How much time necessary to photo-generate Fermi surface from true electron vacuum?

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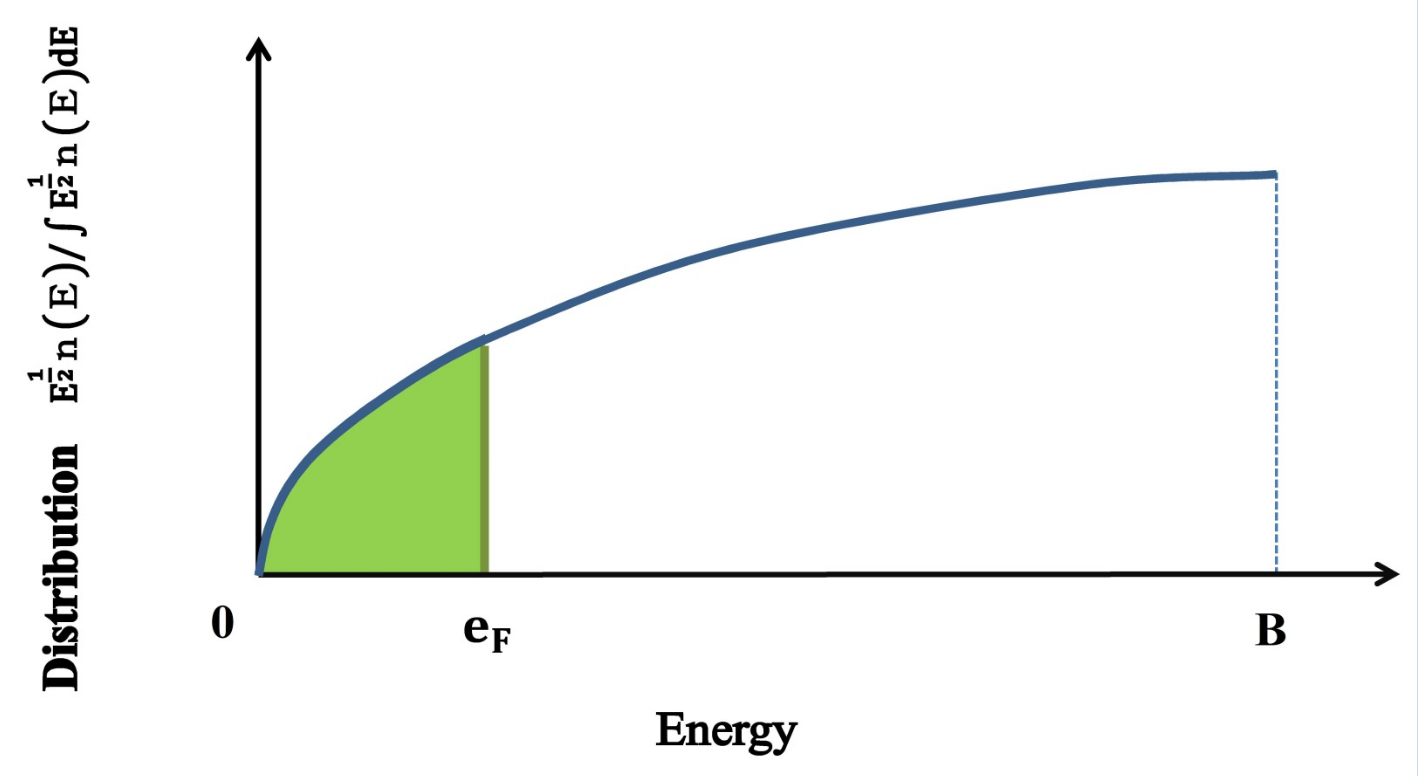
Presence of Fermi surface is the base of solid state science.

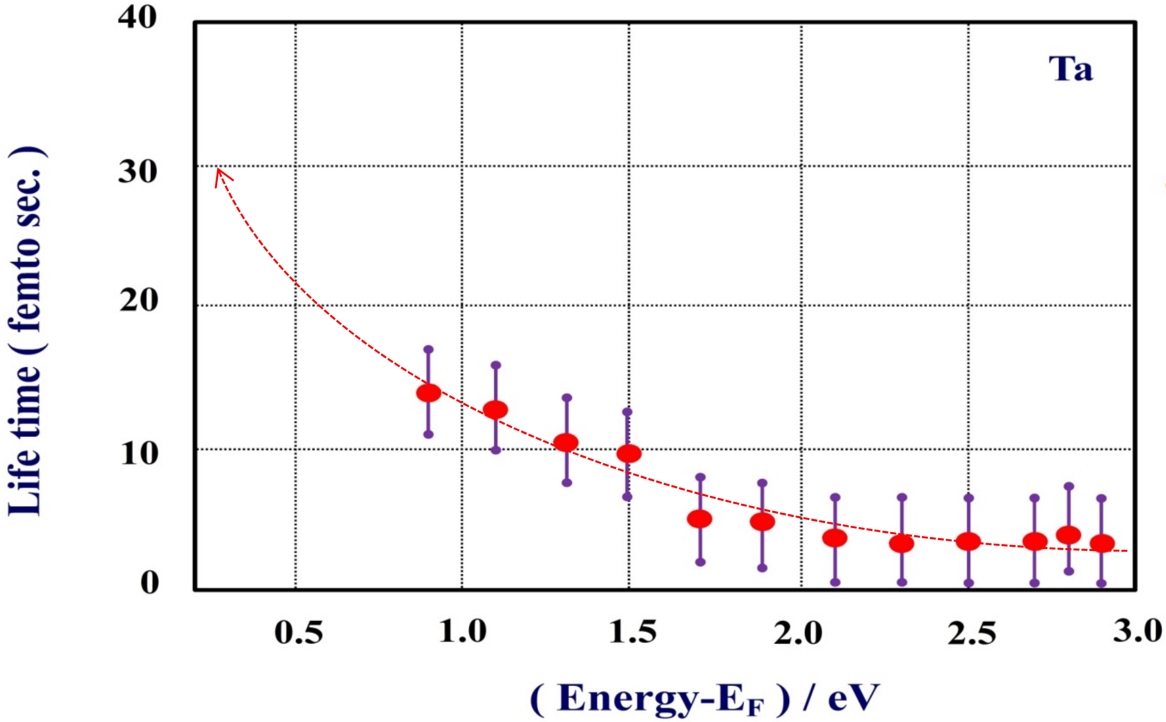


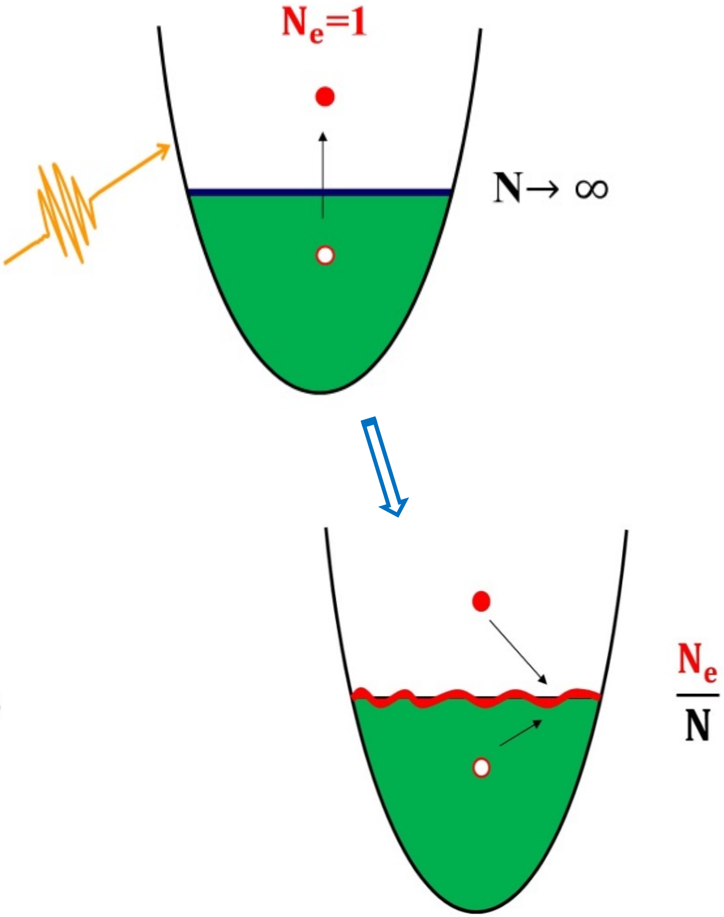
BCS, Kondo, C(S)DW, Plasmon theories are all assumed, it has been already well established.

How much time necessary to photo-generate Fermi surface from true electron vacuum?

Motivation

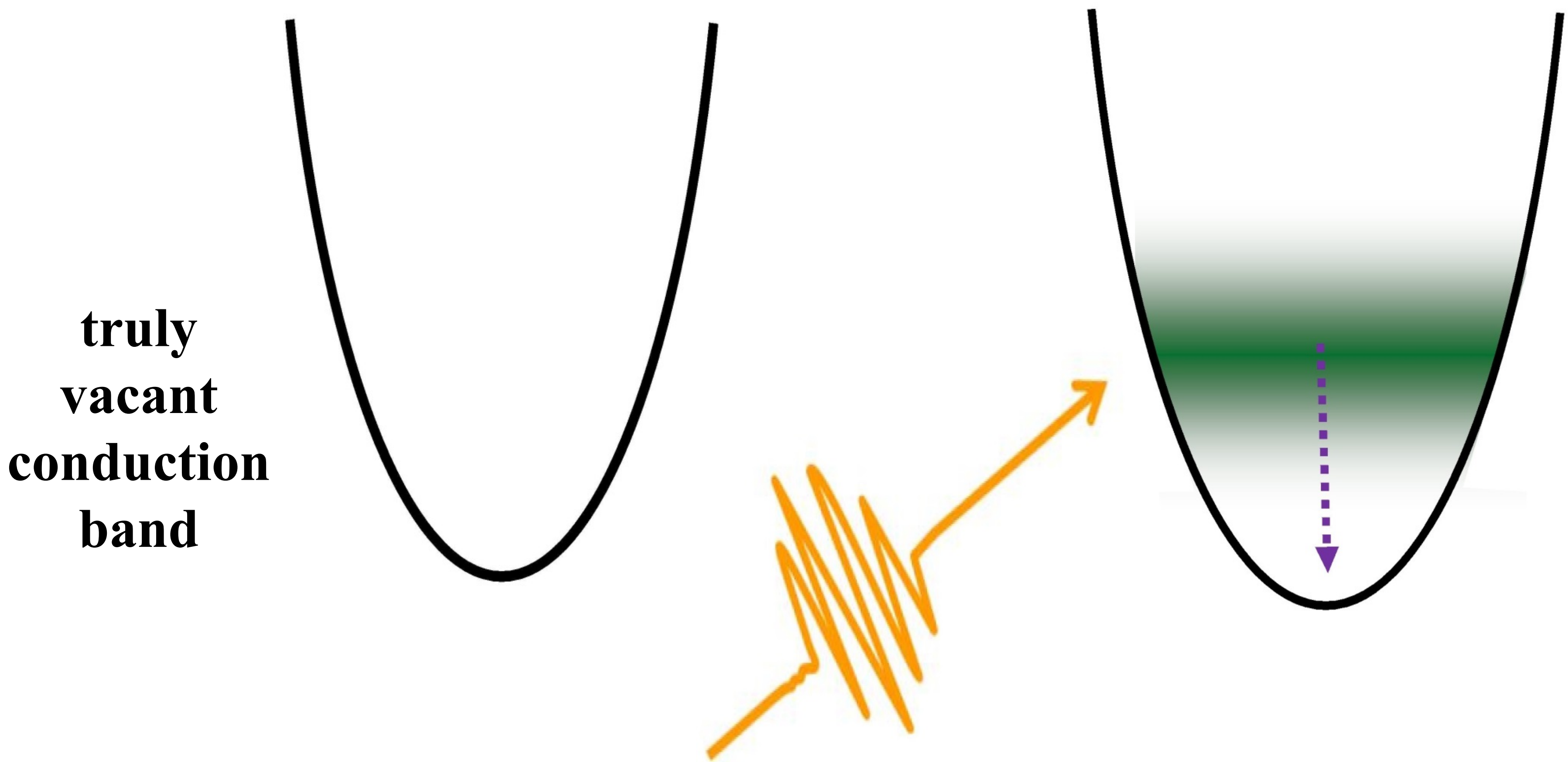
Rapid relaxation dynamics of optically excited electrons in metallic systems, has already been widely investigated. In most cases, however, only a few electrons are excited, while, the main part of electrons is still in the ground state, works as an infinite heart reservoir, resulting in quite rapid relaxation of newly given energy and momentum.



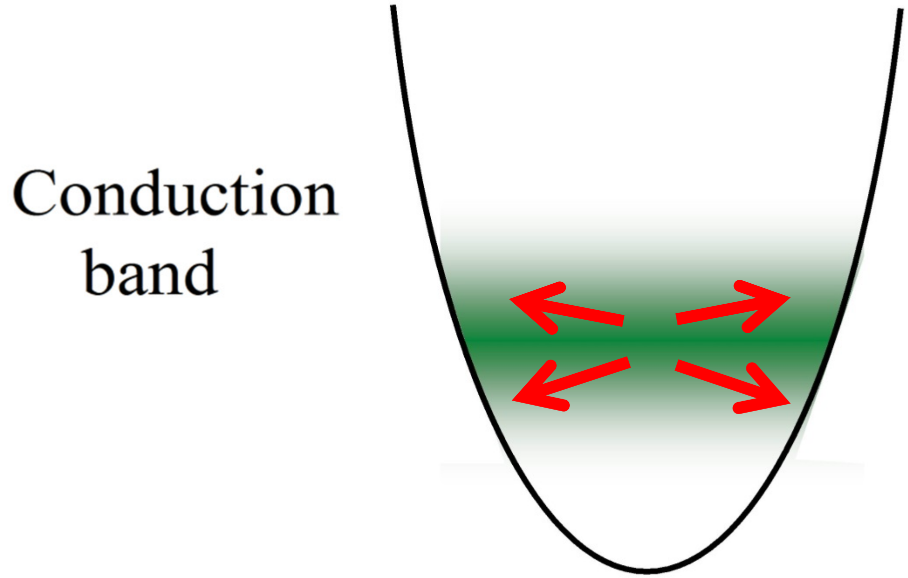


Experiment by V.Zhukov et al, PRB70(2004)233106.

What occurs, if a macroscopic number of electrons are excited, at once, into a truly vacant conduction band, without electronic heat reservoir?



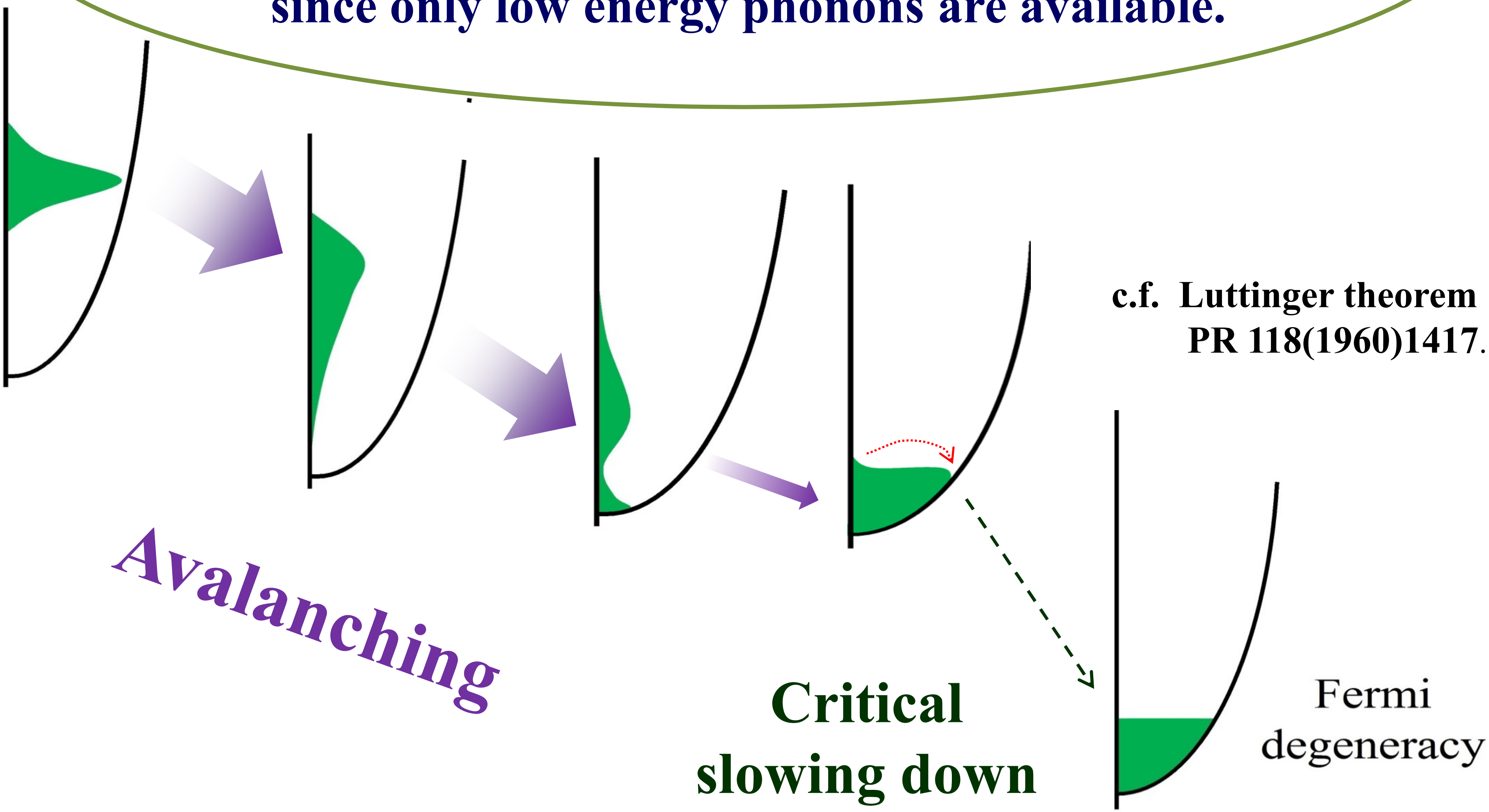
Coulombic inter-electron scatterings within the conduction band, being completely elastic, can give no net energy relaxation.



Intra-band Coulomb scattering

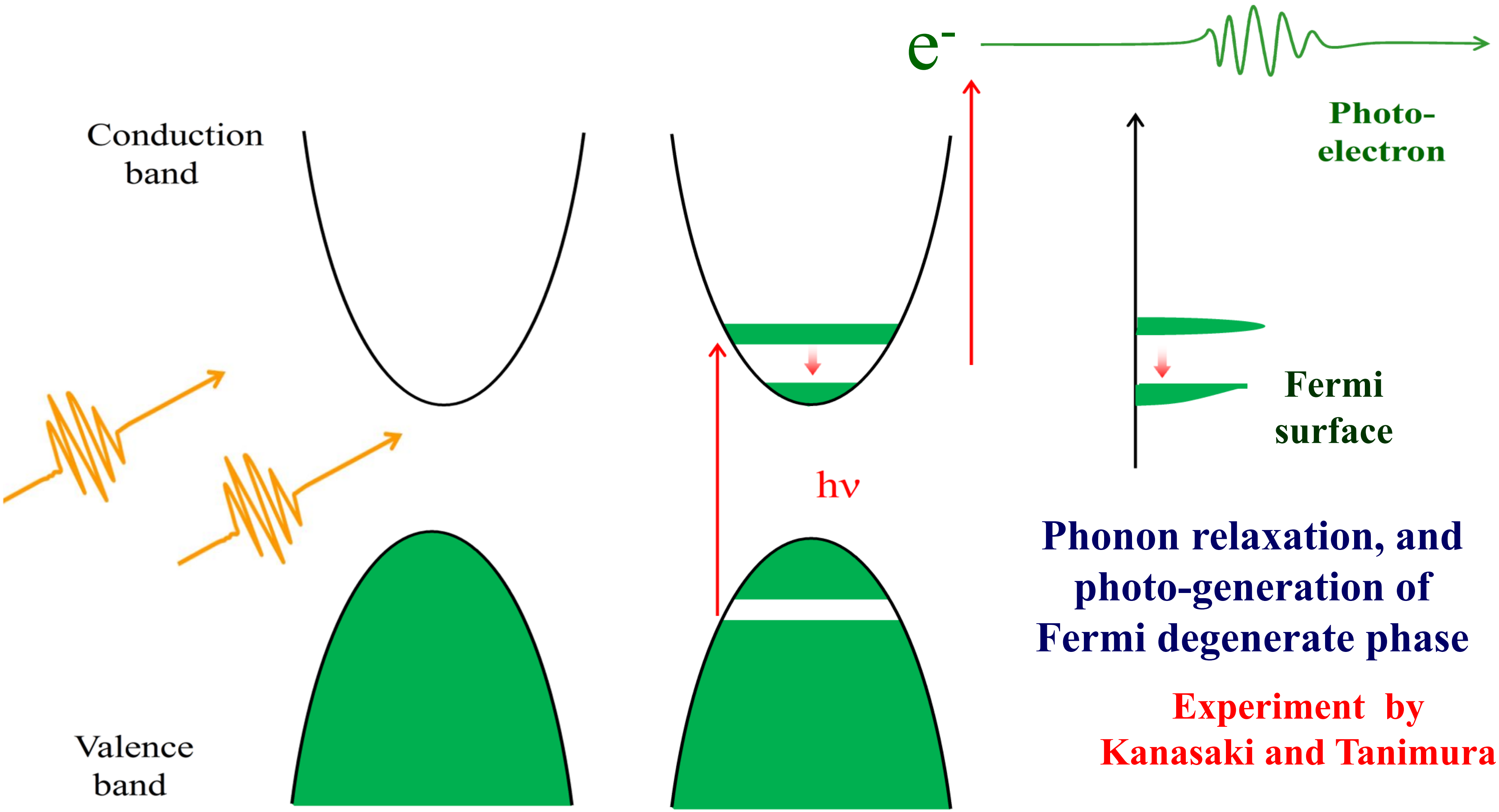
Phonon relaxation, two time regions

avalanching initially, but soon it slows down infinitely,
as approaches Fermi degeneracy,
since only low energy phonons are available.



c.f. Luttinger theorem
PR 118(1960)1417.

Two pulse excitations of GaAs, InP by visible laser
Time resolved photo-emission spectrum of conduction band electron

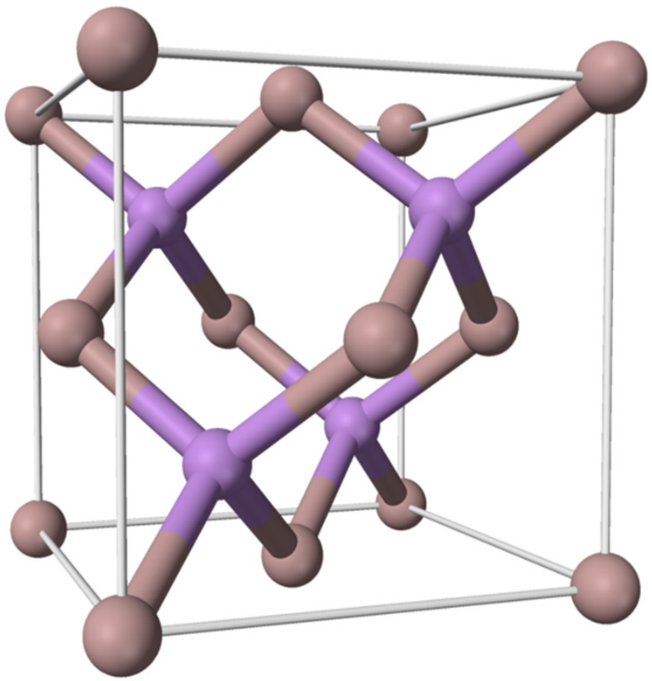


Phonon relaxation, and photo-generation of Fermi degenerate phase

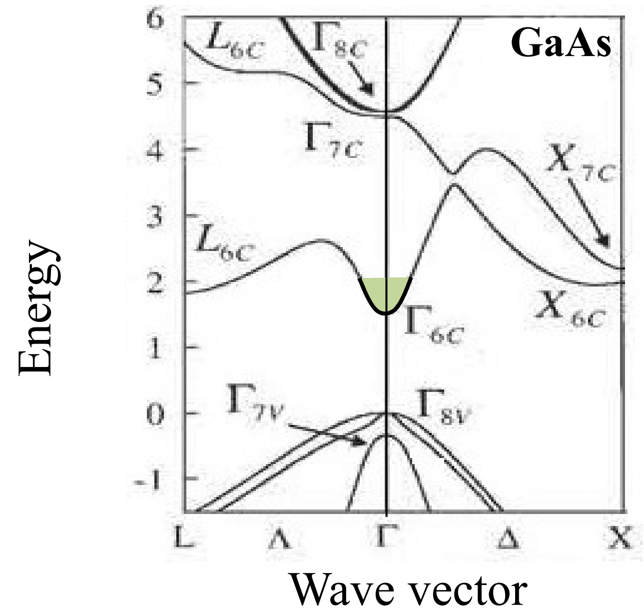
Experiment by Kanasaki and Tanimura

Most simple, but ultimate photo-induced phase transition from true electron vacuum.

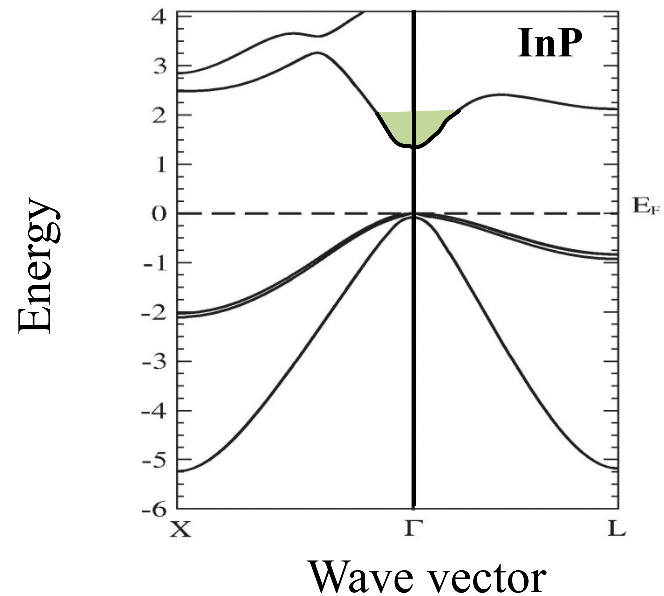
Lattice structure of GaAs, InP



Energy bands

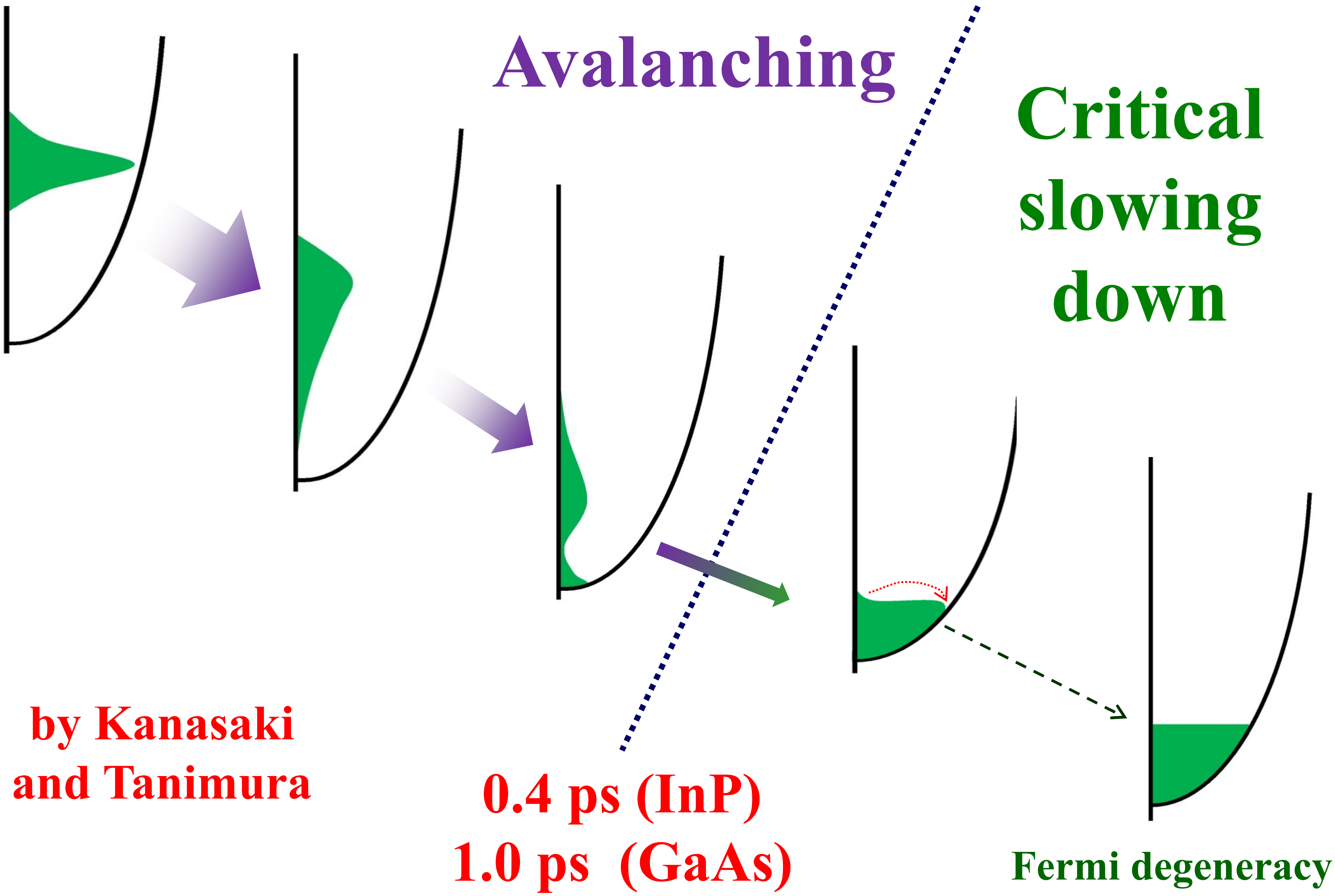


N.Cavassilas et al, PRB 64(2001)115207.



L.Dacal et al, SSC 151(2011)781.

Experimental determination of two time regions



by Kanasaki and Tanimura

Many-electron and acoustic phonon coupled system($\equiv$ H)

$$\mathbf{H = H_0 + H_i, \qquad H_0 \equiv H_e + H_p, \qquad H_i \equiv H_{ep} + H_{ee},}$$

$$\mathbf{H_e \equiv \sum_{k,\sigma(=\alpha,\beta)} (e(k)-\mu)\, a^{+}_{k,\sigma} a_{k,\sigma}, \quad 0 \le e(k) \le B, \quad B = 5\ eV,}$$

$$\mathbf{n_M \equiv (2N)^{-1} \sum_{k,\sigma(=\alpha,\beta)} a^{+}_{k,\sigma} a_{k,\sigma}, \quad n_M = 0.001 \sim 0.005, \quad \mu \equiv}$$ **Chemical potential**

Electron-phonon coupling and Hubbard type weak Coulomb repulsion

$$\mathbf{H_p \equiv \sum_q \omega_q b^{+}_q b_q\,,\ \omega_q \equiv c_s |q|, \ 0 \le \omega_q \le \omega_M (\equiv 24meV),\ c_s = 30 Å/(pico.sec.),}$$

$$\mathbf{H_{ep} \equiv S(2N)^{-1/2} \sum_{q,k,\ \sigma(=\alpha,\beta)} (b^{+}_q + b_{-q}) a^{+}_{k-q,\sigma} a_{k,\sigma}\,, \qquad S \cong 1\ eV}$$

$$\mathbf{H_{ee} \equiv U \sum_{\ell} (n_{\ell,\alpha} - n_M)(n_{\ell,\beta} - n_M)\,, \quad U \cong 1eV,}$$

$$n_{\ell,\sigma} \equiv a^{+}_{\ell,\sigma} a_{\ell,\sigma}, \quad a_{\ell,\sigma} \equiv (2N)^{-1/2} \sum_k e^{ik\cdot\ell} a_{k,\sigma},$$

The whole electronic system is always in a plan wave state only around the bottom of the conduction band minimum, with only a low carrier density, well described by one-boy H_e, and effects of H_i is weak.

Density matrix at a time t $\quad \rho(t)$

$$\rho(t) \to \rho_e(t)\rho_p, \qquad \rho_p \equiv e^{-H_p/k_B T_p}, \quad T_p = 0K$$

Phonon system is always heat reservoir.

$$< n_{\ell,\sigma}(t) > \equiv Tr(n_{\ell,\sigma}\rho(t))/\, Tr(\rho(t)), \quad < \cdots > \equiv Tr(\cdots \rho(t))/\, Tr(\rho(t)),$$

$$< n_{\ell,\sigma}(t) > \to n_M, \qquad \text{independent of time } t$$

The first order effect is always absent

$$< H_i > = < H_{ee} > = < H_{ep} > = 0$$

I . Statistical relaxation theory, Electron temperature$(\equiv T_e(t))$ is always well defined.

At each time t, electron temperature $T_e(t)$ (, $0 \leq T_e(t) \lesssim 200K$) is always well established in electronic system, prescribed by one-body H_e, due to intra-system multiple scattering by H_{ee}, but gradually decreases, as it releases its energy to the phonon system through H_{ep}. We can forget about H_{ee}, except $\mathbf{T_e(t)}$

$$\mathbf{H_i \rightarrow H_{ep}},$$

Density matrix is $$\boldsymbol{\rho(t) \rightarrow \rho_e(t)\rho_p, \ \rho_e(t) \equiv e^{-\frac{H_e}{k_B T_e(t)}}}$$

Total energy decrease of electrons, due to temperature decrease from T_e to $(T_e - \Delta T_e)$, → Electronic heat capacity (, $\equiv C(T_e)$)

$$C(T_e) = \frac{\partial < H_e >}{\partial T_e}, \quad < H_e >= \sum_{k,\sigma(=\alpha,\beta)} (e_k - \mu) < n_{k,\sigma} >, \ n_{k,\sigma} \equiv a^{+}_{k,\sigma} a_{k,\sigma}$$

Fermi distribution: $$< n_{k,\sigma} >= \frac{e^{-(e(k)-\mu)/k_B T_e}}{1+e^{-(e(k)-\mu)/k_B T_e}},$$

where, $\boldsymbol{\mu}(\mathbf{T_e})$ should be determined at given T_e from the self-consistent condition

$$\mathbf{n_M = (2N)^{-1} \sum_{k,\sigma(=\alpha,\beta)} < n_{k,\sigma} >}$$

Thus we get

$$\Delta < H_e(T_e) > = C(T_e)\Delta T_e, \quad C(T_e) \propto T_e,$$

which is well known to be linear at low temperature ? Luttinger, PR **119**(1960)1153.

Total energy increase of phonon system through second order of H_{ep},

within a time interval Δt from t.

Time evolution of $\rho(t+\Delta t)$ from $\rho(t)$

$$\boldsymbol{\rho(t+\Delta t) = e^{-i\Delta tH}\rho_e(t)\rho_p e^{i\Delta tH}}$$

$$< H_p(t+\Delta t) > \equiv \mathrm{Tr}\left(H_p e^{-i\Delta tH}\rho_e(t)\rho_p e^{i\Delta tH}\right)/\mathrm{Tr}\left(\rho_e(t)\rho_p\right),$$

$$\mathbf{e^{-i\Delta tH} = e^{-i\Delta t(H_0+H_i)} = e^{-i\Delta tH_0}\, exp_+\left\{-i\int_0^{\Delta t} d\tau\, \widetilde{H}_i(\tau)\right\}},$$

$\exp_+$ positive chronologically ordered exponential, from left to right

$$e^{i\Delta t H_0} e^{-i\Delta t H} = \exp_+\left\{-i\int_0^{\Delta t} d\tau\, \widetilde{H}_i(\tau)\right\}$$

Here, the interaction representation $\widetilde{O}$ of an operator O is

$$\widetilde{\mathbf{O}}(\boldsymbol{\Delta}\mathbf{t}) \equiv \mathbf{e}^{\,\mathbf{i}\boldsymbol{\Delta}\mathbf{t}\mathbf{H}_0}\mathbf{O}\mathbf{e}^{-\mathbf{i}\boldsymbol{\Delta}\mathbf{t}\mathbf{H}_0}$$

Its straight forward expansion is

$$\mathbf{exp}_+\left\{-\mathbf{i}\int_0^{\Delta t} \mathbf{d}\boldsymbol{\tau}\, \widetilde{\mathbf{H}}_i(\boldsymbol{\tau})\right\} = \mathbf{1} + (-\mathbf{i})\int_0^{\Delta t} \mathbf{d}\boldsymbol{\tau}_1\, \widetilde{\mathbf{H}}_i(\boldsymbol{\tau}_1) + (-\mathbf{i})^2 \int_0^{\Delta t} \mathbf{d}\boldsymbol{\tau}_1 \int_0^{\tau_1} \mathbf{d}\,\boldsymbol{\tau}_2 \widetilde{\mathbf{H}}_i(\boldsymbol{\tau}_1)\widetilde{\mathbf{H}}_i(\boldsymbol{\tau}_2)$$

$$+(-\mathbf{i})^3 \int_0^{\Delta t} \mathbf{d}\boldsymbol{\tau}_1 \int_0^{\tau_1} \mathbf{d}\boldsymbol{\tau}_2 \int_0^{\tau_2} \mathbf{d}\,\boldsymbol{\tau}_3 \widetilde{\mathbf{H}}_i(\boldsymbol{\tau}_1)\widetilde{\mathbf{H}}_i(\boldsymbol{\tau}_2)\widetilde{\mathbf{H}}_i(\boldsymbol{\tau}_3) + \ \ldots\ldots\ldots \quad ,$$

Its complex conjugate

$$\mathbf{e^{i\Delta t H} = e^{i\Delta t(H_0+H_i)} = exp_-\left\{i\int_0^{\Delta t} d\tau'\, \widetilde{H}_i(\tau')\right\} e^{i\Delta t H_0}},$$

exp₋ negative chronologically ordered exponential from right to left.

$$e^{i\Delta t H} e^{-i\Delta t H_0} = \exp_-\left\{i\int_0^{\Delta t} d\tau'\, \widetilde{H}_i(\tau')\right\}$$

Its straight forward expansion is

$$\mathbf{exp_-\left\{i\int_0^{\Delta t} d\tau'\, \widetilde{H}_i(\tau')\right\} = 1 + (i)\int_0^{\Delta t} d\tau'_1\, \widetilde{H}_i(\tau'_1) + (i)^2\int_0^{\Delta t} d\tau'_1 \int_0^{\tau'_1} d\,\tau'_2 \widetilde{H}_i(\tau'_2)\widetilde{H}_i(\tau'_1)}$$

$$\mathbf{+(i)^3\int_0^{\Delta t} d\tau'_1 \int_0^{\tau'_1} d\tau'_2 \int_0^{\tau'_2} d\,\tau'_3 \widetilde{H}_i(\tau'_3)\widetilde{H}_i(\tau'_2)\widetilde{H}_i(\tau'_1) + \dots\dots\dots},$$

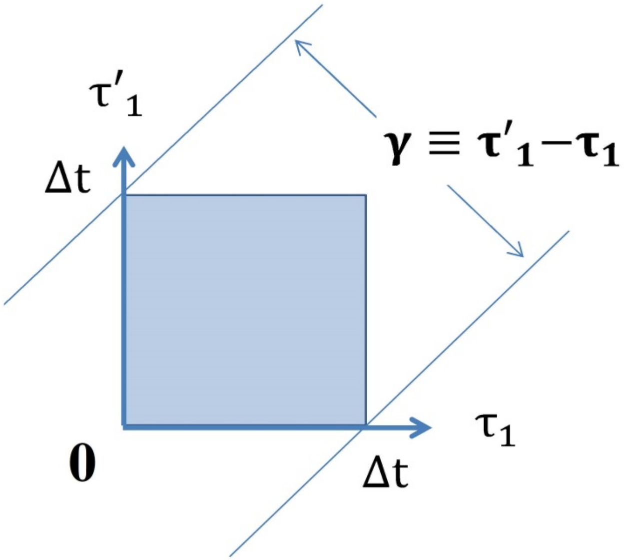
Thus

$$\mathbf{\rho(t+\Delta t) = e^{-i\Delta t H_0}\, exp_+\left\{-i\int_0^{\Delta t} d\tau\, \widetilde{H}_i(\tau)\right\} \rho_e(t)\rho_p exp_-\left\{i\int_0^{\Delta t} d\tau'\, \widetilde{H}_i(\tau')\right\} e^{i\Delta t H_0}},$$

Non zero second order term at phonon vacuum, ρ_p at T_p (= 0 K)

$$< H_p(t+\Delta t) > = \frac{\mathrm{Tr}\left(H_p \int_0^{\Delta t} d\tau_1\, \widetilde{H}_{ep}(\tau_1)\rho_e(t)\rho_p \int_0^{\Delta t} d\tau'_1\, \widetilde{H}_{ep}(\tau'_1)\right)}{\mathrm{Tr}\left(\rho_e(t)\rho_p\right)}$$

$$= \int_0^{\Delta t} d\tau'_1 \int_0^{\Delta t} d\tau_1 \frac{\mathrm{Tr}\left(\widetilde{H}_{ep}(\tau'_1) H_p \widetilde{H}_{ep}(\tau_1)\rho_e(t)\rho_p\right)}{\mathrm{Tr}\left(\rho_e(t)\rho_p\right)} = \Delta t \int_{-\Delta t}^{\Delta t} d\gamma \frac{\mathrm{Tr}\left(\widetilde{H}_{ep}(\gamma) H_p \widetilde{H}_{ep}(0)\rho_e(t)\rho_p\right)}{\mathrm{Tr}\left(\rho_e(t)\rho_p\right)}$$



= .... ,

$\Delta t \to \infty$, long time limit

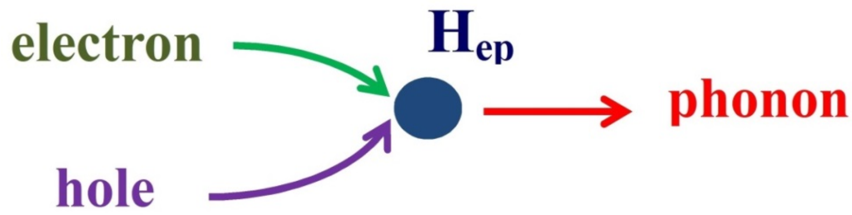
So called golden rule

$$\int_{-\Delta t}^{\Delta t} d\gamma\, e^{-i\bar{e}(k-q)\gamma t}(1-< n_{k-q,\sigma} >)e^{i\bar{e}(k)\gamma t} < n_{k,\sigma} > \omega_q e^{-i\omega_q \gamma}$$

$$= 2\pi\Delta t\omega_q(1-< n_{k-q,\sigma} >) < n_{k,\sigma} > \delta(\omega_q + \bar{e}(k-q) - \bar{e}(k))$$

$$= \Delta t\Gamma(T_e),$$

e-h recombination by phonon emission



$$\Gamma(T_e) = 2\pi\frac{S^2}{2N}\sum_{q,k,\ \sigma(=\alpha,\beta)} \omega_q(1-< n_{k-q,\sigma} >) < n_{k,\sigma} > \delta\left(\omega_q + e(k-q) - e(k)\right)$$

Energy conservation

$$C(T_e)|\Delta T_e| = \Gamma(T_e)\Delta t, \quad \frac{\partial T_e}{\partial t} = -\frac{\Gamma(T_e)}{C(T_e)},$$

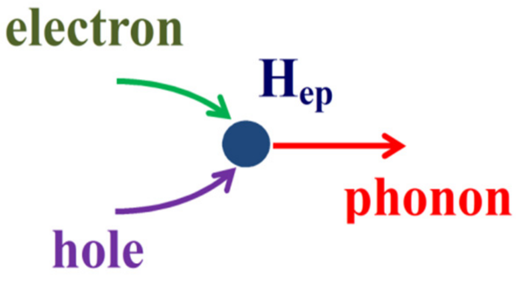
Thus, we finally get the equation for electronic system cooling.

At low temperatures $\Gamma(T_e) \propto T_e^4$

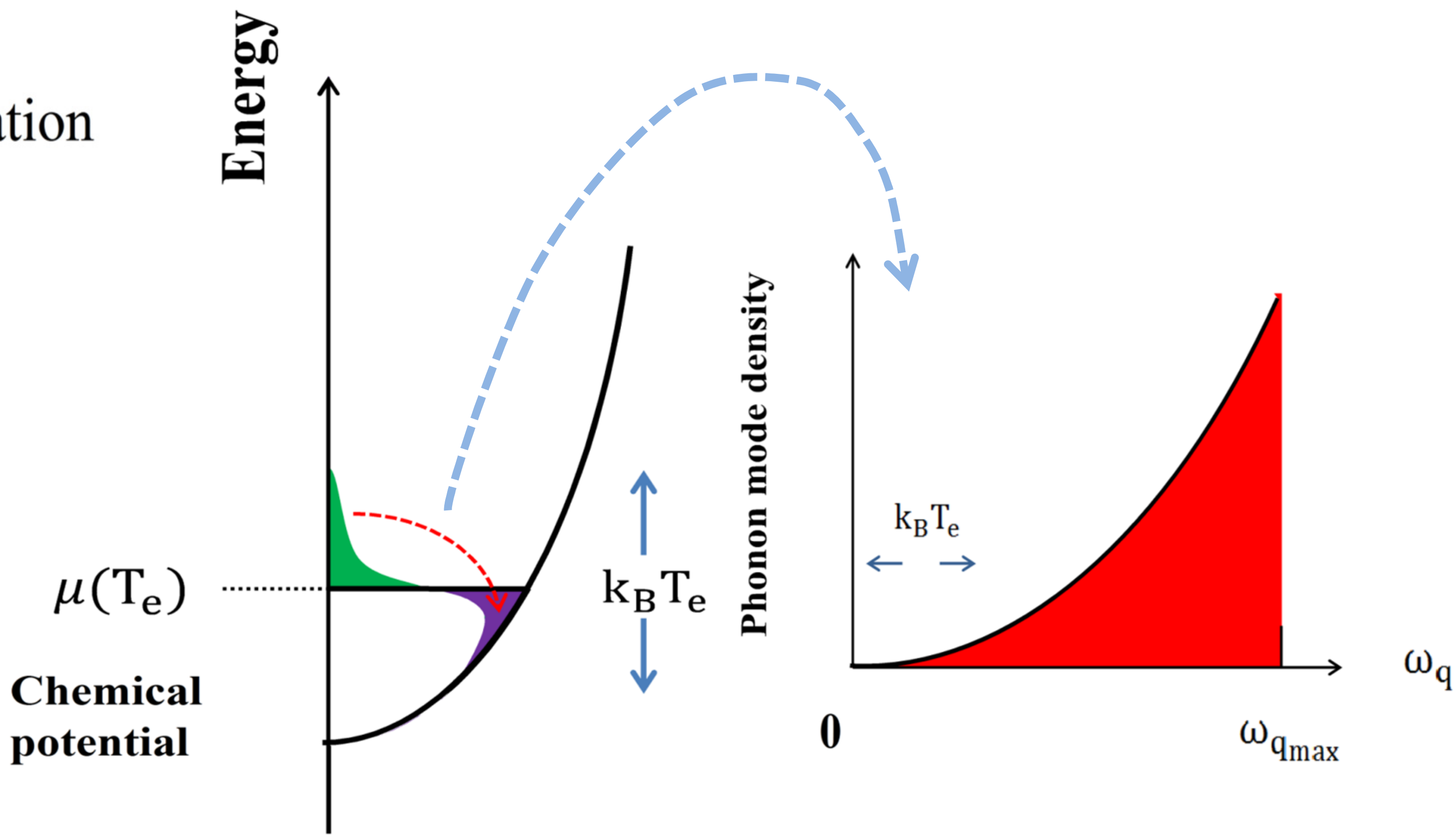
Electron-hole pair number around e_F T_e

acoustic phonon mode density T_e^2

acoustic phonon energy T_e



e-h recombination

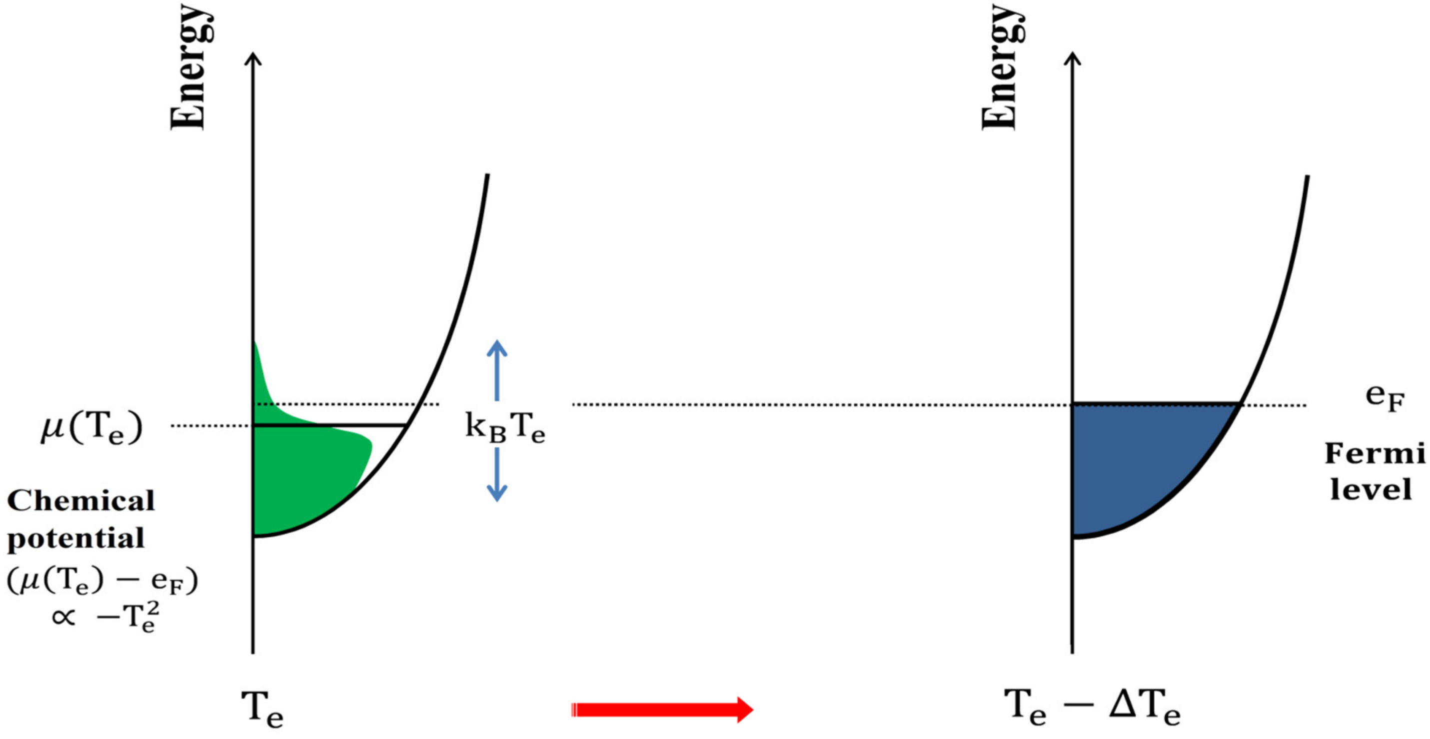


While, total energy decrease of electrons, due to temperature decrease from T_e to $(T_e - \Delta T_e)$, at low temperatures

$$[< H_e(T_e) > - < H_e(T_e - \Delta T_e) >] \propto T_e^2 \rightarrow T_e|\Delta T_e|$$

$$|\text{energy} - e_F| \times (\text{number of electron, or hole around } e_F)$$

$k_B T_e$ $\qquad\qquad$ $k_B T_e / e_F$

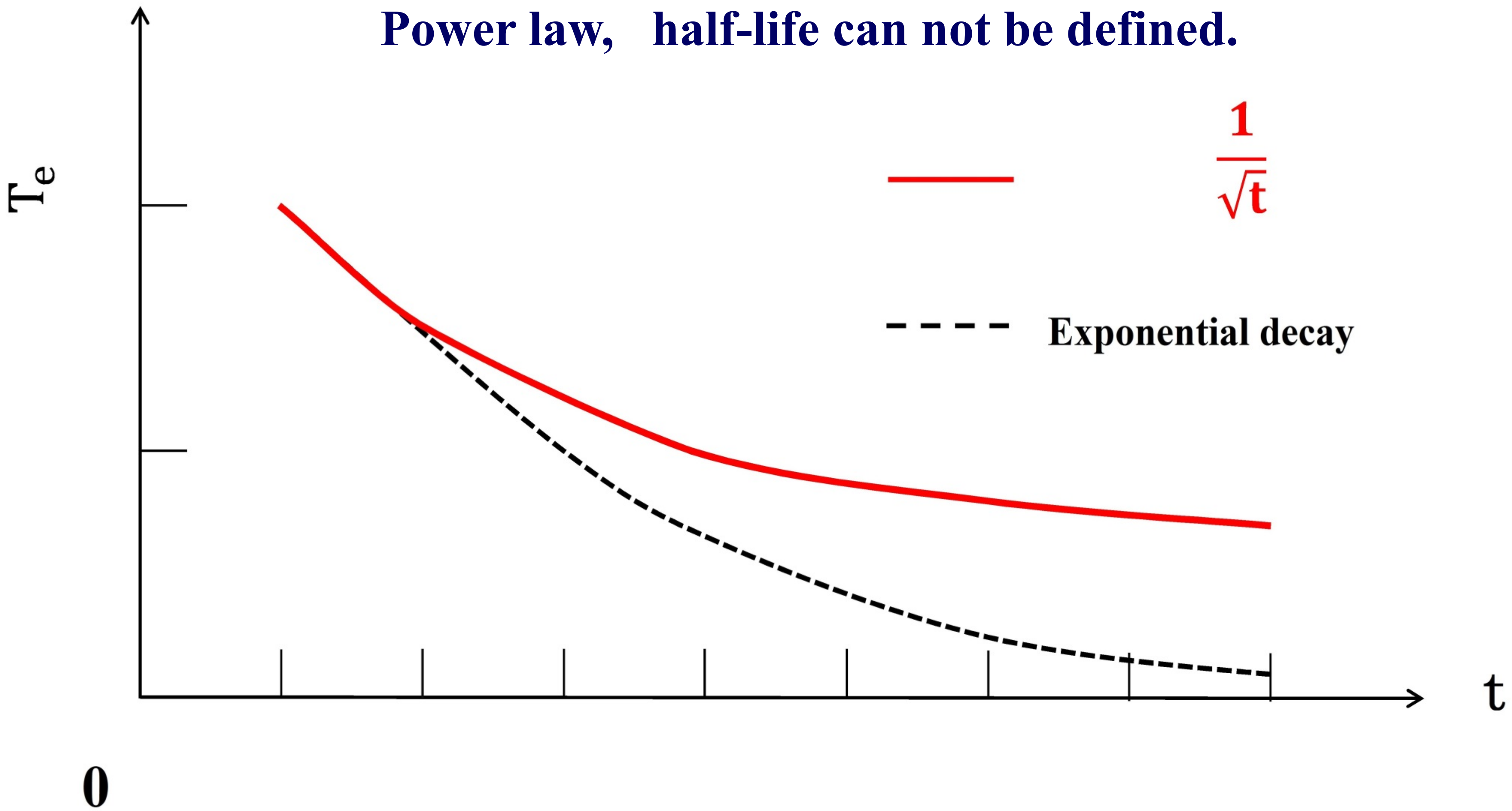


Finally,

$$T_e|\Delta T_e| \propto \Delta t\, T_e^4, \quad \frac{\partial T_e}{\partial t} \propto -T_e^3, \quad T_e \propto t^{-\frac{1}{2}}$$

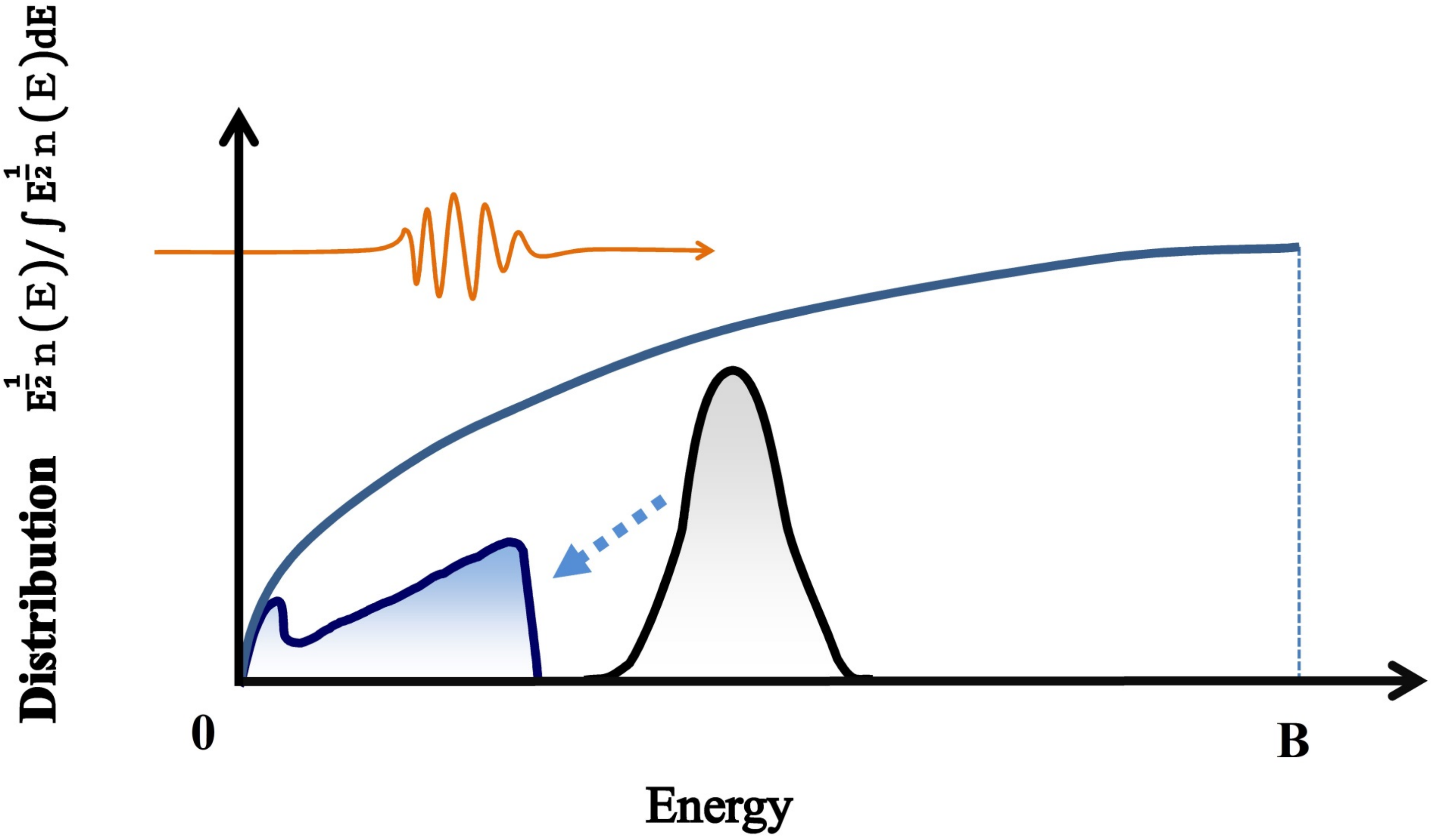
Slowing down of relaxation speed than exponential decay.

Power law, half-life can not be defined.



Iterative theory for real time relaxation dynamic

without electron temperature approximation



Ⅱ.Iterative theory for relaxation dynamics without electron temperature approximation

We now recover the full interactions for multiple scattering.

$$\mathbf{H_i = (H_{ep} + H_{ee}),}$$

$$\boldsymbol{\rho}(\mathbf{t}) \rightarrow \boldsymbol{\rho}_{\mathbf{e}}(\mathbf{t})\boldsymbol{\rho}_{\mathbf{p}}, \quad \boldsymbol{\rho}_{\mathbf{p}} \equiv \mathbf{e}^{-\frac{\mathbf{H_p}}{\mathbf{k_B T_p}}}, \ \mathbf{T_p = 0K}$$

$\boldsymbol{\rho}_{\mathbf{e}}(\mathbf{t})$ is now non-equilibrium state starting from the photo-excitation.

$$\tilde{\rho}(t+\Delta t) = \exp_{+}\left\{-i\int_0^{\Delta t} d\tau\, \tilde{H}_i(\tau)\right\}\rho_e(t)\rho_p \exp_{-}\left\{i\int_0^{\Delta t} d\tau'\, \tilde{H}_i(\tau')\right\}$$

Second order time evolution Δt from the transient state at t

$$\tilde{\boldsymbol{\rho}}(\mathbf{t}+\Delta\mathbf{t}) = \boldsymbol{\rho}_{\mathbf{e}}(\mathbf{t})\boldsymbol{\rho}_{\mathbf{p}}$$

$$+\int_0^{\Delta t} d\tau_1\, \tilde{H}_{ep}(\tau_1)\rho_e(t)\rho_p \int_0^{\Delta t} d\tau'_1\, \tilde{H}_{ep}(\tau'_1)$$

$$-\int_0^{\Delta t} d\tau_1 \int_0^{\tau_1} d\tau_2\, \tilde{H}_{ep}(\tau_1)\tilde{H}_{ep}(\tau_2)\ \rho_e(t)\rho_p\ -\rho_e(t)\rho_p \int_0^{\Delta t} d\tau'_1 \int_0^{\tau'_1} d\tau'_2\ \tilde{H}_{ep}(\tau'_2)\,\tilde{H}_{ep}(\tau'_1)$$

$$+\int_0^{\Delta t} d\tau_1\, \tilde{H}_{ee}(\tau_1)\, \rho_e(t)\rho_p \int_0^{\Delta t} d\tau'_1 \tilde{H}_{ee}(\tau'_1)$$

$$-\int_0^{\Delta t} d\tau_1 \int_0^{\tau_1} d\tau_2\, \tilde{H}_{ee}(\tau_1)\tilde{H}_{ee}(\tau_2)\ \rho_e(t)\rho_p\ -\rho_e(t)\rho_p \int_0^{\Delta t} d\tau'_1 \int_0^{\tau'_1} d\tau'_2\ \tilde{H}_{ee}(\tau'_2)\,\tilde{H}_{ee}(\tau'_1)$$

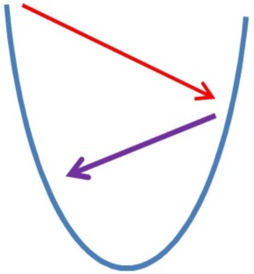
What we want know finally is the time evolution of electron number $\mathbf{n}_{\mathbf{k},\boldsymbol{\sigma}}(\mathbf{t}+\Delta\mathbf{t})$

$$\frac{\partial n_{k,\sigma}(t+\Delta t)}{\partial \Delta t},\quad n_{k,\sigma}(t+\Delta t) \equiv \langle n_{k,\sigma} \rangle$$

Rate equation for $n_{k,\sigma}(t)$

Gain of $n_{k,\sigma}(t)$ proportional to $(1-<n_{k,\sigma}>)$, loss proportional to $<n_{k,\sigma}>$.

$$\frac{\partial n_{k,\sigma}(t)}{\partial t} = (1-<n_{k,\sigma}(t)>)(\Gamma^{+}_{ep,k}(t) + \Gamma^{+}_{ee,k}(t)) - <n_{k,\sigma}(t)> (\Gamma^{-}_{ep,k}(t) + \Gamma^{-}_{ee,k}(t))$$



$|k, \sigma>$

$$\Gamma^{+}_{ep,k} \equiv \pi S^2 N^{-1} \sum_q <n_{k+q,\sigma}> \delta(e(k) + \omega_q - e(k+q))$$

$$\Gamma^{-}_{ep,k} \equiv \pi S^2 N^{-1} \sum_q (1-<n_{k+q,\sigma}>) \delta(e(k+q) + \omega_q - e(k))$$

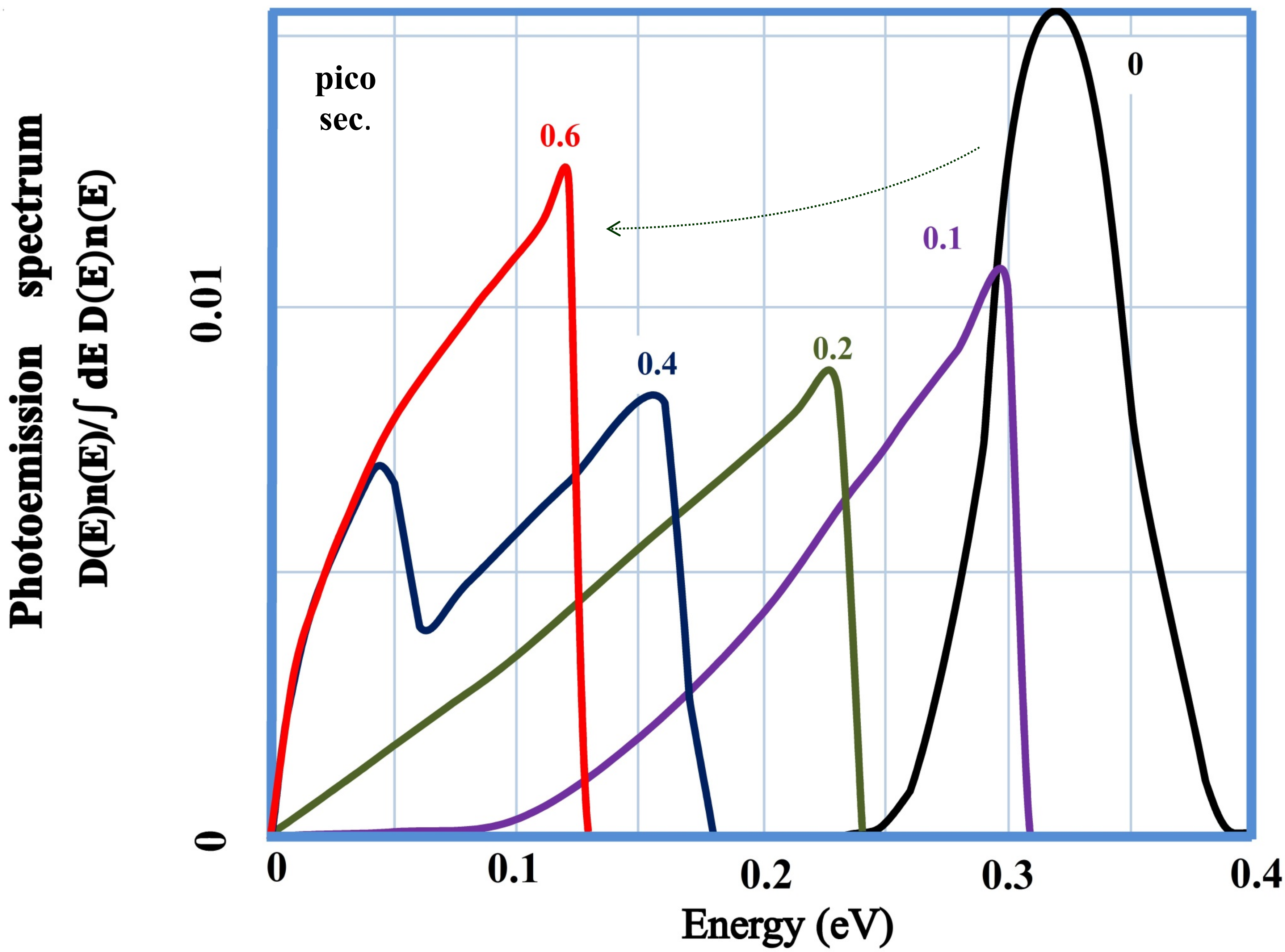
$$\Gamma^{+}_{ee,k} \equiv 2\pi U^2 N^{-2} \sum_{k'} (1-<n_{k',-\sigma}>) \sum_{k''} <n_{k-k'',\sigma}><n_{k'+k'',-\sigma}> \delta(e(k) + e(k') - e(k-k'') - e(k'+k''))$$

$$\Gamma^{-}_{ee,k} \equiv 2\pi U^2 N^{-2} \sum_{k'} <n_{k',-\sigma}> \sum_{k''} (1-<n_{k+k'',\sigma}>)(1-<n_{k'-k'',-\sigma}>) \delta(e(k+k'') + e(k'-k'') - e(k) - e(k'))$$

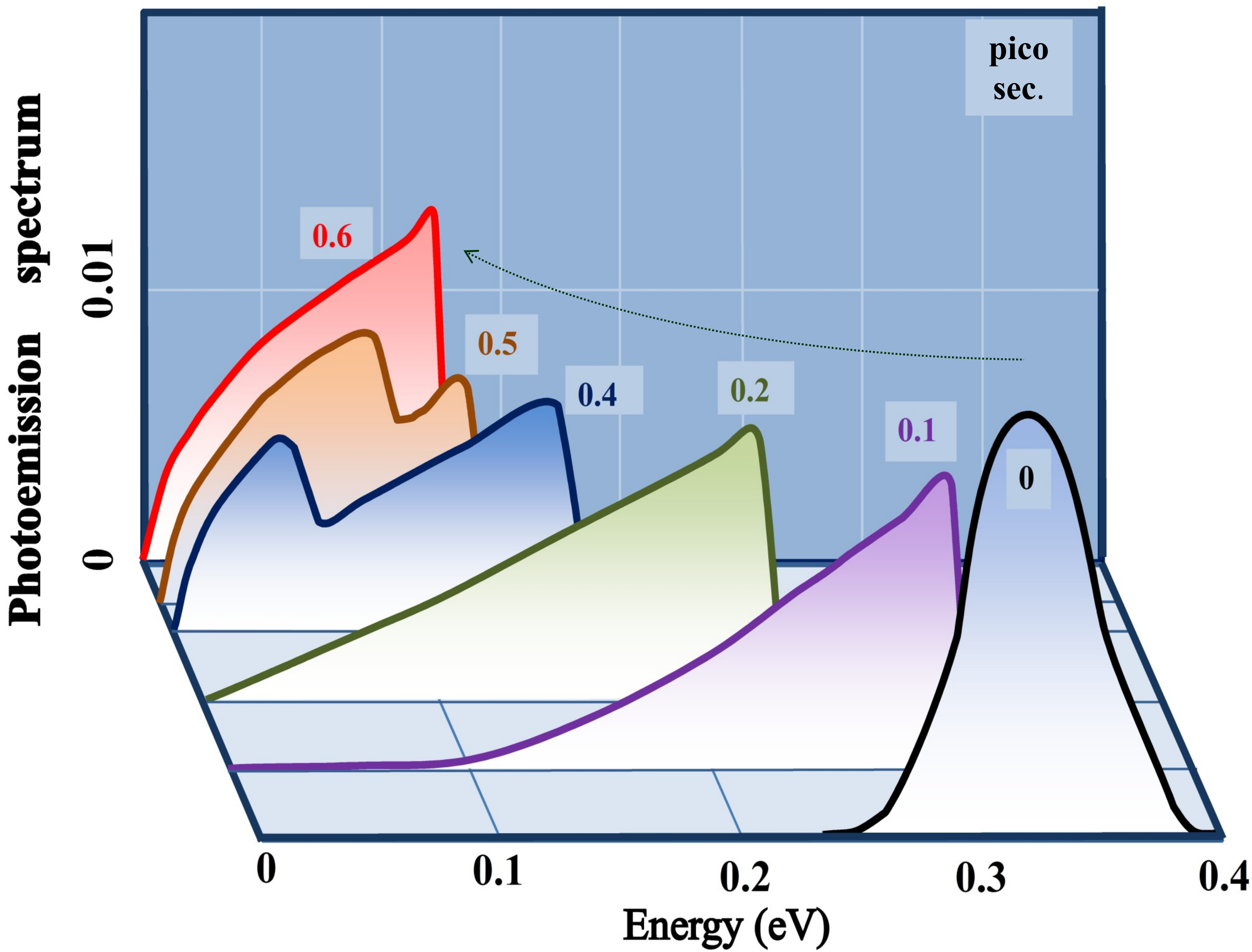
Iterative theory for dynamics

$$n_{k,\sigma}(t+\Delta t) = n_{k,\sigma}(t) + \Delta t\{(1-<n_{k,\sigma}(t)>)(\Gamma^{+}_{ep,k}(t) + \Gamma^{+}_{ee,k}(t)) - <n_{k,\sigma}(t)> (\Gamma^{-}_{ep,k}(t) + \Gamma^{-}_{ee,k}(t))\}$$

$$n_{k,\sigma}(t+2\Delta t) = n_{k,\sigma}(t+\Delta t) + \Delta t\{(1-<n_{k,\sigma}(t+\Delta t)>)(\Gamma^{+}_{ep,k}(t+\Delta t) + \Gamma^{+}_{ee,k}(t+\Delta t)) - <n_{k,\sigma}(t+\Delta t)> (\Gamma^{-}_{ep,k}(t+\Delta t) + \Gamma^{-}_{ee,k}(t+\Delta t))\}$$

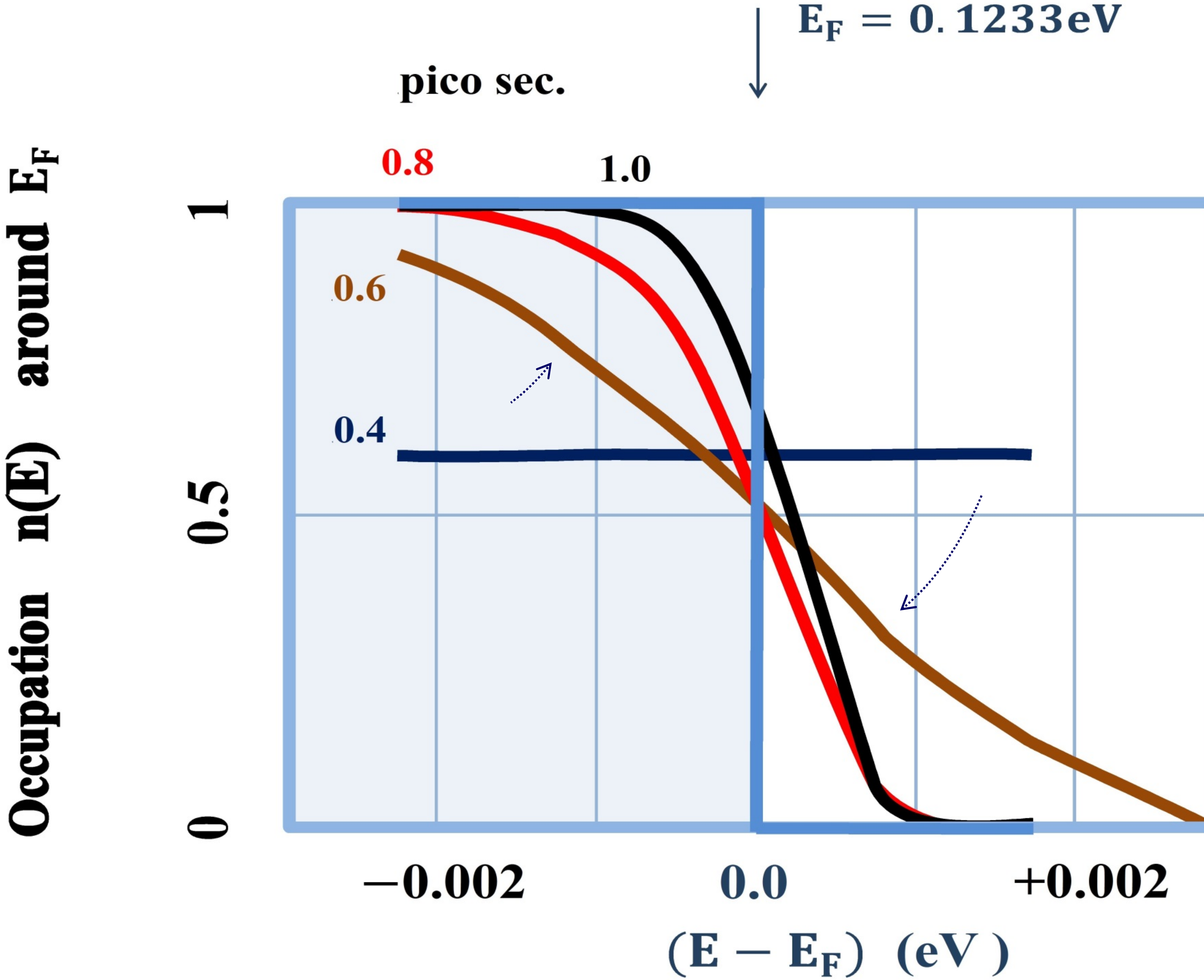


Photoemission spectrum
D(E)n(E)/∫ dE D(E)n(E)
0.01
0
pico
sec.
0.6
0.4
0.2
0.1
0
0
0.1
0.2
0.3
0.4
Energy (eV)



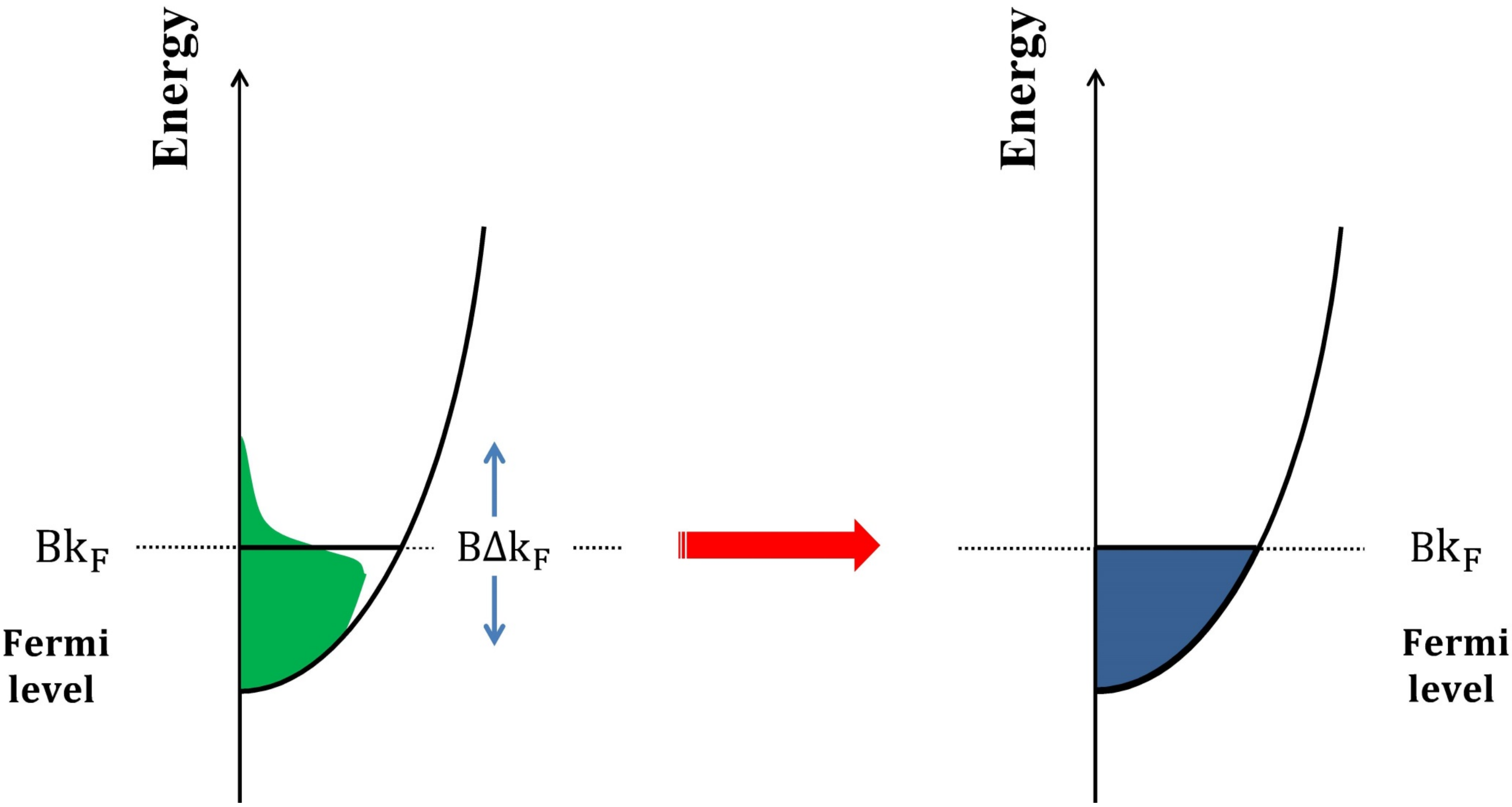
pico
sec.
0.6
0.5
0.4
0.2
0.1
0
Photoemission spectrum
0.01
0
0
0.1
0.2
0.3
0.4
Energy (eV)

Occupation around E_F



Residual photo-excitation energy($\equiv \Delta E_r$) at final stage

$$\Delta E_r \propto (\Delta k_F)^2$$



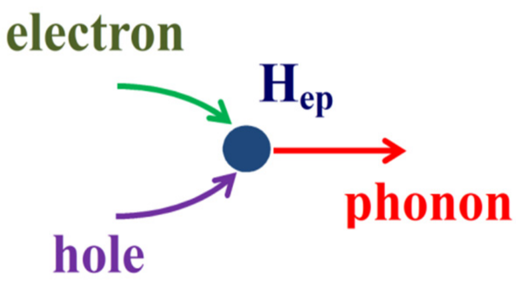
Final stage of phonon relaxation

Transition rate ($\equiv \Gamma$), $\Gamma \propto (\Delta k_F)^4$

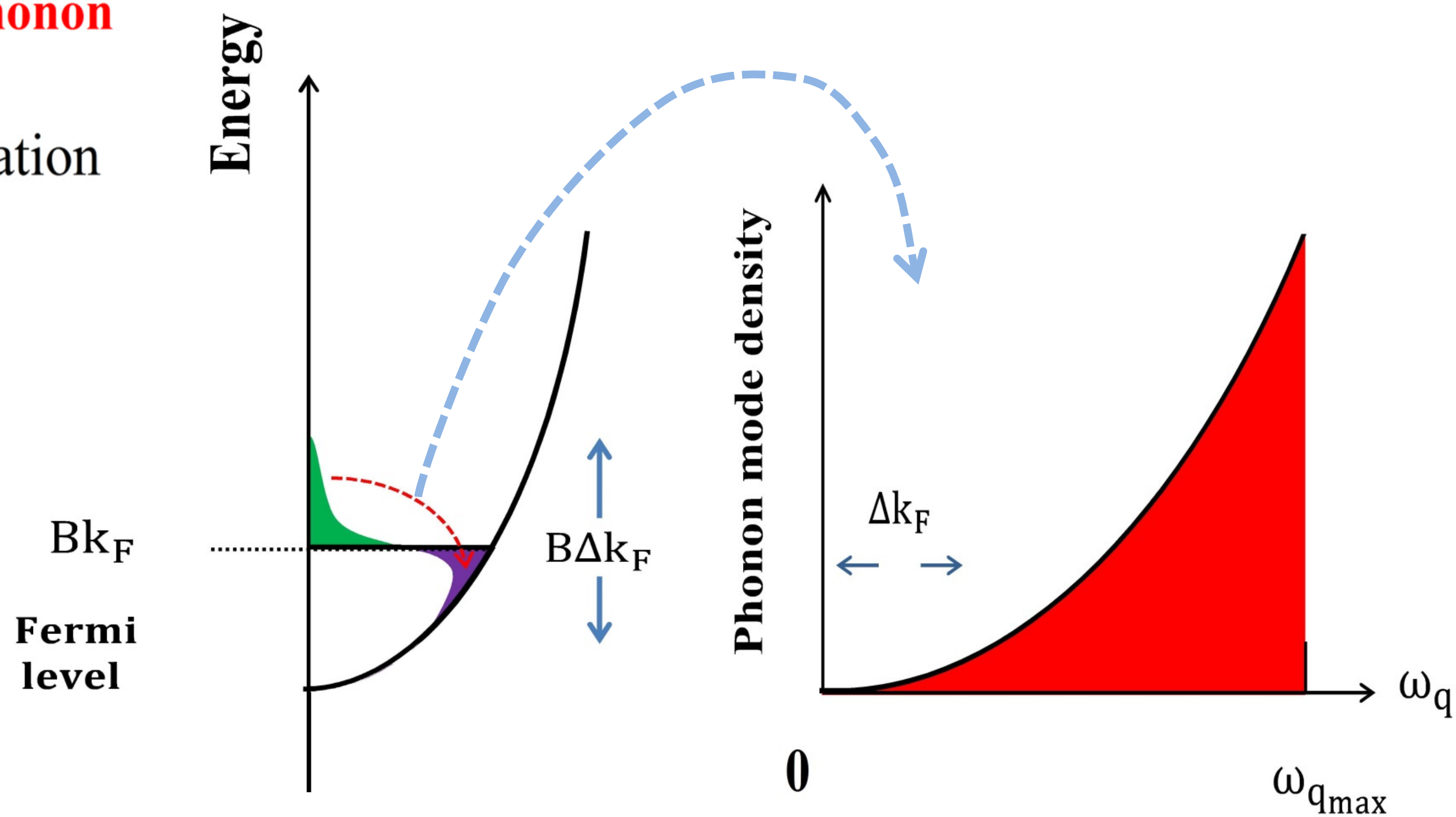
Electron – hole number $\propto \Delta k_F$

Phonon mode density $\propto (\Delta k_F)^2$

Phonon energy $\propto \Delta k_F$



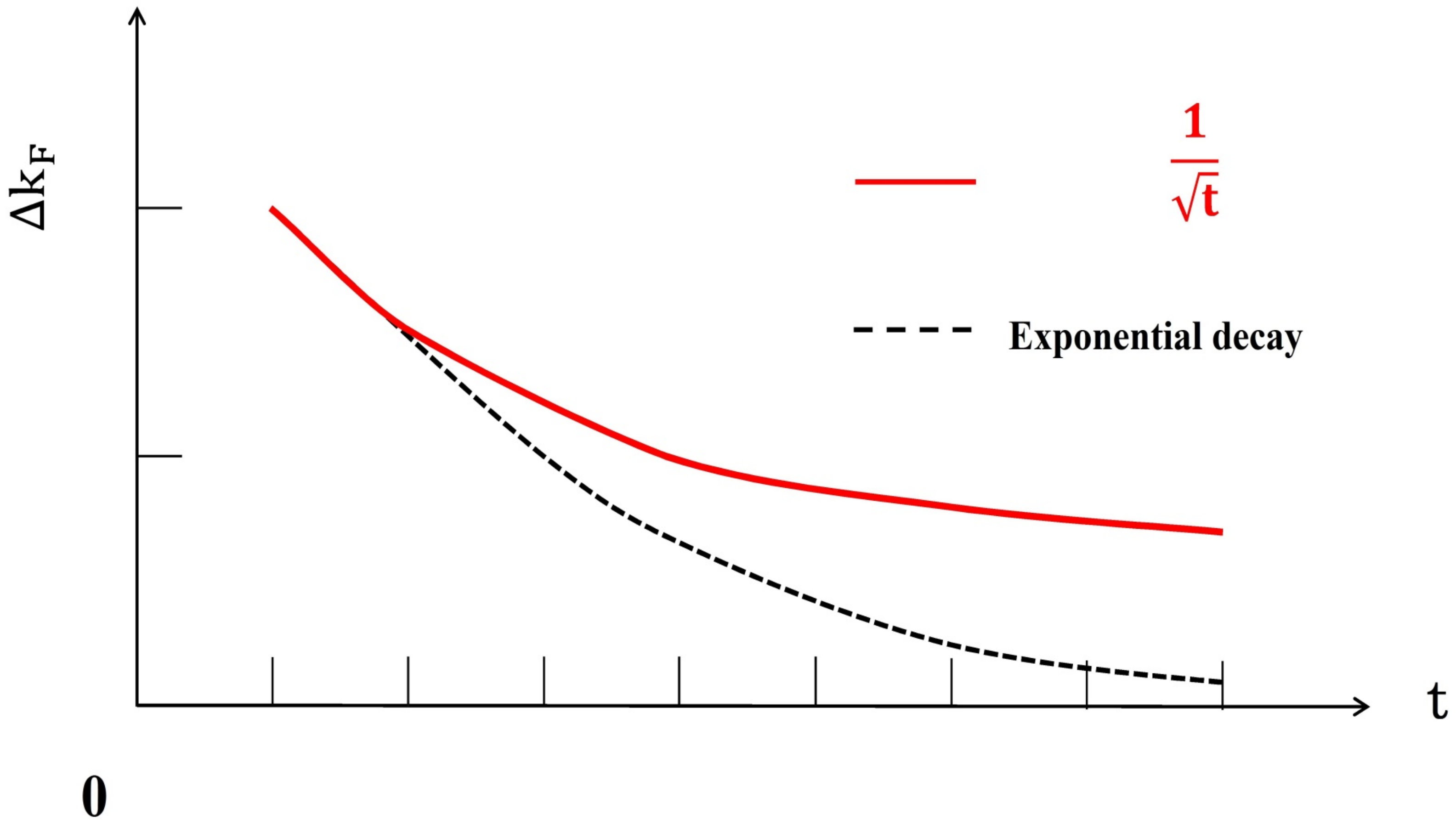
e-h recombination



Finally

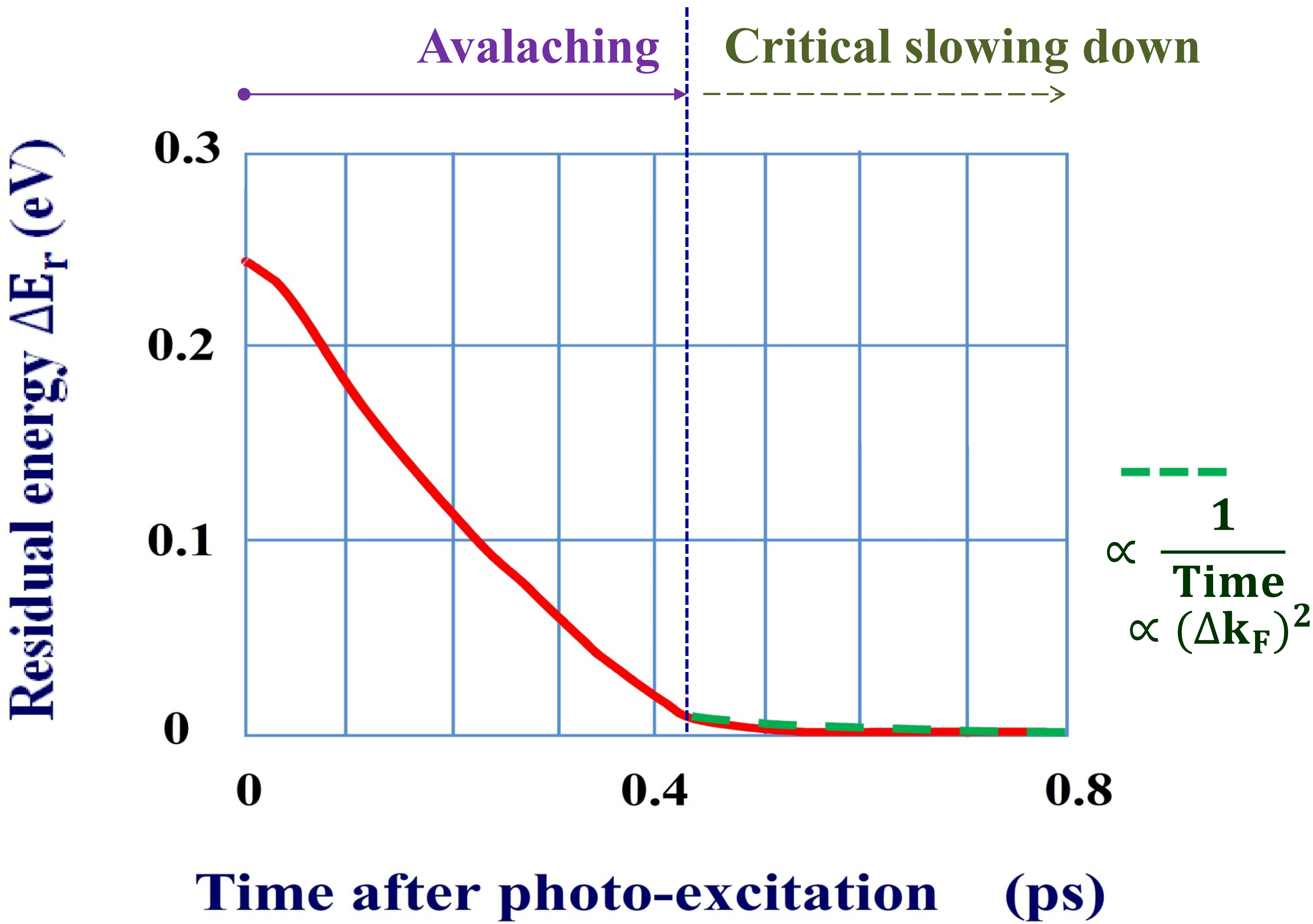
$$(\Delta k_F)^2 \propto \Delta t (\Delta k_F)^4, \quad \frac{\partial(\Delta k_F)}{\partial t} \propto (\Delta k_F)^3, \quad \Delta k_F \propto t^{-\frac{1}{2}}$$

Slowing down of relaxation speed than exponential decay.

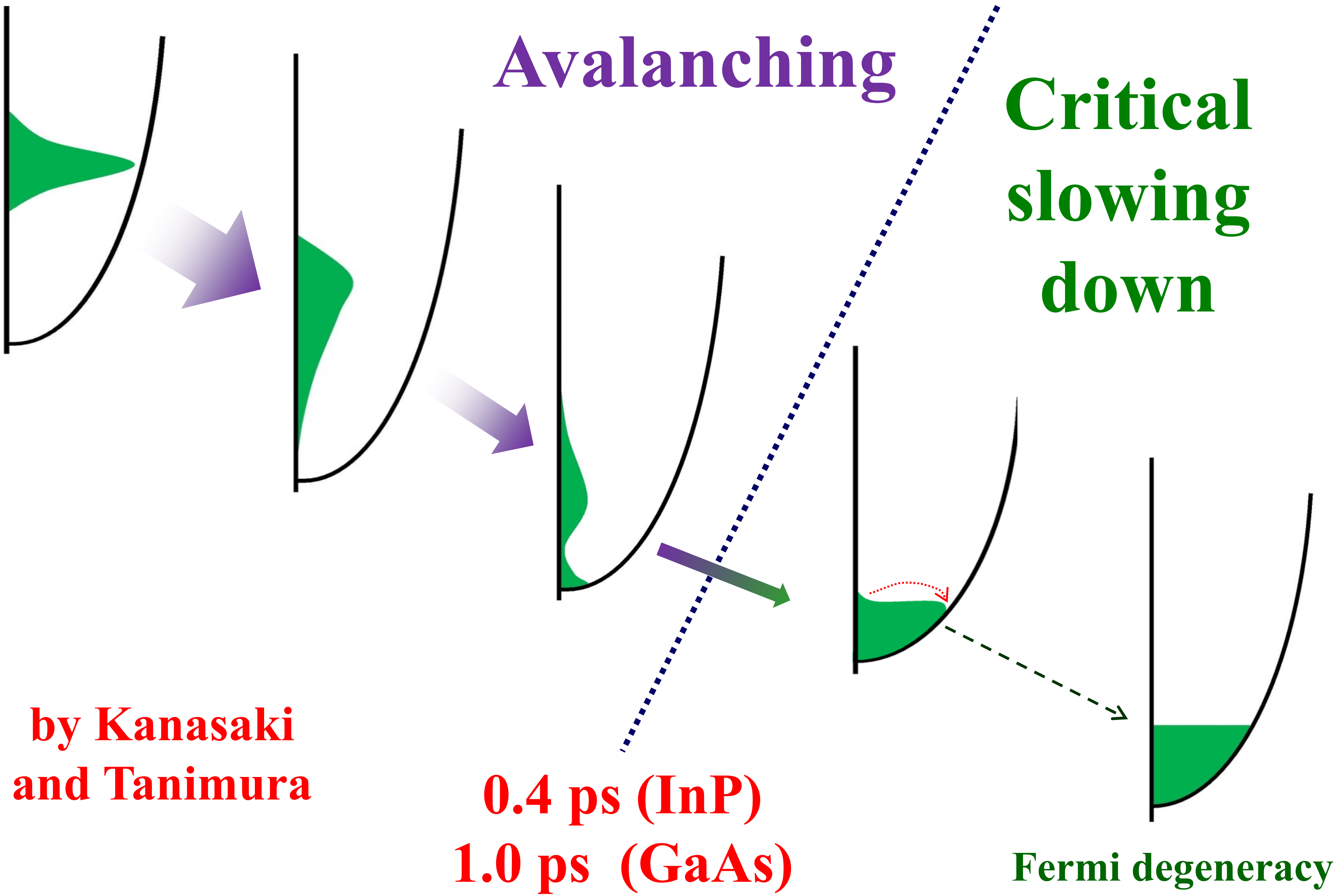


Relaxation dynamics of residual excitation energy ΔE_r, Theory

Two time regions

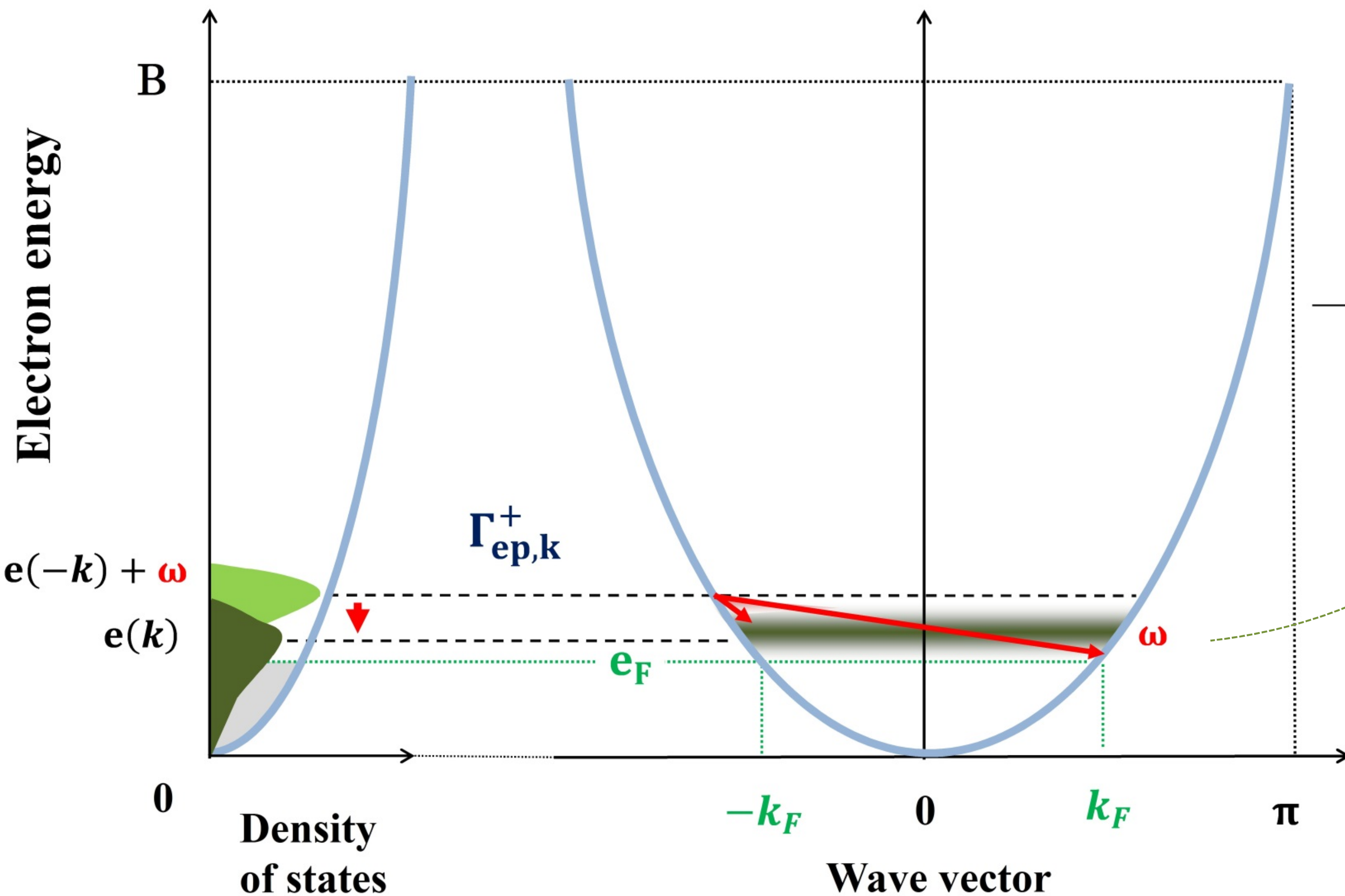


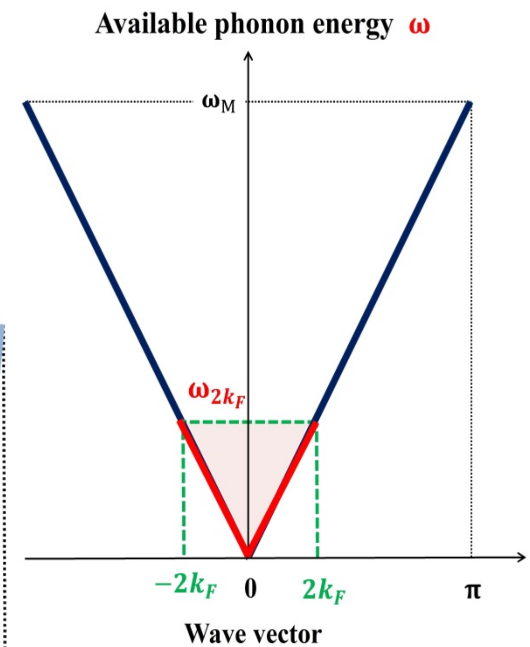
Experimental determination of two time regions



Avalanching speed rapidly increases as electron density (, k_F) increases.

$$\Gamma^{+}_{\mathrm{ep,k}} \propto k_F\,\omega$$



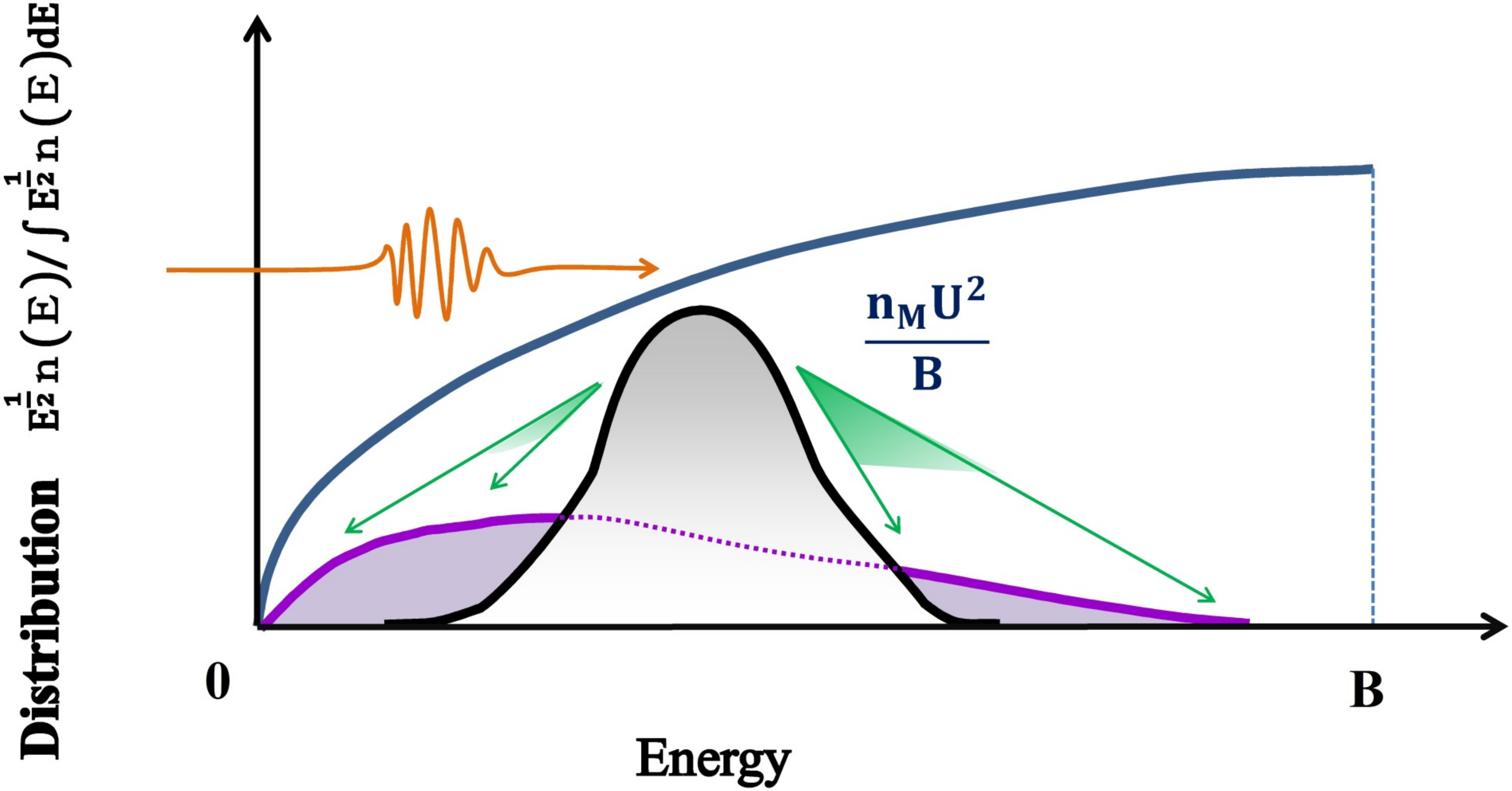


Available ω also increases as k_F increases.

How much time necessary to photo-generate Fermi surface from true electron vacuum?

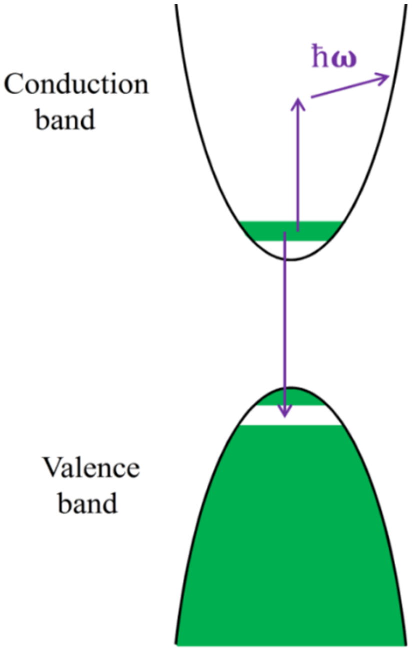
Never terminates.

Initial stage Coulombic elastic scattering



$n_M \lesssim 10^{-3}$, rare than e-ph

Decay channels

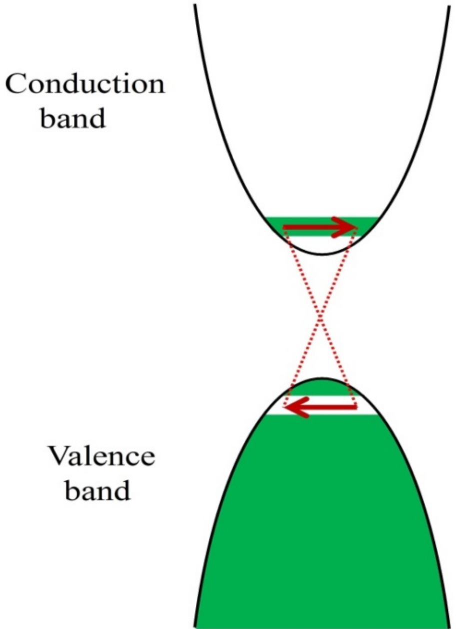


1. Radiative recombination of e-h pair,
   10^{-9} sec

2. Momentum, charge and spin fluctuations

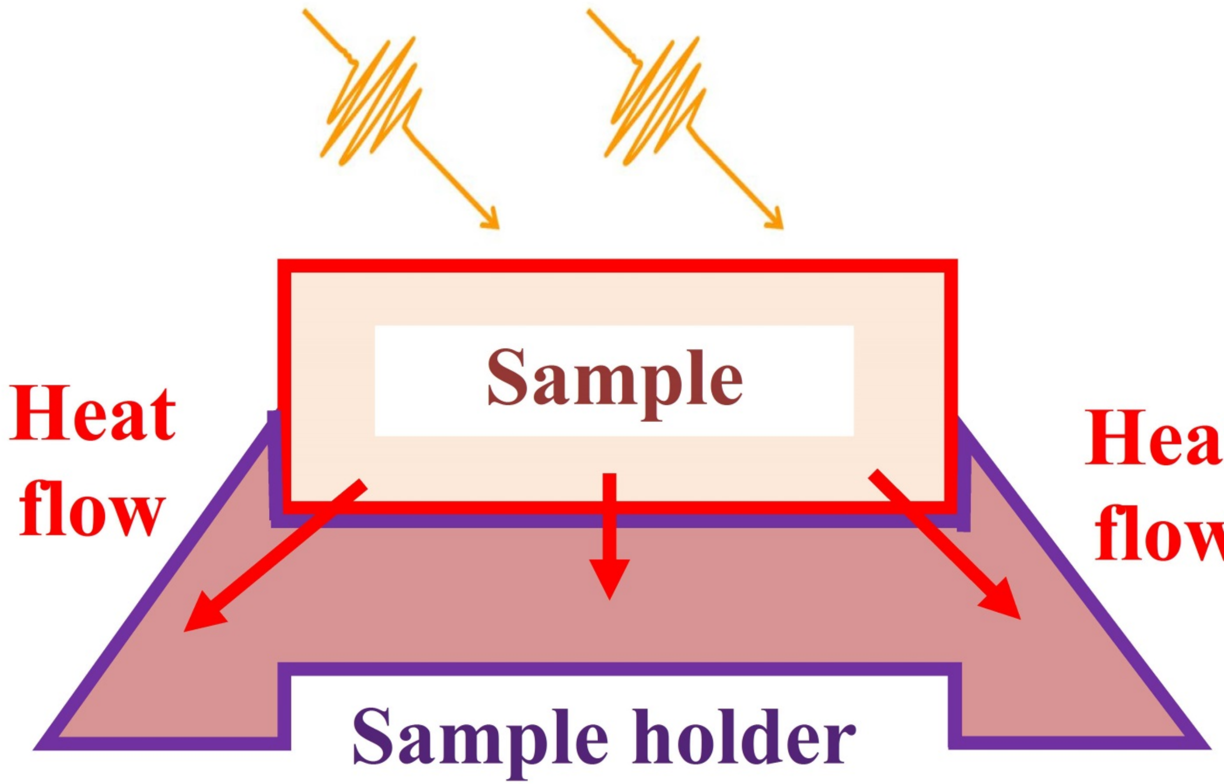
   give no energy dissipation

3. Auger recombination of e-h pair with no energy dissipation,
   10^{-12} sec



4. Inter-band Coulomb scattering, similarly to the intra-band one, gives no dissipation.

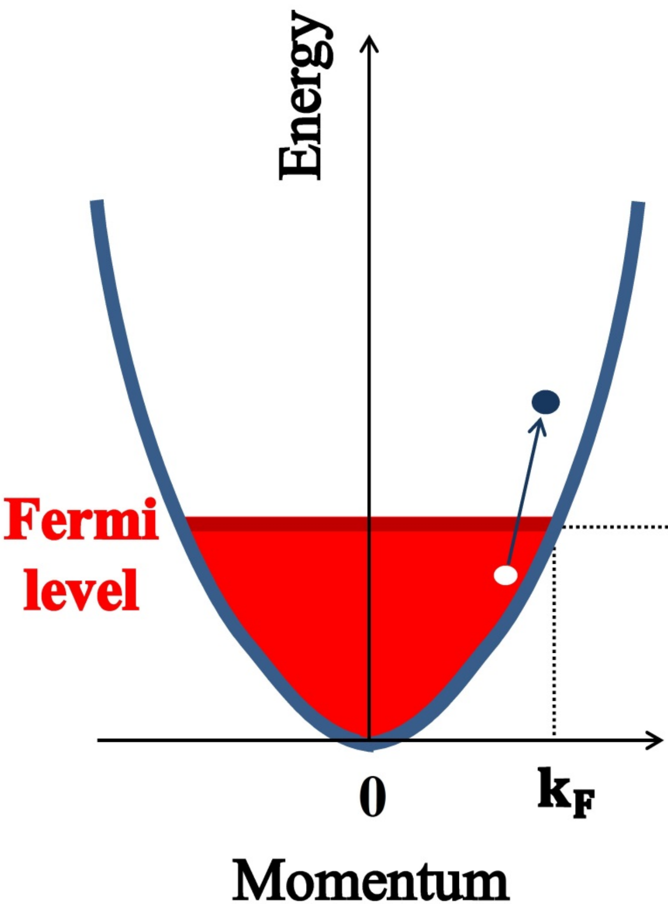
Importance of thermodynamic boundary condition

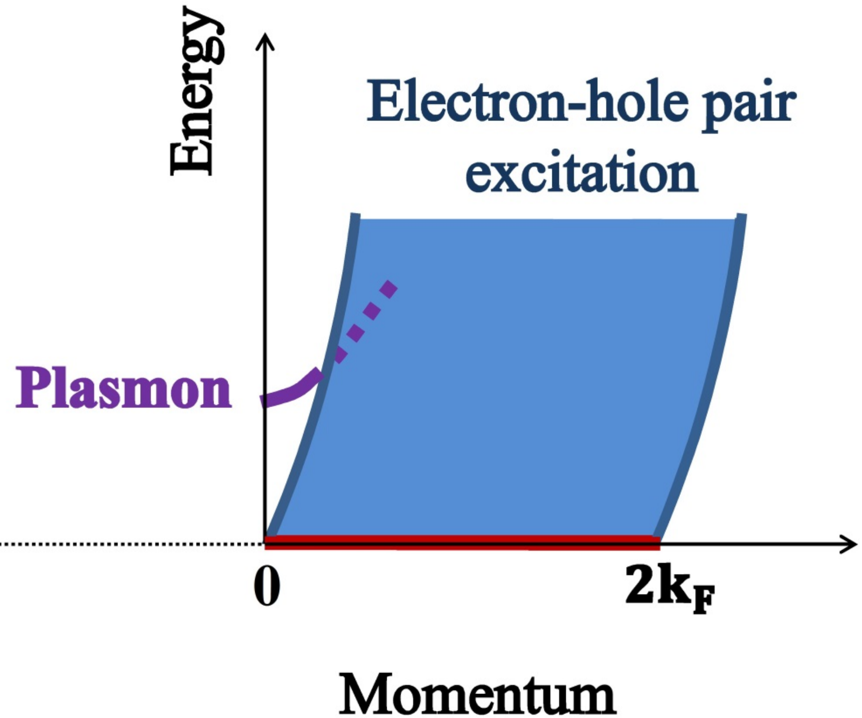


Uncontrolled boundary condition gives uncontrolled experimental results.

Plasmon

is the coulombic anti-bound state between electron-hole, above the well-established Fermi distribution.





P. Anderson, Phys. Rev. 112(1958)1900.