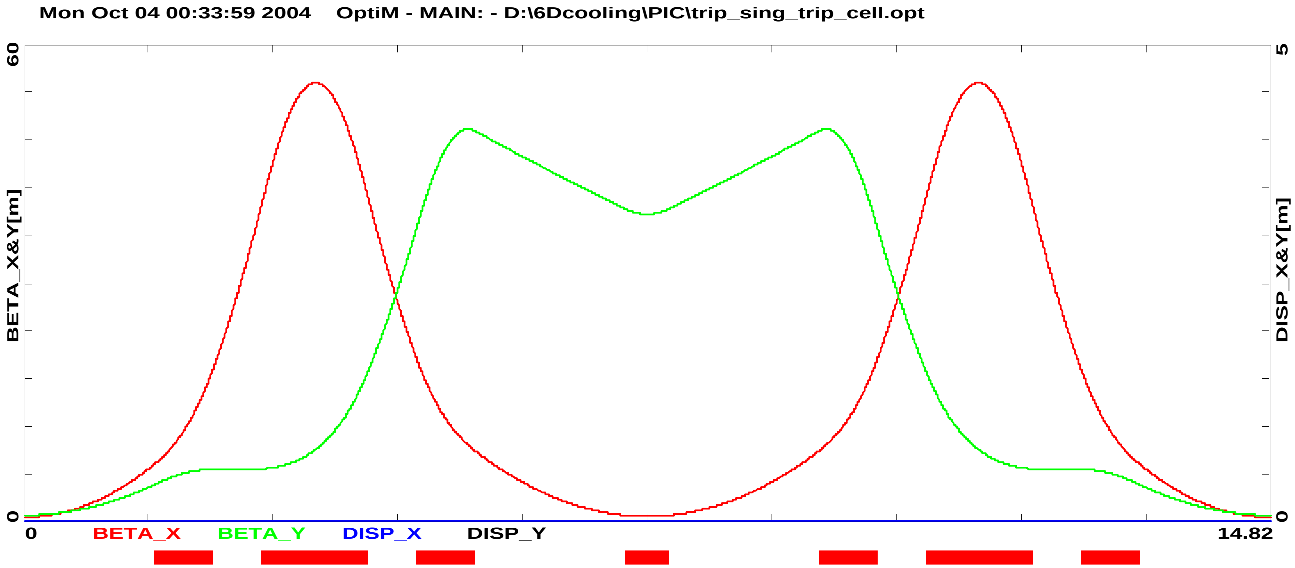
- Coordinate and angle are uncoupled - resulting beam emittance has a wide spread in x' and narrow spread in x.

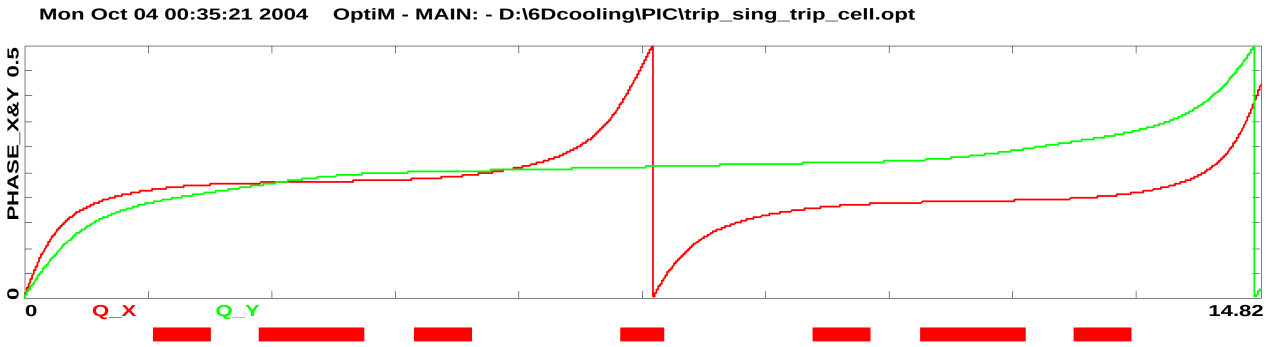
$$x_{s_0+s} = \pm e^{-\lambda s} x_{s_0}$$

$$x'_{s_0+s} = \pm e^{\lambda s} x'_{s_0}$$

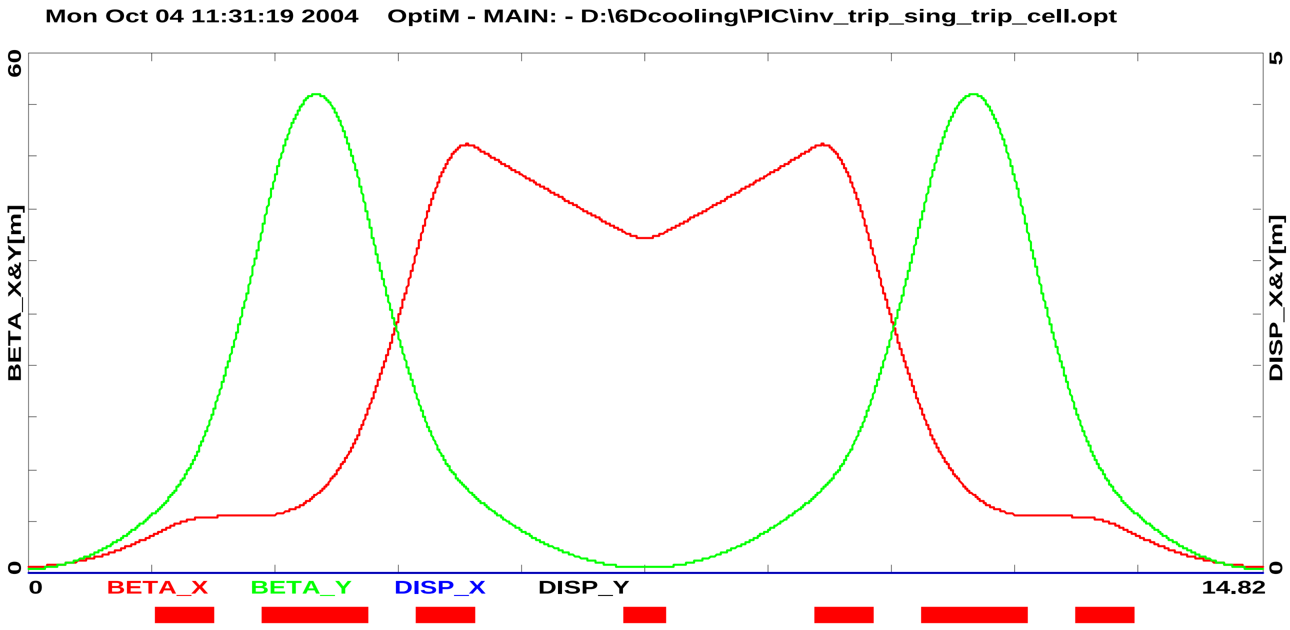
x x' = const

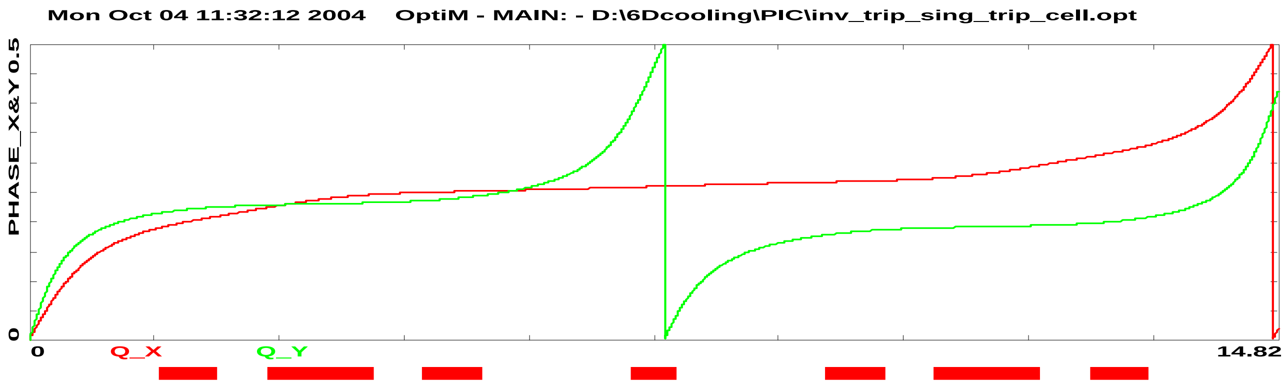
Resonantly perturbed uniform triplet lattice (H)



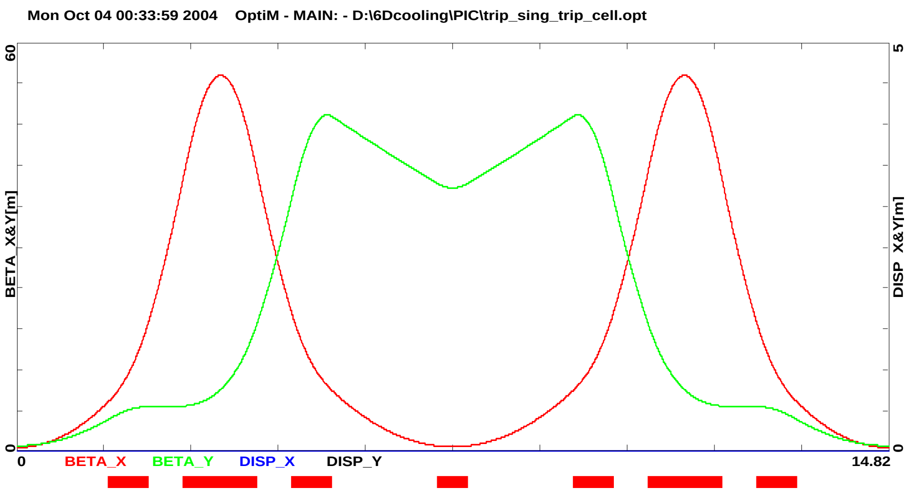


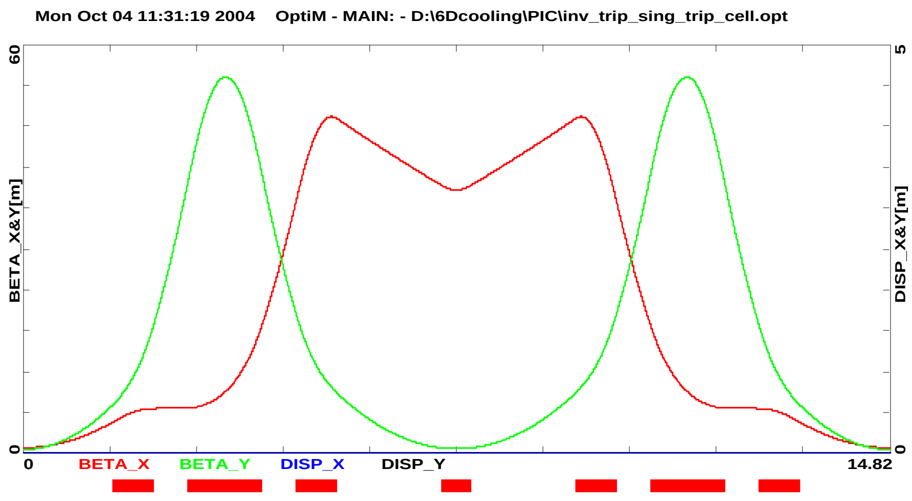
Resonantly perturbed uniform triplet lattice (V)

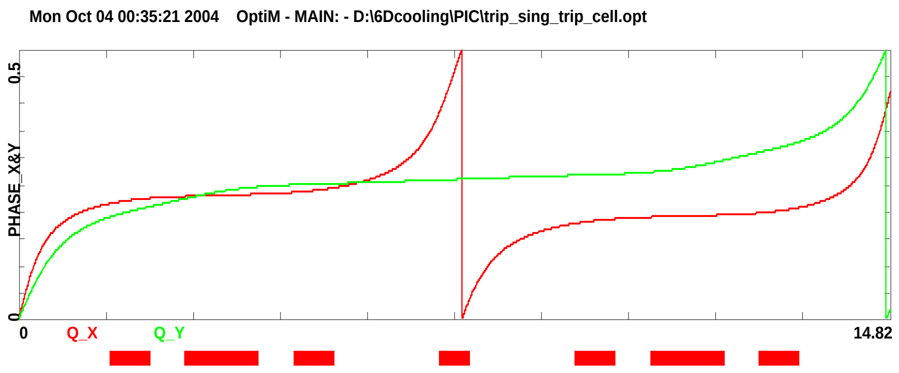


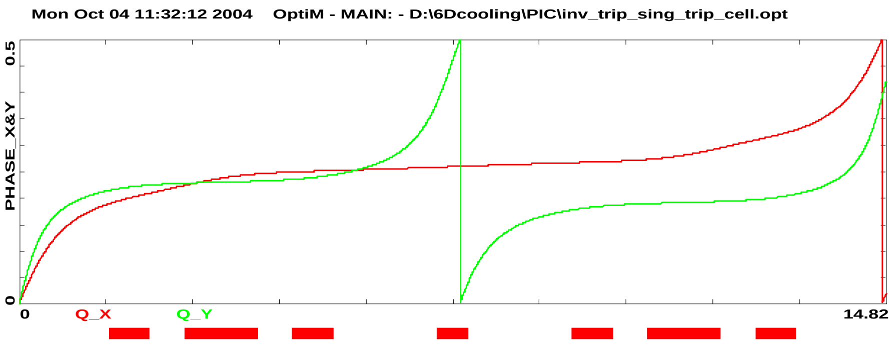


Symmetrized double cell ($\Delta\phi_x = 3\pi = \Delta\phi_y$)

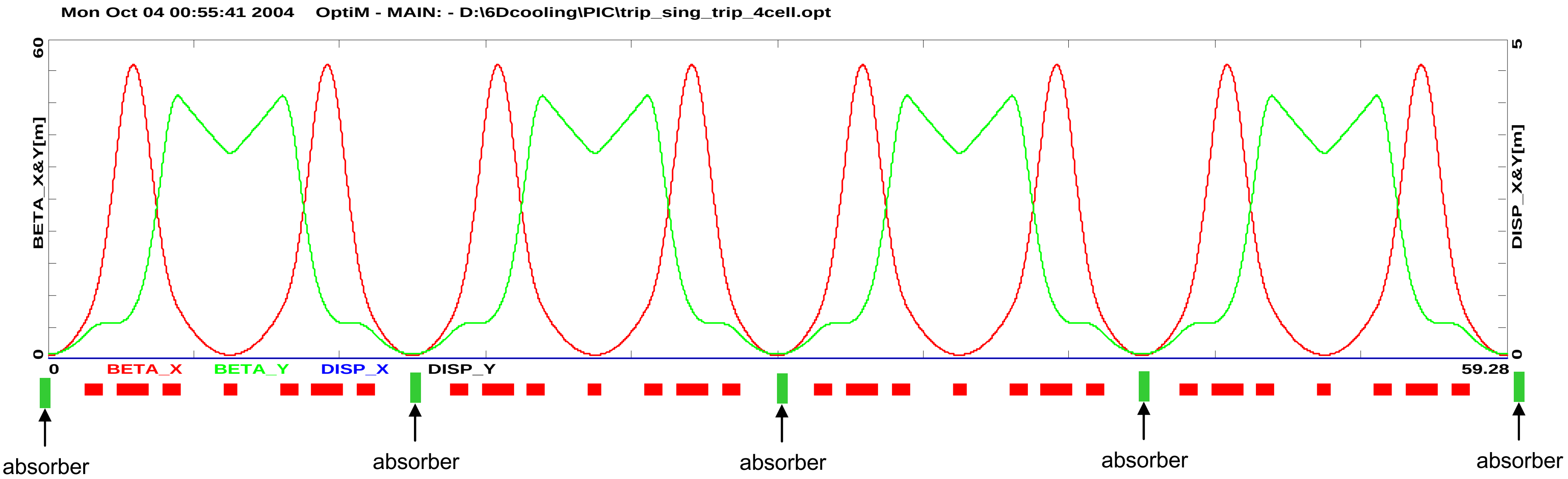








4-cell resonant channel



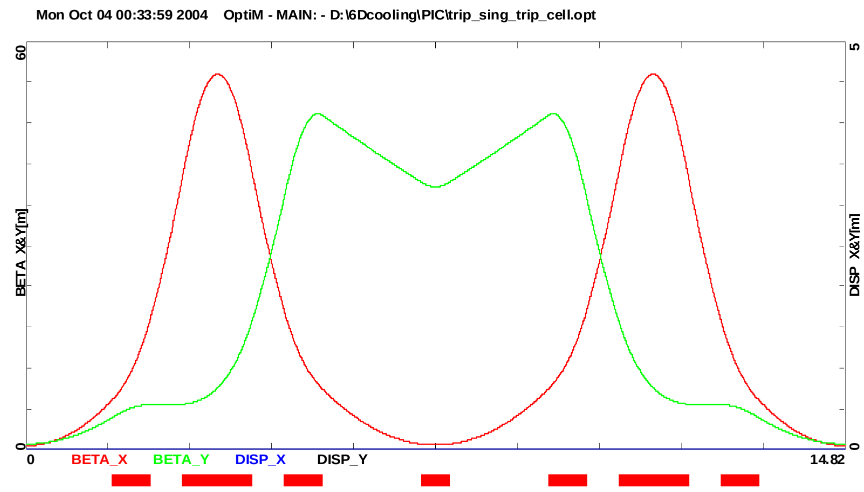
- Uniform triplet lattice resonantly perturbed by a singlet
- Absorber placed half way between triplets

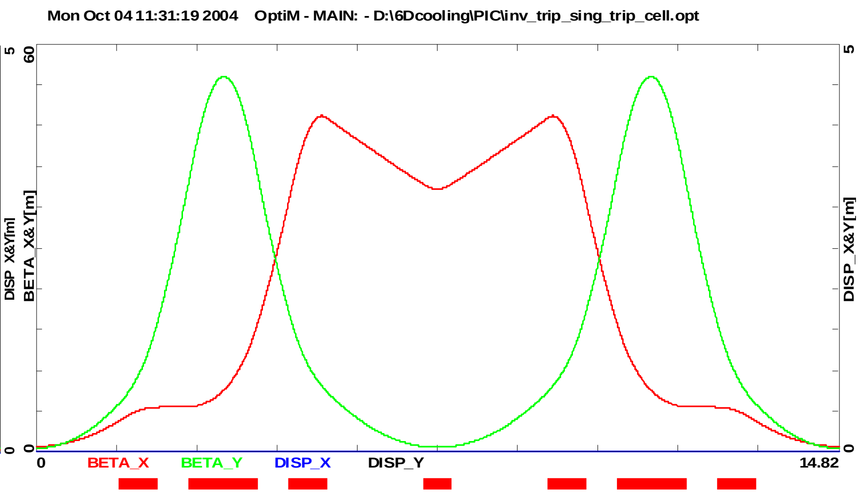
Final cooling – initial beam parameters

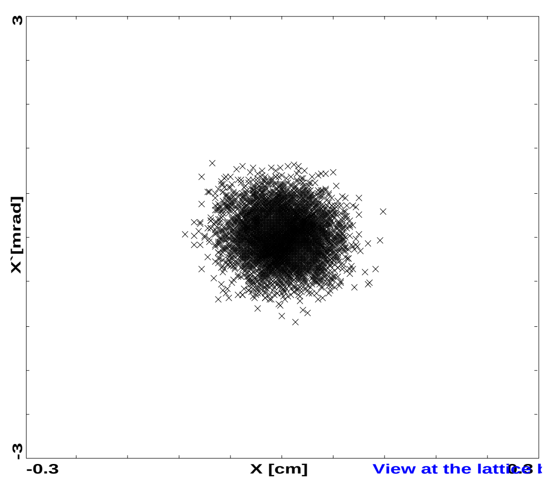
p = 287 MeV/c

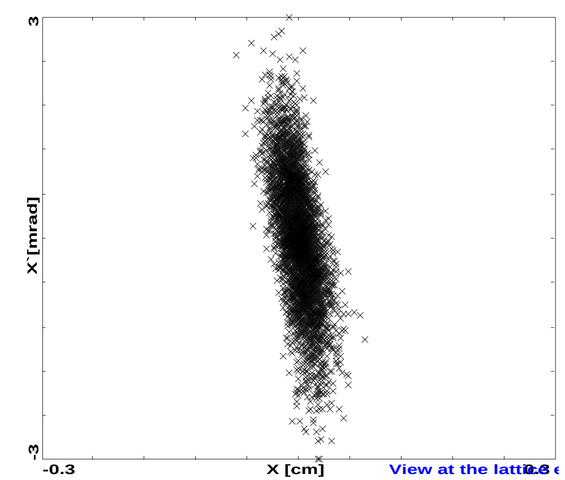
after helical cooling channel		ε_{rms}
normalized emittance: $\varepsilon_x/\varepsilon_y$	mm·mrad	**300**
longitudinal emittance: ε_l ($\varepsilon_l = \sigma_{\Delta p}\ \sigma_z/m_\mu c$) momentum spread: $\sigma_{\Delta p/p}$ bunch length: σ_z	mm mm	**7** **0.03** **100**

Angular 'shearing' of the transverse phase-space

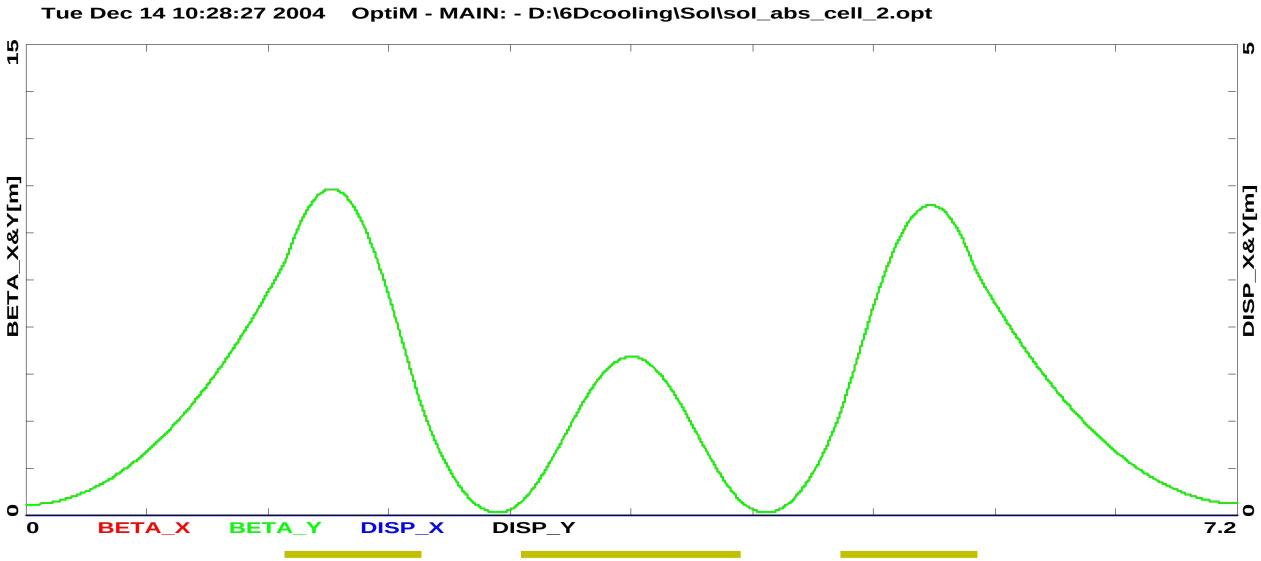


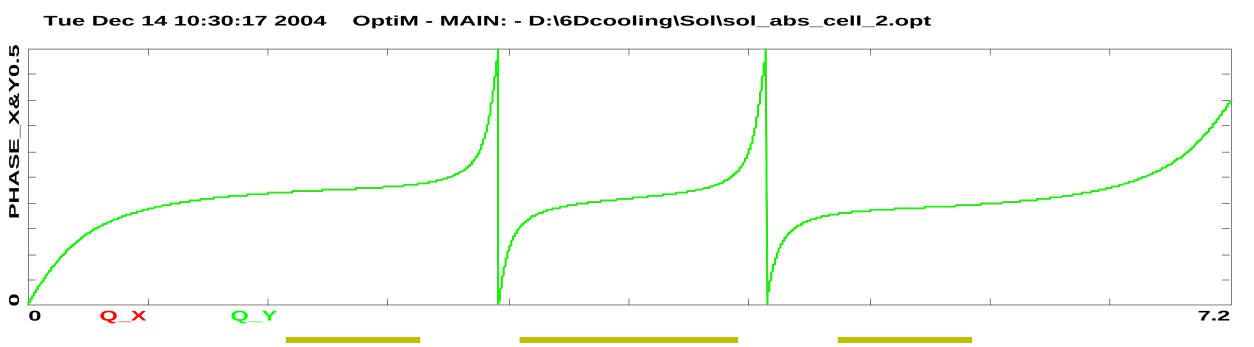






Solenoid cell





c1	L[cm]=130	B[kG]=38.1	Aperture[cm]=10
c2	L[cm]=80	B[kG]=-34.3	Aperture[cm]=10

'soft-edge' solenoid model

- Zero aperture solenoid - ideal linear solenoid transfer matrix:

$$M_{sol} = \begin{bmatrix} \frac{1+\cos(kL)}{2} & \frac{\sin(kL)}{k} & \frac{\sin(kL)}{2} & \frac{1-\cos(kL)}{k} \\ -\frac{k\sin(kL)}{4} & \frac{1+\cos(kL)}{2} & -k\frac{1-\cos(kL)}{4} & \frac{\sin(kL)}{2} \\ -\frac{\sin(kL)}{2} & -\frac{1-\cos(kL)}{k} & \frac{1+\cos(kL)}{2} & \frac{\sin(kL)}{k} \\ k\frac{1-\cos(kL)}{4} & -\frac{\sin(kL)}{2} & -\frac{k\sin(kL)}{4} & \frac{1+\cos(kL)}{2} \end{bmatrix}$$

$k = eB_0/pc$

'soft-edge' solenoid – edge effect

- Non-zero aperture - correction due to the finite length of the edge :

  - It decreases the solenoid total focusing – via the effective length of:

$$L = \frac{1}{B_0} \int_{-\infty}^{\infty} B_z(s)\, ds$$

  - It introduces axially symmetric edge focusing at each solenoid end:

$$\Phi_{edge} = \frac{1}{2}\left(\int_{-\infty}^{\infty} B_z^2(s)\, ds - B_0^2 L \right) = -\frac{k^2 a}{8} \qquad k = eB_0/pc$$

  - axially symmetric quadrupole

  - $\mathbf{M}_{soft\ sol} = \mathbf{M}_{edge}\, \mathbf{M}_{sol}\, \mathbf{M}_{edge}$

$$\mathbf{M}_{edge} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\Phi_{edge} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\Phi_{edge} & 1 \end{bmatrix}$$

'soft-edge' solenoid – nonlinear effects

- Nonlinear focusing term $\Delta F \sim \mathcal{O}(r^2)$ follows from the scalar potential:
  - Scalar potential in a solenoid

$$\phi(r,z) = \phi_0 + B_z \cdot z + \left(\frac{d}{dz}B_z\right)\cdot\frac{2\cdot z^2 - r^2}{4} + \left(\frac{d^2}{dz^2}B_z\right)\cdot\frac{z\cdot\left(2\cdot z^2 - 3\cdot r^2\right)}{12} + \left(\frac{d^3}{dz^3}B_z\right)\cdot\frac{8\cdot z^4 - 24\cdot z^2\cdot r^2 + 3\cdot r^4}{192}$$

  - Solenoid B-fields

$$B_z(r,z) = B_z - \left(\frac{d^2}{dz^2}B_z\right)\cdot\frac{r^2}{4}$$

$$B_r(r,z) = -\frac{r}{2}\cdot\left(\frac{d}{dz}B_z\right) + \frac{r^3}{16}\cdot\left(\frac{d^3}{dz^3}B_z\right)$$

'soft-edge' solenoid – nonlinear effects

- In tracking simulations the first nonlinear focusing term, $\Delta F \sim \mathcal{O}(r^2)$ is also included:

$$\Phi \equiv \frac{1}{F} \approx \left(\frac{e}{2pc}\right)^2 \left(\int B^2 ds + \frac{r^2}{2}\int B'^2 ds\right) \approx L\left(\frac{eB_0}{2pc}\right)^2\left(1+\frac{r^2}{3aL}\right)$$

  - Nonliner focusing at r = 20 cm for 1 m long solenoid with 25 cm aperture radius

$$\frac{\Delta F}{F} \approx \frac{r^2}{2}\frac{\int B'^2 ds}{\int B^2 ds} \approx \frac{r^2}{3aL} \xrightarrow{L/r=4,\, r\approx 0.8a} 0.07$$

Thin absorber with re-acceleration

- Ionization cooling due to energy loss (-Δp) in a thin absorber followed by immediate re-acceleration (Δp) can be described as:

$$\Delta\theta_\perp = -\theta_\perp \frac{\Delta p}{p}$$

  - The corresponding canonical transfer matrix can be written as

$$M_{abs} = K \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-\frac{\Delta p}{p} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1-\frac{\Delta p}{p} \end{bmatrix} K^{-1}$$

$$K = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -k/2 & 0 \\ 0 & 0 & 1 & 0 \\ k/2 & 0 & 0 & 1 \end{bmatrix} \qquad \hat{\mathbf{x}} \equiv \begin{bmatrix} x \\ p_x \\ y \\ p_y \end{bmatrix} \qquad \mathbf{x} \equiv \begin{bmatrix} x \\ \theta_x \\ y \\ \theta_y \end{bmatrix}$$

$$k = eB_z / pc \qquad \hat{\mathbf{x}} = K\mathbf{x}$$

Final cooling – initial beam parameters

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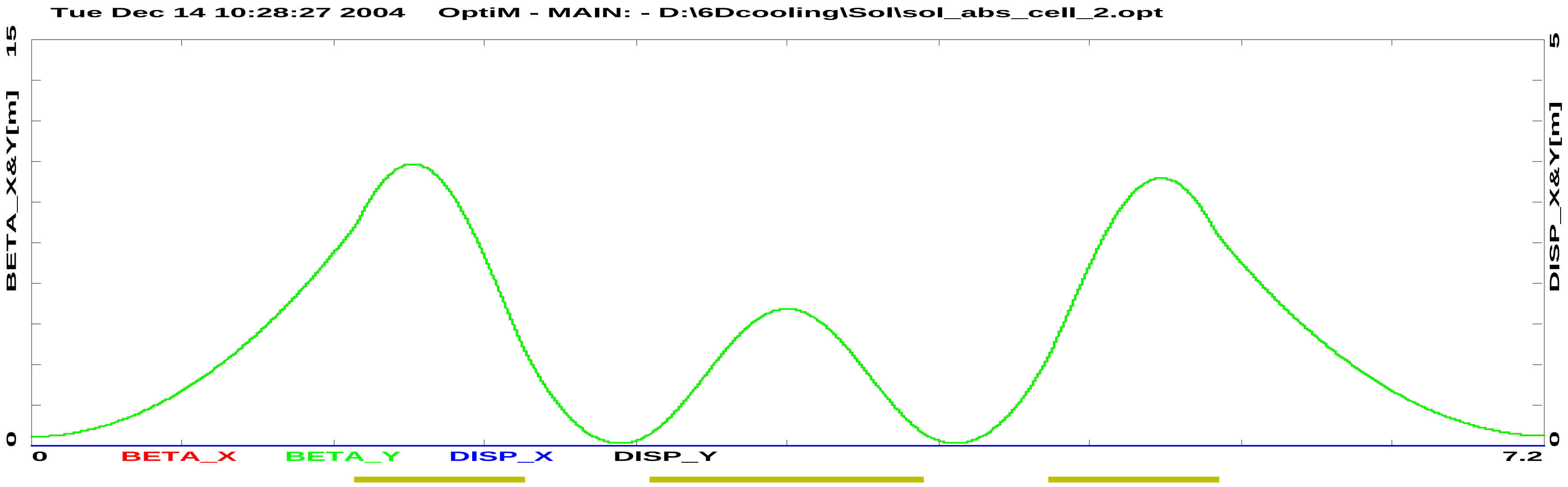
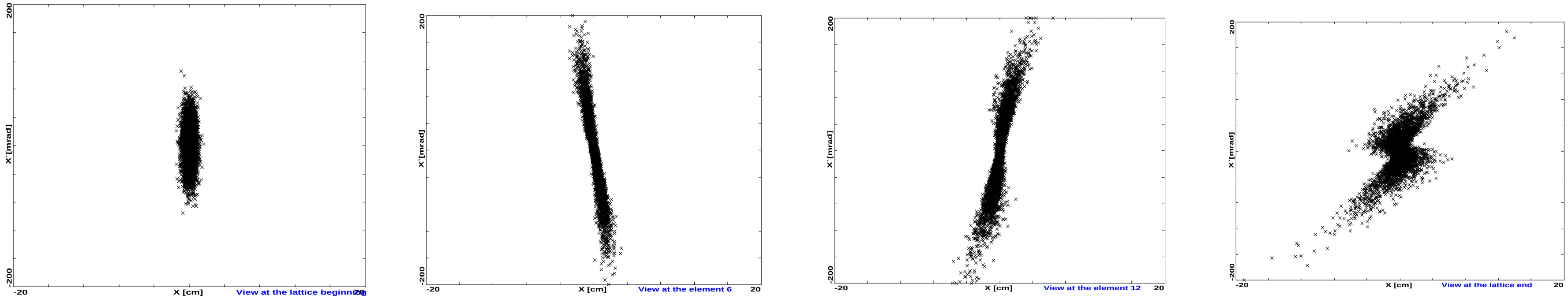
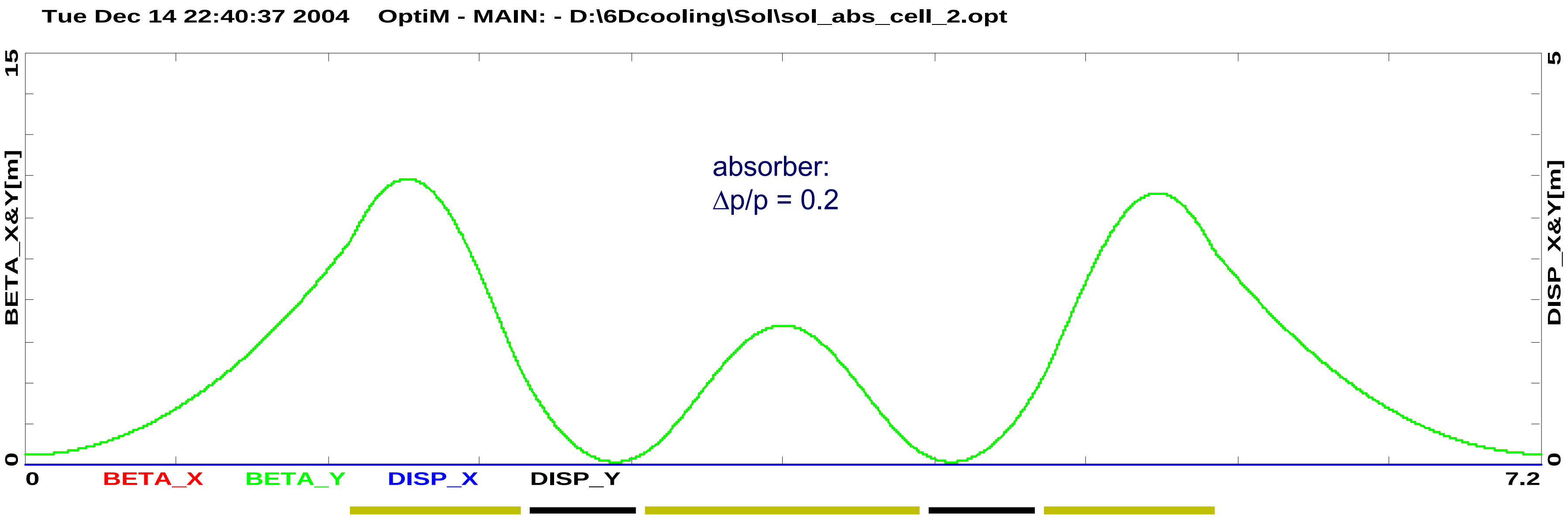
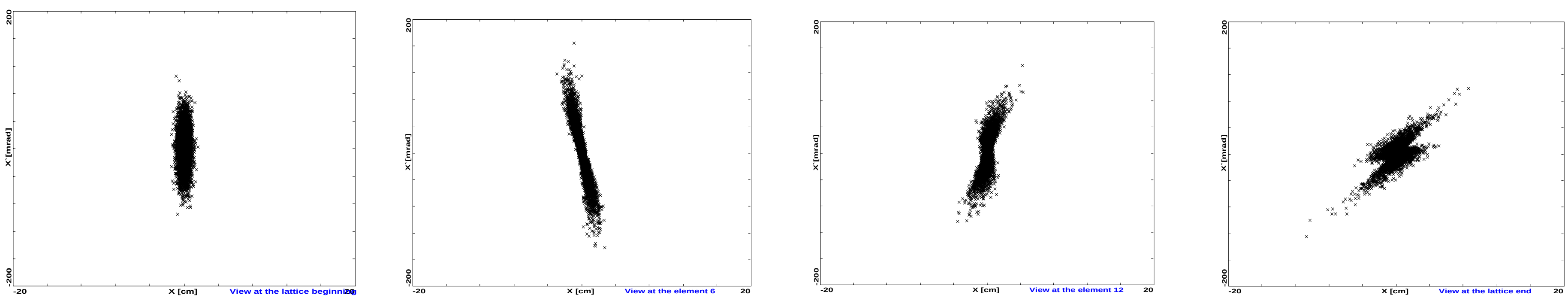
Solenoid cell, no absorber ($\Delta\phi = \pi$) – particle tracking



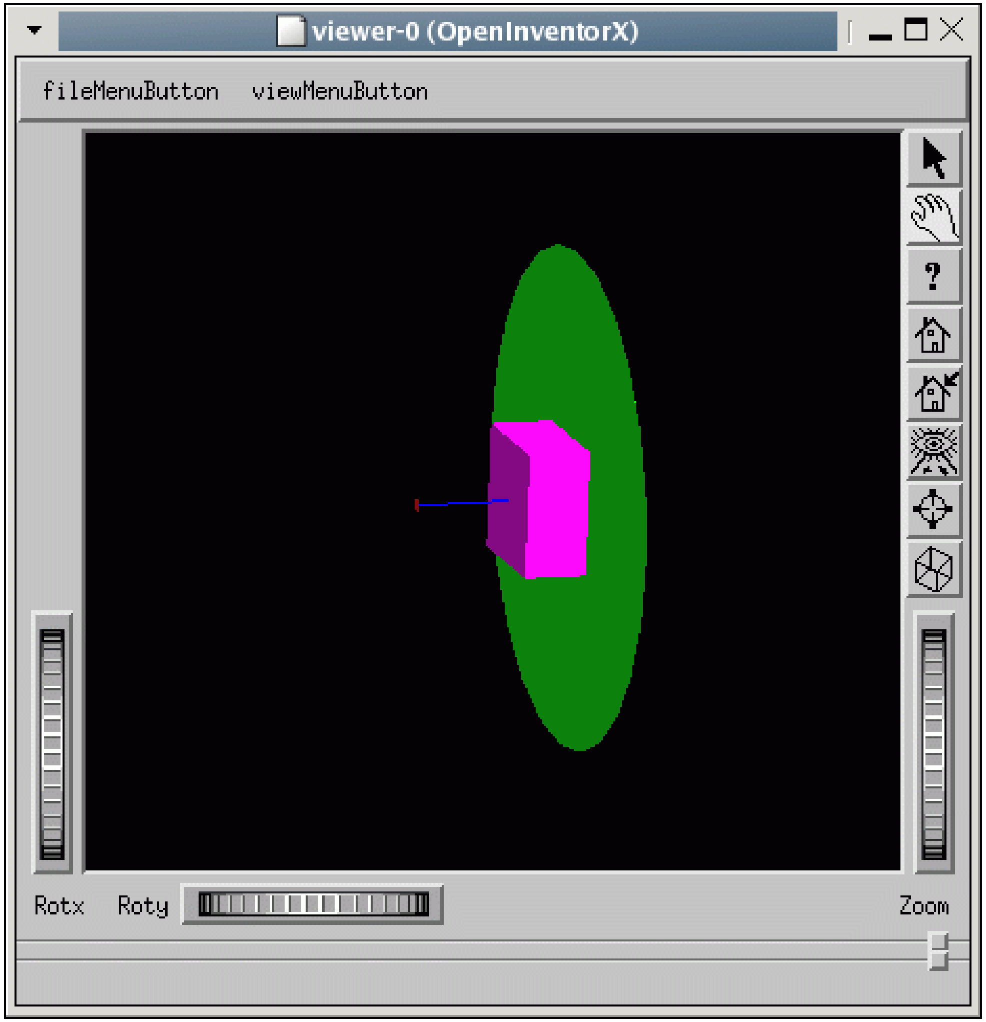


Solenoid cell with absorber ($\Delta\phi = \pi$) – particle tracking





Absorber – G4BL model

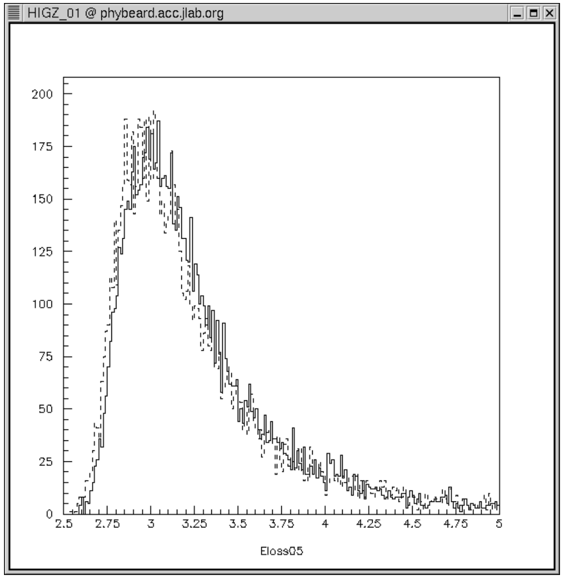


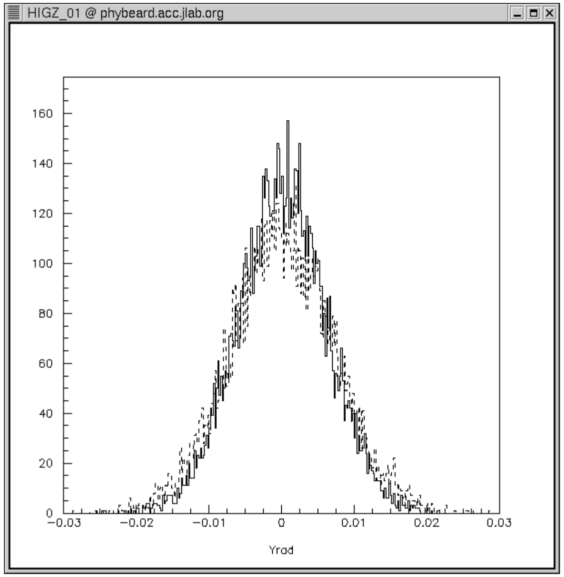
50 cm long box of gaseous Hydrogen at 200 atm

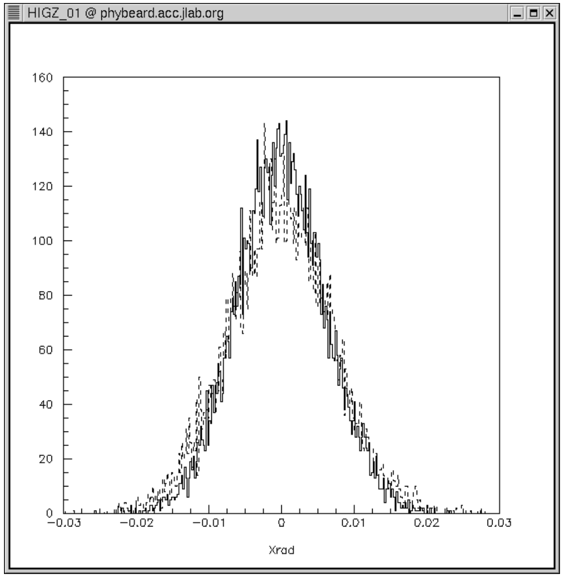
E = 200 MeV

p = 287 MeV/c

Absorber – G4BL simulation



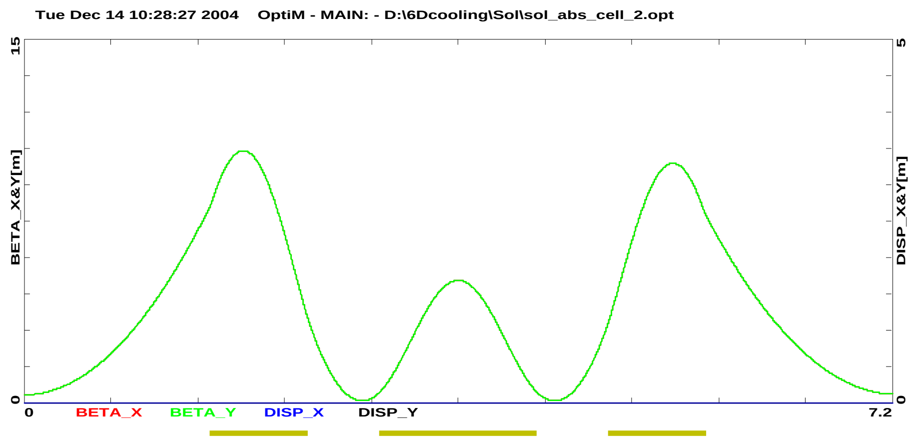


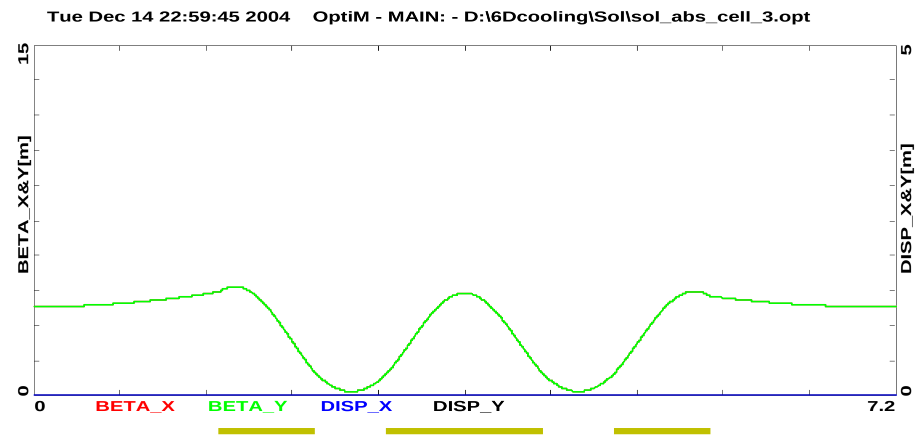


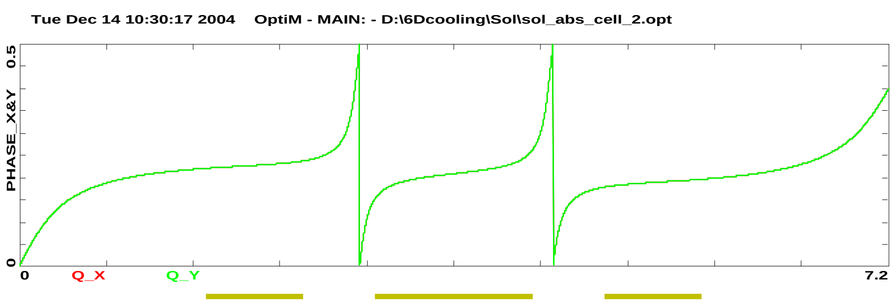
Initial test beam - muons at p = 287 MeV/c

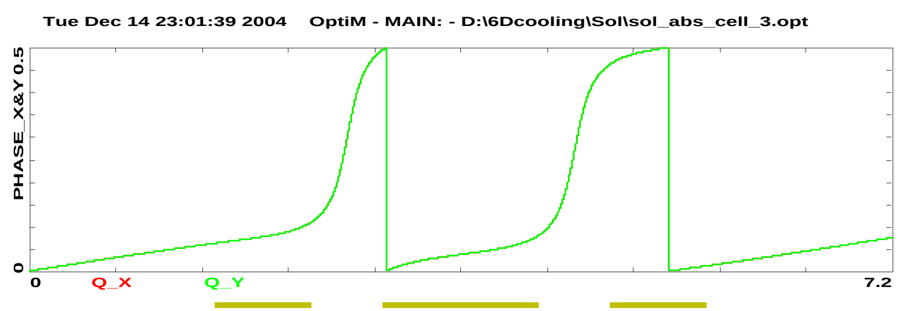
parallel beam with no momentum spread

Solenoid cell – resonant vs non-resonant





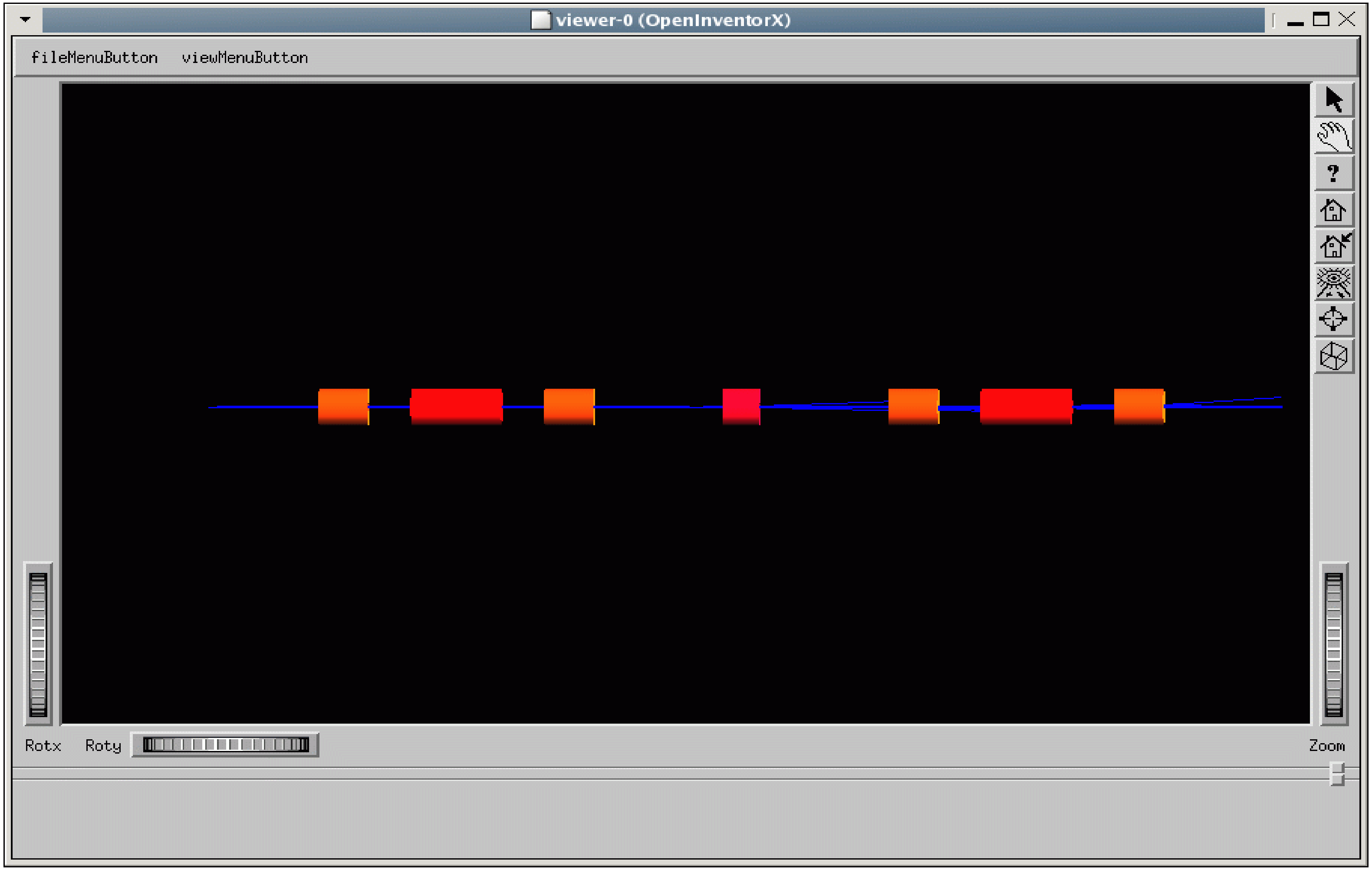




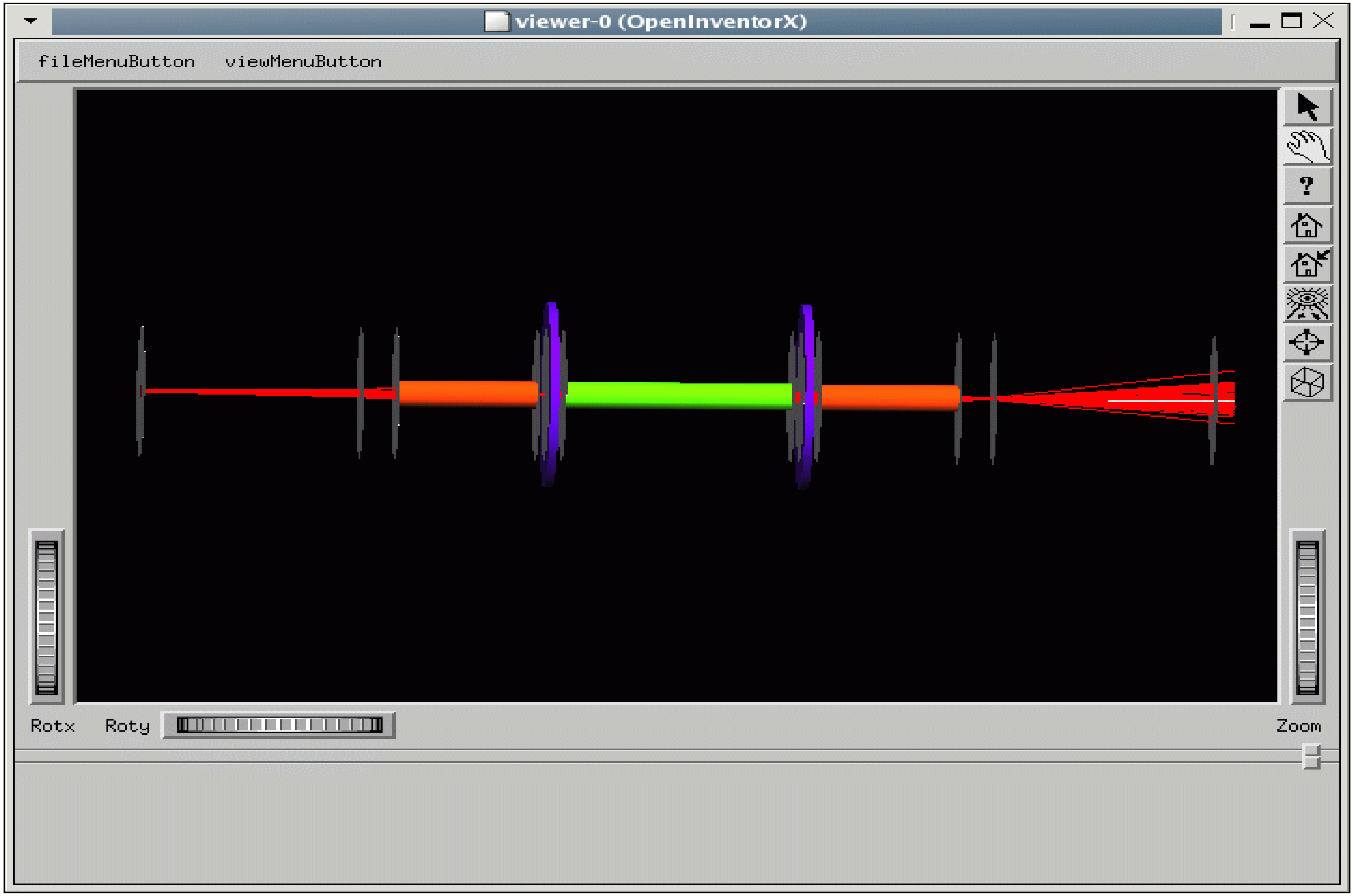
L[cm]=130 B[kG]=38.1
L[cm]=80 B[kG]=-34.3

L[cm]=130 B[kG]=32.9
L[cm]=80 B[kG]=-32.1

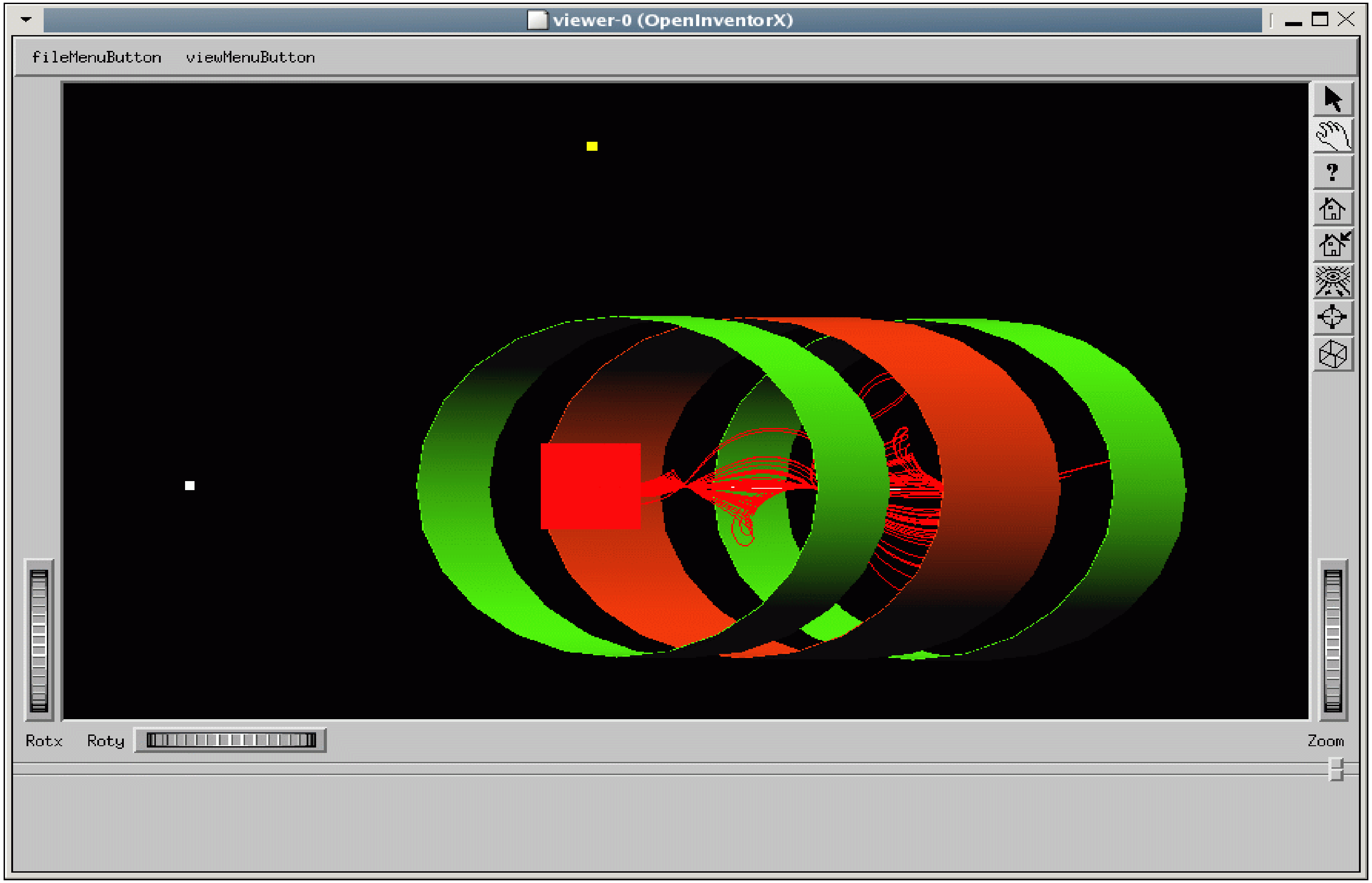
Quadrupole cell – G4BL view



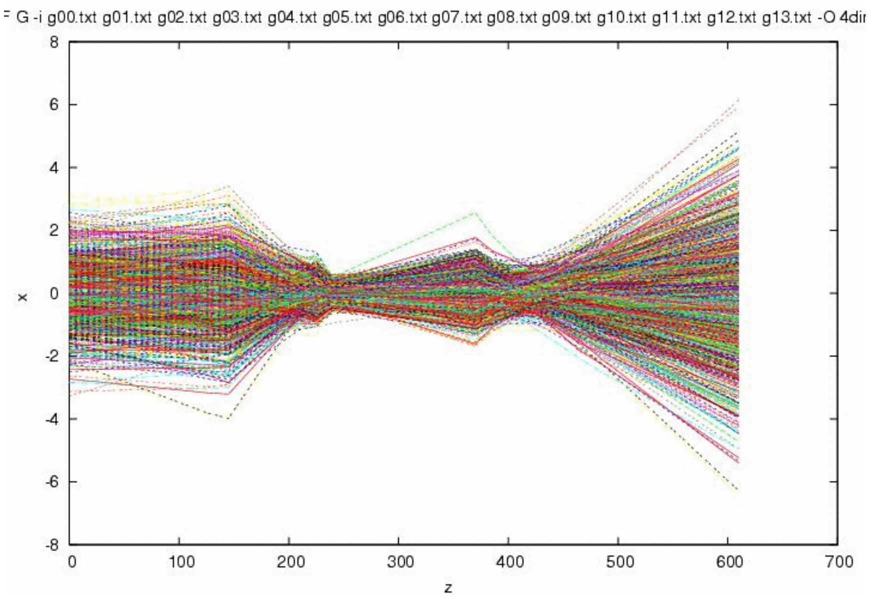
Solenoid cell – G4BL view

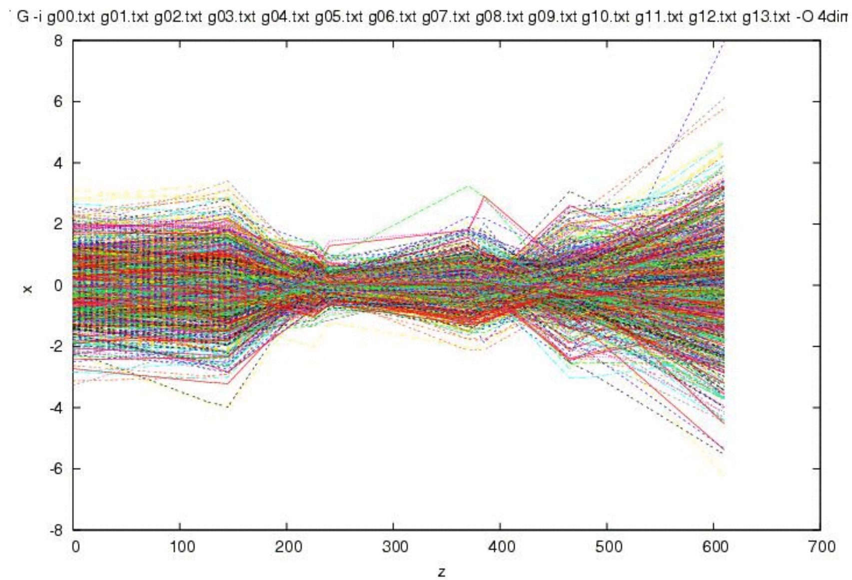


Solenoid cell – G4BL view



Solenoid cell – G4BL trajectories





Summary

- Present status…
  - Prototype PIC lattices - quadrupole and solenoid channels
    - Lattices with absorbers studied via transfer matrix code
    - Building and testing G4BL tools
  - Initial G4BL study of a solenoid channel
- Future work…
  - G4beamline simulation of a qudrupole channel with absorbers
  - G4beamline simulation of a solenoid channel with absorbers
  - G4beamline simulation with absorbers followed by RF cavities
  - Emittance implementation – ecalc9